1 Introduction to Statistics

You're a citizen scientist who has started collecting data about rising water in the river next to where you live. For months, you painstakingly measure the water levels and enter your findings into a notebook. But at the end of it, what exactly do you have? What can all this data tell us?

In this lesson, we'll explore how we can use NumPy to analyze data. We'll learn different methods to calculate common statistical properties of a dataset, such as finding the mean and standard deviation. By the end, you'll be able to do basic analysis of a dataset and understand how we can use statistics to come to conclusions about data.

The statistical concepts that we'll cover include:

- Mean
- Median
- Percentiles
- Interquartile Range
- Outliers
- Standard Deviation

To start, we'll be analyzing single-variable datasets. One way to think of a single-variable dataset is that it contains answers to a question. For instance, we might ask 100 people, "How tall are you?" Their heights in inches would form our dataset.

For our purposes, we'll be organizing our datasets into NumPy arrays. To learn more about NumPy arrays, take our course Learn NumPy: Introduction.

1.1 Code

Introduction to Statistics with NumPy

After the river in your town flooded during a recent hurricane, you've become interested in collecting data about the its height. Every day for the past month, you walk to the river, measure the height of the water, and enter this information into a notebook.

Let's look at how you can use NumPy functions to analyze your dataset.

First, we'll import the NumPy module, so we can use its statistical calculation functions.

import numpy as np

 $water_h eight = np.array([4.01, 4.03, 4.27, 4.29, 4.19, 4.15, 4.16, 4.23, 4.29, 4.19, 4.00, 4.22, 4.25, 4.19, 4.10, 4.14, 4.14, 4.15, 4.16, 4.23, 4.29, 4.19, 4.00, 4.22, 4.25, 4.19, 4.10, 4.14, 4.$

Let's use the function np.mean() to find the average water height:

 $np.mean(water_height)$

5.25100000000000003

But wait! We should sort our data to see if there could be any measurements to throw our data off, or represent a deviation from the mean:

```
np.sort(water_height)
```

```
\begin{array}{l} \operatorname{array}([\ 3.99,\ 4.\ \ ,\ 4.01,\ 4.03,\ 4.03,\ 4.04,\ 4.08,\ 4.08,\ 4.1\ \ ,\ 4.11,\ 4.14,\ 4.15,\ 4.16,\\ 4.19,\ 4.19,\ 4.19,\ 4.22,\ 4.23,\ 4.23,\ 4.23,\ 4.25,\ 4.27,\ 4.29,\ 4.29,\ 6.18,\ 8.19,\\ 11.2\ \ ,\ 14.03,\ 14.2\ \ ]) \end{array}
```

Looks like that thunderstorm might have impacted the average height! Let's measure the median to see if its more representative of the dataset:

 $np.median(water_height)$

4.190000000000000004

While the median tells us where half of our data lies, let's look at a value closer to the end of the dataset. We can use percentiles to use a data points position and get its value:

np.percentile(water_height, 75)

4.26499999999999997

So far, we've gotten a good idea about specific values. But what about the spread of our data? Let's calculate the standard deviation to understand how similar or how different each data point is:

 $np.std(water_height)$

2.784585367099861

Great! Just using a few simple functions we've been able to quickly calculate several important measurements and can begin analyzing our dataset.

2 Mean

The first statistical concept we'll explore is mean, also commonly referred to as an average. The mean is a useful measurement to get the center of a dataset. NumPy has a built-in function to calculate the average or mean of arrays: np.mean

Let's say we want to find the average number of pounds of produce a person purchases per week. We administered a survey and received 1,000 responses:

```
survey_r esponses = [5, 10.2, 4, .3...6.6]
```

We can then transform the dataset into a NumPy array and use the function np.mean to calculate the average:

 $;;; survey_array = np.array(survey_responses) >>> np.mean(survey_array)5.220$

2.1 Mean with NumPy

We can also use np.mean to calculate the percent of array elements that have a certain property.

As we know, a logical operator will evaluate each item in an array to see if it matches the specified condition. If the item matches the given condition, the item will evaluate as True and equal 1. If it does not match, it will be False and equal 0.

When np.mean calculates a logical statement, the resulting mean value will be equivalent to the total number of True items divided by the total array length.

In our produce survey example, we can use this calculation to find out the percentage of people who bought more than 8 pounds of produce each week:

```
ζζζ np.mean(survey<sub>a</sub> rray > 8)0.2
```

The logical statement survey $_{a}rray > 8evaluates which survey answers were greater than 8, and assigns the model of the survey answers were greater than 8.$

2.2 Mean with Multiple Dimension Arrays

Calculating the Mean of 2D Arrays

If we have a two-dimensional array, np.mean can calculate the means of the larger array as well as the interior values.

