

Coursework 3: Graph Algorithms and Complexity Theory

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Tutorial Session: Thursday 1pm

Explanation: A *hamiltonian path* in an undirected graph is a path that contains all vertices of the graph (without repetition). Similarly, a *hamiltonian cycle* is a cycle that contains all vertices of the graph. A graph with hamiltonian path is *traceable*, and a graph with hamiltonian cycle is *hamiltonian*.

1. Specify decision problems **HP** and **HC** dealing with hamiltonian paths and hamiltonian cycles in undirected graphs.
2. Show $\mathbf{HP} \leq_m^p \mathbf{HC}$ by completing the following tasks:
 - (a) Construct a polynomial transformation f from **HP** to **HC**.
 - (b) Show for all graphs G that $G \in Y_{HP} \Rightarrow f(G) \in Y_{HC}$.
 - (c) Show for all graphs G that $f(G) \in Y_{HC} \Rightarrow G \in Y_{HP}$.
3. Show $\mathbf{HC} \leq_m^p \mathbf{HP}$ by completing the following tasks:
 - (a) Construct a polynomial transformation f from **HC** to **HP**.
 - (b) Show for all graphs G that $G \in Y_{HC} \Rightarrow g(G) \in Y_{HP}$.
 - (c) Show for all graphs G that $g(G) \in Y_{HP} \Rightarrow G \in Y_{HC}$.