Coursework 3: Graph Algorithms and Complexity Theory

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Tutorial Session: Thursday 1pm

Explanation: A *hamiltonian path* in an undirected graph is a path that contains all vertices of the graph (without repetition). Similarly, a *hamiltonian cycle* is a cycle that contains all vertices of the graph. A graph with hamiltonian path is *traceable*, and a graph with hamiltonian cycle is *hamiltonian*.

1. Specify decision problems **HP** and **HC** dealing with hamiltonian paths and hamiltonian cycles in undirected graphs.

   HP:
   Input: An undirected graph $G = (V, E)$.
   Question: Does G contain a Hamiltonian Path?

   HC:
   Input: An undirected graph $G = (V, E)$.
   Question: Does G contain a Hamiltonian Circuit?

2. Show **HP** $\leq_m^p$ **HC** by completing the following tasks:

   (a) Construct a polynomial transformation f from **HP** to **HC**. Let $G = (V, E)$ be an input for a Hamiltonian Path. Let $f(G) = G' = (V', E)$, where $V' = V \cup \{v\} v \notin V$ and $E' = E \cup \{\{v, w\} | w \in v\}$. This is in polynomial time as adding a vertex takes constant time and adding edges $\{v, w\}$ for all $w \in V$ takes $|V|$ amount of time. In total this operation takes $|V| + c$ amount of time, which is polynomial.

   (b) Show for all graphs G that $G \in Y_{HP} \Rightarrow f(G) \in Y_{HC}$.

   (c) Show for all graphs G that $f(G) \in Y_{HC} \Rightarrow G \in Y_{HP}$.

3. Show **HC** $\leq_m^p$ **HP** by completing the following tasks:

   (a) Construct a polynomial transformation f from **HC** to **HP**.

(b) Show for all graphs G that $G \in Y_{HC} \Rightarrow g(G) \in Y_{HP}$.

(c) Show for all graphs G that $g(G) \in Y_{HP} \Rightarrow G \in Y_{HC}$.