Let's imagine a game of ring toss at a carnival. In this game, you have three different chances to get all three rings onto a stick. In our ring tossarray, each interior array (the arrays within the still three rings) and the still three rings onto a stick. In our ring tossarray, each interior array (the arrays within the still three rings) are the still three rings onto a stick.

First, we can use np.mean to find the mean across all the arrays:

To find the means of each interior array, we specify axis 1 (the "rows"):

```
i.i.i. np.mean(ring<sub>t</sub>oss, axis = 1) array([0.33333333, 0.33333333, 0.66666667])
```

To find the means of each index position (i.e, mean of all 1st tosses, mean of all 2nd tosses, ...), we specify axis 0 (the "columns"):

```
iii np.mean(ring<sub>t</sub> oss, axis = 0) array([0.66666667, 0., 0.66666667])
```

3 Outliers

As we can see, the mean is a helpful way to quickly understand different parts of our data. However, the mean is highly influenced by the specific values in our data set. What happens when one of those values is significantly different from the rest?

Values that don't fit within the majority of a dataset are known as outliers. It's important to identify outliers because if they go unnoticed, they can skew our data and lead to error in our analysis (like determining the mean). They can also be useful in pointing out errors in our data collection.

When we're able to identify outliers, we can then determine if they were due to an error in sample collection or whether or not they represent a significant but real deviation from the mean.

Suppose we want to determine the average height for 3rd graders. We measure several students at the local school, but accidentally measure one student in centimeters rather than in inches. If we're not paying attention, our dataset could end up looking like this:

```
[50, 50, 51, 49, 48, 127]
```

In this case, 127 would be an outlier.

Some outliers aren't the result of a mistake. For instance, suppose that one of our 3rd graders had skipped a grade and was actually a year younger than everyone else in the class:

```
[50, 50, 51, 49, 48, 45]
```

She might be significantly shorter at 45", but her height would still be an outlier.

Suppose that another student was just unusually tall for his age:

```
[50, 50, 51, 49, 48, 58.5]
```

His height of 58.5" would also be an outlier.

3.1 Sorting and Outliers

Sorting and Outliers

One way to quickly identify outliers is by sorting our data, Once our data is sorted, we can quickly glance at the beginning or end of an array to see if some values lie far beyond the expected range. We can use the NumPy function np.sort to sort our data.

Let's go back to our 3rd grade height example, and imagine an 8th grader walked into our experiement:

```
iii heights = np.array([49.7, 46.9, 62, 47.2, 47, 48.3, 48.7])
```

If we use np.sort, we can immediately identify the taller student since their height (62") is noticeably outside the range of the dataset:

```
¿¿¿ np.sort(heights) array([46.9, 47., 47.2, 48.3, 48.7, 49.7, 62])
```

4 Median

Another key metric that we can use in data analysis is the median. The median is the middle value of a dataset that's been ordered in terms of magnitude (from lowest to highest).

Let's look at the following array:

```
np.array([1, 1, 2, 3, 4, 5, 5])
```

In this example, the median would be 3, because it is positioned half-way between the minimum value and the maximum value.

If the length of our dataset was an even number, the median would be the value halfway between the two central values. So in the following example, the median would be 3.5:

```
np.array([1, 1, 2, 3, 4, 5, 5, 6])
```

But what if we had a very large dataset? It would get very tedious to count all of the values. Luckily, NumPy also has a function to calculate the median, np.median:

```
iii my_array = np.array([50, 38, 291, 59, 14]) >>> np.median(my_array)50.0
```

5 Mean vs Median

In a dataset, the median value can provide an important comparison to the mean. Unlike a mean, the median is not affected by outliers. This becomes important in skewed datasets, datasets whose values are not distributed evenly.

6 Percentiles

As we know, the median is the middle of a dataset: it is the number for which 50

This type of point is called a percentile. The Nth percentile is defined as the point N

Let's look at the following array:

$$d = [1, 2, 3, 4, 4, 4, 6, 6, 7, 8, 8]$$

There are 11 numbers in the dataset. The 40th percentile will have 40

In NumPy, we can calculate percentiles using the function np.percentile, which takes two arguments: the array and the percentile to calculate.

