

# Coursework 3: Graph Algorithms and Complexity Theory

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Tutorial Session: Thursday 1pm

**Explanation:** A *hamiltonian path* in an undirected graph is a path that contains all vertices of the graph (without repetition). Similarly, a *hamiltonian cycle* is a cycle that contains all vertices of the graph. A graph with hamiltonian path is *traceable*, and a graph with hamiltonian cycle is *hamiltonian*.

1. Specify decision problems **HP** and **HC** dealing with hamiltonian paths and hamiltonian cycles in undirected graphs.

**HP:**

Input: An undirected graph  $G = (V, E)$ .

Question: Does  $G$  contain a Hamiltonian Path?

**HC:**

Input: An undirected graph  $G = (V, E)$ .

Question: Does  $G$  contain a Hamiltonian Circuit?

2. Show  $\mathbf{HP} \leq_m^p \mathbf{HC}$  by completing the following tasks:
  - (a) Construct a polynomial transformation  $f$  from **HP** to **HC**.
  - (b) Show for all graphs  $G$  that  $G \in Y_{HP} \Rightarrow f(G) \in Y_{HC}$ .
  - (c) Show for all graphs  $G$  that  $f(G) \in Y_{HC} \Rightarrow G \in Y_{HP}$ .
3. Show  $\mathbf{HC} \leq_m^p \mathbf{HP}$  by completing the following tasks:
  - (a) Construct a polynomial transformation  $g$  from **HC** to **HP**.
  - (b) Show for all graphs  $G$  that  $G \in Y_{HC} \Rightarrow g(G) \in Y_{HP}$ .
  - (c) Show for all graphs  $G$  that  $g(G) \in Y_{HP} \Rightarrow G \in Y_{HC}$ .