Here's how we would use NumPy to calculate the 40th percentile of array d:

$$ξξξ d = np.array([1, 2, 3, 4, 4, 4, 6, 6, 7, 8, 8])$$
 $ξξξ np.percentile(d, 40)$ 4.00

Some percentiles have specific names:

The 25th percentile is called the first quartile The 50th percentile is called the median The 75th percentile is called the third quartile

The minimum, first quartile, median, third quartile, and maximum of a dataset are called a five-number summary. This set of numbers is a great thing to compute when we get a new dataset.

The difference between the first and third quartile is a value called the interquartile range. For example, say we have the following array:

$$d = [1, 2, 3, 4, 4, 4, 6, 6, 7, 8, 8]$$

We can calculate the 25th and 75th percentiles using np.percentile:

```
np.percentile(d, 25) ¿¿¿ 3.5 np.percentile(d, 75) ¿¿¿ 6.5
```

Then to find the interquartile range, we subtract the value of the 25th percentile from the value of the 75th:

```
6.5 - 3.5 = 3
50
```

7 Standard Deviation

While the mean and median can tell us about the center of our data, they do not reflect the range of the data. That's where standard deviation comes in.

Similar to the interquartile range, the standard deviation tells us the spread of the data. The larger the standard deviation, the more spread out our data is from the center. The smaller the standard deviation, the more the data is clustered around the mean.

8 Review of NumPy

Let's review! In this lesson, you learned how to use NumPy to analyze single-variable datasets. Here's what we covered:

Using the np.sort method to locate outliers. Calculating central positions of a dataset using np.mean and np.median. Understanding the spread of our data using percentiles and the interquartile range. Finding the standard deviation of a dataset using np.std.

In our next lesson, we'll continue our exploration of NumPy and see how we can use it to analyze different statistical distributions. Follow the checkpoints below to practice what you just learned!

9 Distributions

A university wants to keep track of the popularity of different programs over time, to ensure that programs are allocated enough space and resources. You work in the admissions office and are asked to put together a set of visuals that show these trends to interested applicants. How can we calculate these distributions? Would we be able to see trends and predict the popularity of certain programs in the future? How would we show this information?

In this lesson, we are going to learn how to use NumPy to analyze different distributions, generate random numbers to produce datasets, and use Matplotlib to visualize our findings.

This lesson will cover:

How to generate and graph histograms How to identify different distributions by their shape Normal distributions How standard deviations relate to normal distributions Binomial distributions

9.1 Statistical Distributions with NumPy

Imagine that you work as an admissions officer at a university. Part of your job is to collect, analyze, and visualize data that's relevant to interested applicants.

Recently, you've become interested in how histograms can show different distributions of populations and even occurences. You think that histograms would be useful in visualizing different trends, such as changes in department numbers and participation in extracurriculars. You also want to learn more about how you can use randomly generated distributions to make statistical calculations and predict the probability of future events, such as the sucess of your ultimate frisbee team.

For this lesson, we'll be using NumPy to calculate distributions and Matplotlib to graph our calculations.

import numpy as np from matplotlib import pyplot as plt

One set of data you want to analyze is enrollment in different degree programs. By looking at histograms of the number of years students are enrolled in a program, you can identify what programs are becoming more popular, which are falling out of favor, and which have steady, continual enrollment.

First, let's look at how many hundreds of students decide to enroll in Codecademy University and how many years they've been enrolled.

```
total_e nroll ment = [1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5]
```

 $plt.hist(total_enrollment, bins = 5, range = (1,6))plt.title('StudentEnrollment(CodecademyUniversity)')$

The histogram above shows the University's total enrollment, which is fairly consistent. This is a uniform distribution and is what the University wants to

see. Total enrollment is staying at a good level.

Now let's take a look at the enrollment specifically for students seeking a degree in History:

```
history_e nroll ment = [1, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5]
```

What does this histogram tell us? Well this is somewhat skewed left dataset, we can see that there are a lot more students who have been enrolled for 3 or 4 years over 1 and 2 years. This indicates that the History program is becoming less popular. Where are all the students going then?

The school recently invested a lot of money in a new building for the Computer Science Department. Let's take a look at enrollment and see if the investment is paying off.

```
cs_enrollment = [1, 1, 1, 1, 1, 2, 2, 2, 2, 3, 4, 4]
```

```
plt.hist(cs_enrollment, bins = 5, range = (1,6)) \\ plt.title('StudentEnrollment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienceDepartment(ComputerScienc
```

It looks like enrollment has skyrocketed for the Computer Science department in recent years. This could be because the University invested in the department, or it could be a sign that the sought after job skills in the real world are changing. Whatever the reason, the histograms let us clearly see the trends.

Interested applicants would like to know what kinds of SAT scores accepted students had. You've previously calculated that the mean score is 1250, with a standard deviation of 50.

Rather than gather every students score, you take what you know about the data and use a random number generator to generate a model.

```
sat_scores = np.random.normal(1250, 50, size = 100000)
```

 $plt.hist(sat_scores, bins = 1000, range = (800, 1600)) \\ plt.title('AdmittedStudentSATScores') \\ plt.xlabel('SATScores') \\ plt.xlabel('SATScores')$

95

```
mean = 1250 one_s td = 50
```

 $two_below = (mean - 2 * one_std)printtwo_below$

1150

Looks like they're just below it! Better re-take that test.

One of the big draws to your school is your excellent ultimate frisbee team. The team wins about 70

 $ultimate = np.random.binomial (50, 0.70, size=10000) \ plt.hist (ultimate, range=(0, 50), bins=50, normed=True) \ plt.xlabel ('Number of Games') \ plt.ylabel ('Frequency') \ plt.show()$

Since it's a little hard to see from the graph, let's calculate exactly what chance they have of winning 40 games:

ultimate = np.random.binomial(50, 0.70, size=10000) np.mean(ultimate == 40)

0.0410000000000000002

Hmm, looks like it might be tough for the team to reach that number of wins, given the current data - but even more of a reason for this applicant to sign up and help the team improve!

10 Histograms

When we first look at a dataset, we want to be able to quickly understand certain things about it:

Do some values occur more often than others? What is the range of the dataset (i.e., the min and the max values)? Are there a lot of outliers?

We can visualize this information using a chart called a histogram.

For instance, suppose that we have the following dataset:

$$d = [1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 5]$$

A simple histogram might show us how many 1's, 2's, 3's, etc. we have in this dataset. Value Number of Samples 1 3 2 5 3 2 4 4 5 1

When graphed, our histogram would look like this:

Suppose we had a larger dataset with values ranging from 0 to 50. We might not want to know exactly how many 0's, 1's, 2's, etc. we have.

Instead, we might want to know how many values fall between 0 and 5, 6 and 10, 11 and 15, etc.

These groupings are called bins. All bins in a histogram are always the same size. The width of each bin is the distance between the minimum and maximum values of each bin. In our example, the width of each bin would be 5.

For a dataset like this, our histogram table would look like this: Bin Number of Values (0, 5) 2 (6, 10) 10 (11, 15) 11 (46, 50) 3

And if we were to graph this histogram, it would look like this:

We can graph histograms using a Python module known as Matplotlib. We're not going to go into detail about Matplotlib's plotting functions, but if you're interested in learning more, take our course Introduction to Matplotlib.

For now, familiarize yourself with the following syntax to draw a histogram:

This imports the plotting package. We only need to do this once. from matplotlib import pyplot as plt This plots a histogram plt.hist(data)

This displays the histogram plt.show()

When we enter plt.hist with no keyword arguments, matplotlib will automatically make a histogram with 10 bins of equal width that span the entire range of our data.

If you want a different number of bins, use the keyword bins. For instance, the following code would give us 5 bins, instead of 10:

```
plt.hist(data, bins=5)
```

If you want a different range, you can pass in the minimum and maximum values that you want to histogram using the keyword range. We pass in a tuple of two numbers. The first number is the minimum value that we want to plot and the second value is the number that we want to plot up to, but not including.

For instance, if our dataset contained values between 0 and 100, but we only wanted to histogram numbers between 20 and 50, we could use this command:

We pass 51 so that our range includes 50 plt.hist(data, range=(20, 51))

Here's a complete example:

```
from matplotlib import pyplot as plt d = \text{np.array}([1,\,1,\,1,\,2,\,2,\,2,\,2,\,2,\,3,\,3,\,4,\,4,\,4,\,4,\,5]) plt.hist(d, bins=5, range=(1, 6)) plt.show()
```

11 Examples of Distributions

Histograms and their datasets can be classified based on the shape of the graphed values. In the next two exercises, we'll look at two different ways of describing histograms.

One way to classify a dataset is by counting the number of distinct peaks present in the graph. Peaks represent concentrations of data. Let's look at the following examples:

A unimodal dataset has only one distinct peak. histogram

A bimodal dataset has two distinct peaks. This often happens when the data contains two different populations. histogram

A multimodal dataset has more than two peaks. histogram

A uniform dataset doesn't have any distinct peaks.

Most of the datasets that we'll be dealing with will be unimodal (one peak). We can further classify unimodal distributions by describing where most of the numbers are relative to the peak.

A symmetric dataset has equal amounts of data on both sides of the peak. Both sides should look about the same. histogram

A skew-right dataset has a long tail on the right of the peak, but most of the data is on the left. histogram

A skew-left dataset has a long tail on the left of the peak, but most of the data is on the right. histogram

The type of distribution affects the position of the mean and median. In heavily skewed distributions, the mean becomes a less useful measurement.

SYMMETRIC histogram

SKEW-RIGHT histogram

SKEW-LEFT histogram

12 Normal Distributions

The most common distribution in statistics is known as the normal distribution, which is a symmetric, unimodal distribution.

Lots of things follow a normal distribution:

The heights of a large group of people Blood pressure measurements for a group of healthy people Errors in measurements

Normal distributions are defined by their mean and standard deviation. The mean sets the "middle" of the distribution, and the standard deviation sets the "width" of the distribution. A larger standard deviation leads to a wider distribution. A smaller standard deviation leads to a skinnier distribution.

Here are a few examples of normal distributions with different means and standard deviations:

 $normal_distribution$

As we can see, each set of data has the same "shape", but with slight differences depending on their mean and standard deviation.

We can generate our own normally distributed datasets using NumPy. Using these datasets can help us better understand the properties and behavior of different distributions. We can also use them to model results, which we can then use as a comparison to real data.

In order to create these datasets, we need to use a random number generator. The NumPy library has several functions for generating random numbers, including one specifically built to generate a set of numbers that fit a normal

distribution: np.random.normal. This function takes the following keyword arguments:

loc: the mean for the normal distribution scale: the standard deviation of the distribution size: the number of random numbers to generate

```
a = np.random.normal(0, 1, size=100000)
```

If we were to plot this set of random numbers as a histogram, it would look like this:

13 Standard Deviation and Normal Distribution

We know that the standard deviation affects the "shape" of our normal distribution. The last exercise helps to give us a more quantitative understanding of this.

Suppose that we have a normal distribution with a mean of 50 and a standard deviation of 10. When we say "within one standard deviation of the mean", this is what we are saying mathematically:

```
lower_bound = mean - std = 50 - 10 = 40
```

$$upper_bound = mean + std = 50 + 10 = 60$$

It turns out that we can expect about 68

As we saw in the previous exercise, no matter what mean and standard deviation we choose, 68

In fact, here are a few more helpful rules for normal distributions:

689599.7

14 Binomial Distribution

It's known that a certain basketball player makes 30

However, he actually made 4 free throws out of 10 or 40

The binomial distribution can help us. It tells us how likely it is for a certain number of "successes" to happen, given a probability of success and a number of trials.

In this example:

The probability of success was 30The number of trials was 10 (he took 10 shots) The number of successes was 4 (he made 4 shots)

The binomial distribution is important because it allows us to know how likely a certain outcome is, even when it's not the expected one. From this graph, we can see that it's not that unlikely an outcome for our basketball player to get

4 free throws out of 10. However, it would be pretty unlikely for him to get all 10.

binomiall

There are some complicated formulas for determining these types of probabilities. Luckily for us, we can use NumPy - specifically, its ability to generate random numbers. We can use these random numbers to model a distribution of numerical data that matches the real-life scenario we're interested in understanding.

For instance, suppose we want to know the different probabilities of our basketball player making 1, 2, 3, etc. out of 10 shots.

NumPy has a function for generating binomial distributions: np.random.binomial, which we can use to determine the probability of different outcomes.

The function will return the number of successes for each "experiment".

It takes the following arguments:

N: The number of samples or trials P: The probability of success size: The number of experiments

Let's generate a bunch of "experiments" of our basketball player making 10 shots. We choose a big N to be sure that our probabilities converge on the correct answer.

Let's generate 10,000 "experiments" N = 10 shots P = 0.30 (30)

a = np.random.binomial(10, 0.30, size=10000)

Now we have a record of 10,000 experiments. We can use Matplotlib to plot the results of all of these experiments:

plt.hist(a, range=(0, 10), bins=10, normed=True) plt.xlabel('Number of "Free Throws"') plt.ylabel('Frequency') plt.show()

binomiall

15 Binomial Distribution and Probability

Let's return to our original question:

Our basketball player has a 30

We can calculate a different probability by counting the percent of experiments with the same outcome, using the np.mean function.

Remember that taking the mean of a logical statement will give us the percent of values that satisfy our logical statement.

Let's calculate the probability that he makes 4 baskets:

a = np.random.binomial(10, 0.30, size=10000) np.mean(a == 4)

When we run this code, we might get:

įį 0.1973

Remember, because we're using a random number generator, we'll get a slightly different result each time. With the large *size we chose, the calculated probability should be accurate to about 2 decimal places.*

So, our basketball player has a roughly 20

This suggests that what we observed wasn't that unlikely. It's quite possible that he hasn't got any better; he just got lucky.

16 Review

Let's review! In this lesson, you learned how to use NumPy to analyze different distributions and generate random numbers to produce datasets. Here's what we covered:

What is a histogram and how to map one using Matplotlib How to identify different dataset shapes, depending on peaks or distribution of data The definition of a normal distribution and how to use NumPy to generate one using NumPy's random number functions The relationships between normal distributions and standard deviations The definition of a binomial distribution

Now you can use NumPy to analyze and graph your own datasets! You should practice building your intuition about not only what the data says, but what conclusions can be drawn from your observations.

np.random.normal(loc = 16.3, scale = 3.3, size = 1000)

17 Quiz

Which of the following are the correct keyword arguments for generating a random distribution using np.random.binomial? N, P, size

What is a histogram? A chart that creates equally spaced bins and counts how many values from our dataset fall into each bin.

How many peaks does a unimodal dataset have? One

In a normal distribution, how much of the data lies within one standard deviation? 68

The average height of a male giraffe is 16.3 feet with a standard deviation of 3.3 feet. Which of the following will generate a random distribution of 1000 male giraffe heights using np.random.normal? np.random.normal(loc = 16.3, scale = 3.3, size = 1000)

Why do we use binomial distributions? Because they are effective at helping us understand the different probabilities that an event will occur.

18 Project Election

```
import codecademylib import numpy as np from matplotlib import pyplot as
plt
survey_responses = ['Ceballos', 'Kerrigan', 'Ceballos', 'Ceballos', 'Ceballos', 'Kerrigan', 'Kerrigan', 'Ceballos', 'Ceballos', 'Ceballos', 'Kerrigan', 'Ceballos', 'Kerrigan', 'Ceballos', 'Ceballo
total_ceballos = sum([1forninsurvey_responses if n == 'Ceballos'])print(total_ceballos)
percentage_ceballos = 100 * total_ceballos/float(len(survey_responses))
print(percentage_ceballos)
plt.hist(possible_surveys, range = (0, 1), bins = 20)plt.show()
ceballos_loss_surveys = np.mean(possible_surveys < 0.5) * 100
print(ceballos_loss_surveys)
possible_s urveys_len = float(len(possible_s urveys))incorrect_p redictions = len(possible_s urveys[possible_s urveys])incorrect_p redictions = len(possible_s urveys[possible_s urveys[
[0.5] ceballos<sub>l</sub>oss<sub>s</sub> urveys<sub>2</sub> = incorrect_p redictions/possible_s urveys_l enprint(ceballos<sub>l</sub>oss<sub>s</sub> urveys<sub>2</sub>)
large_survey = np.random.binomial(float(7000), 0.54, size = 10000)/float(7000)
ceballos_loss_new = np.mean(large_survey < 0.5)print(ceballos_loss_new)
large_s urvey s_l en = float(len(large_s urvey))incorrect_p redictions = len(large_s urvey[large_s urvey < 
[0.5] ceballos<sub>l</sub>oss<sub>s</sub>urveys<sub>n</sub>ew<sub>2</sub> = incorrect_p redictions/large_s urveys_l enprint(ceballos<sub>l</sub>oss<sub>s</sub>urveys_new<sub>2</sub>)
```

19 Project Muncies

```
import codecademylib import numpy as np calorie_s tats = np.genfromtxt('cereal.csv', delimiter =',')calorie<math>_s tats_s orted = np.sort(calorie_s tats) average_calories = np.mean(calorie_s tats)median_calories = <math>np.median(calorie_s tats_s orted)nth_percentile = np.percentile(calorie_s tats_s orted, 4)more_calories = <math>np.mean(calorie_s tats > 60)* 100calorie_s tats = np.std(calorie_s tats)
```

"' CrunchieMunchies are lit. They are more healthy than 96"'

 $print(calorie_std)print(more_calories)print(nth_percentile)print(average_calories)print(median_calories)print(nth_percentile)print(average_calories)print(median_calories)print(nth_percentile)print(average_calories)print(median_calories)print(nth_percentile)print(average_calories)print(median_calories)print(nth_percentile)print(average_calories)print(median_calories)print(nth_percentile)print(average_calories)print(median_calories)print(nth_percentile)print(average_calories)print(median_calories)prin$

19.1 csv

```
70,\ 120,\ 70,\ 50,\ 110, 110,\ 110,\ 130,\ 90,\ 90,\ 120,\ 110,\ 120,\ 110,\ 110,\ 110,\ 110,\ 100,\ 110,\ 110,\ 110,\ 110,\ 110,\ 120,\ 120,\ 120,\ 110,\ 100,\ 110,\ 100,
```

 $110,\ 120,\ 120,\ 110,\ 110,\ 110,\ 140,\ 110,\ 100,\ 110,\ 100,\ 150,\ 150,\ 160,\ 100,\ 120,\ 140,\ 90,\ 130,\ 120,\ 100,\ 50,\ 50,\ 100,\ 100,\ 120,\ 100,\ 90,\ 110,\ 110,\ 80,\ 90,\ 90,\ 110,\ 110,\ 140,\ 100,\ 110,\ 110,\ 100,\ 110$

20 Bakery

```
import numpy as np (Flour—Sugar—Eggs—Milk—Butter) cupcakes = np.array([2, 0.75, 2, 1, 0.5]) recipes = np.genfromtxt('recipes.csv', delimiter=',') print(recipes) eggs = recipes[:,2] print(eggs == 1) cookies = recipes[2,:] print(cookies) double_batch = cupcakes * 2 grocery_list = double_batch + cookies print(grocery_list)
```

20.1 csv

2,0.75,2,1,0.5 1,0.125,1,1,0.125 2.75,1.5,1,0,1 4,0.5,2,2,0.5

21 Introduction To ScyPy

Say you work for a major social media website. Your boss always says "data drives all our decisions" and it seems to be true. Metrics are collected on all users of your website, terabytes of data stored in replicated databases.

One day, your boss wants to know if college students are engaging in your website. You pull up the records for users in that age bracket and look at them one by one. The first person only spent half a second on your website before closing the tab — that doesn't look good. But the second person was on the site for thirty minutes! That's a running average of 15 minutes site time per user, but you still have half a million records to look at.

On top of that, you need to compare it against other age brackets (and the average overall). That's going to take a lot of time if you do it all by hand, and you're still not sure what your methodology for proving college students spend enough time on your website to be "engaged".

When conducting data analysis, we want to say something meaningful about our data. Often, we want to know if a change or difference we see in a dataset is "real" or if it's just a normal fluctuation or a result of the specific sample of people we have chosen to measure. A difference we observe in our data is only important if we can be reasonably sure that it is representative of the population as a whole, and reasonably sure that our result is repeatable.

This question of whether a difference is significant or not is essential to making decisions based on that difference. Some instances where this might come up include:

Performing an A/B test — are the different observations really the results of different conditions (i.e., Condition A vs. Condition B)? Or just the result of random chance? Conducting a survey — is the fact that men gave slightly different responses than women a real difference between men and women? Or just the result of chance?

In this lesson, we will cover the fundamental concepts that will help us create tests to measure our confidence in our statistical results:

Sample means and population means The Central Limit Theorem Why we use hypothesis tests What errors we can come across and how to classify them

Are the Millennials Engaged?

You work at the global megacorp social network SpyPy. SpyPy has 1.5 billion daily users, and you want to make sure that people in the millennial age bracket are engaging with your website. Your boss seems particularly frazzled by this question, and he's put it on you to find out. You decide that "engagement" means spending more than the average of seven minutes on the website. You fire up your data-science stack in Python and first check the average time – which turns out to be near 11 whole minutes! But you can't really tell if they're really spending more time or if it's just random chance that a few of your users left the browser open and walked away. You write the following code:

import spypy from scipy.stats import ttest₁ samp

 $millennial_times = spypy.get_site_times_for_demographic('millennial')t_stat, p_val = ttest_1samp(millennial_times, 7)$

if $\mathbf{p}_val < .05: print" The Millennials are engaged!" else: print" The Millennials are not engaged: (!" The Millennial$

The Millennials are engaged!

SpyPy: We're Significantly Different

Well that's great news! Millennials are, for the most part, spending around 10 minutes on your website. But before you break out the champagne glasses your boss is in a frenzy again, this time about Metropolitan Statistical Areas (MSAs). You are tasked with finding if people in cooler climates post more pictures on SpyPy than people in warmer climates. You cross corroborate with weather data and run a statistical test on the info.

from scipy.stats import $ttest_ind$

 $warmer_weather_picture_count = spypy.get_number_pictures_for_climate('hot')colder_weather_picture_count = spypy.get_number_pictures_for_climate('cold')$

 $t_s tat, p_v al = ttest_i nd(warmer_w eather_picture_count, colder_w eather_picture_count)$

 $if \ p_val < .05: print" People from colder climates posta different number of pictures compared to people from print" Climatedoesn't appear to a ffect the number of pictures posted"$

Climate doesn't appear to affect the number of pictures posted

SpyPy: Because We Care About Your Data

Seems like climate probably doesn't really affect the number of times people post pictures. Not really sure why that would've been the case anyway. SpyPy has a new feature that you think will get people to interact with the website for longer: SpyPy Stories. It is preliminarily being launched to 8 million users and the internal goal is to get 2 million people to post SpyPy Stories in the first week. Unfortunately, only 1,997,893 people posted SpyPy Stories this week. We want to know if this is a significant difference from our goal – did we pretty much meet it or did we seriously miss? You know how to answer this question:

from scipy.stats import $binom_t est$

 $number_o f_t rials = 8000000 expected_s uccesses = 2000000 actual_s uccesses = 1997893 expected_s uccess_rate = float(expected_s uccesses)/float(number_o f_t rials)$

 $\begin{aligned} \mathbf{p}_v a l &= binom_t est(actual_successes, n = number_o f_t rials, p = expected_success_r ate) if p_v a l < \\ 0.05 &: print"We didn'thit our target by a significant amount"else : print"We just missed our target by a very snown and the print of the$

We just missed our target by a very small amount!

Looks like we came very close to hitting our target for SpyPy Stories! You've saved the day so many times already! Your boss comes by to thank you for all the hard work you put in today and says you've made significant contributions to the team. You tell her you're not sure if that's true, but you definitely have a way of finding out.

22 Sample and Population Means

Suppose you want to know the average height of an oak tree in your local park. On Monday, you measure 10 trees and get an average height of 32 ft. On Tuesday, you measure 12 different trees and reach an average height of 35 ft. On Wednesday, you measure the remaining 11 trees in the park, whose average height is 31 ft. Overall, the average height for all trees in your local park is 32.8 ft.

The individual measurements on Monday, Tuesday, and Wednesday are called samples. A sample is a subset of the entire population. The mean of each sample is the sample mean and it is an estimate of the population mean.

Note that the sample means (32 ft., 35 ft., and 31 ft.) were all close to the population mean (32.8 ft.), but were all slightly different from the population mean and from each other.

For a population, the mean is a constant value no matter how many times it's recalculated. But with a set of samples, the mean will depend on exactly what samples we happened to choose. From a sample mean, we can then extrapolate the mean of the population as a whole. There are many reasons we might use sampling, such as:

- We don't have data for the whole population.
- We have the whole population data, but it is so large that it is infeasible to analyze.
- We can provide meaningful answers to questions faster with sampling.

When we have a numerical dataset and want to know the average value, we calculate the mean. For a population, the mean is a constant value no matter how many times it's recalculated. But with a set of samples, the mean will depend on exactly what samples we happened to choose. From a sample mean, we can then extrapolate the mean of the population as a whole.

23 Central Limit Theorem

Perhaps, this time, you're a tailor of school uniforms at a middle school. You need to know the average height of people from 10-13 years old in order to know which sizes to make the uniforms. Knowing the best decisions are based on data, you set out to do some research at your local middle school.

Organizing with the school, you measure the heights of some students. Their average height is 57.5 inches. You know a little about sampling and decide that measuring 30 out of the 300 students gives enough data to assume 57.5 inches is roughly the average height of everyone at the middle school. You set to work with this dimension and make uniforms that fit people of this height, some smaller and some larger.

Unfortunately, when you go about making your uniforms many reports come back saying that they are too small. Something must have gone wrong with your decision-making process! You go back to collect the rest of the data: you measure the sixth graders one day (56.7, not so bad), the seventh graders after that (59 inches tall on average), and the eighth graders the next day (61.7 inches!). Your sample mean was so far off from your population mean. How did this happen?

Well, your sample selection was skewed to one direction of the total population. It looks like you must have measured more sixth graders than is representative

of the whole middle school. How do you get an average sample height that looks more like the average population height?

In the previous exercise, we looked at different sets of samples taken from a population and how the mean of each set could be different from the population mean. This is a natural consequence of the fact that a set of samples has less data than the population to which it belongs. If our sample selection is poor then we will have a sample mean seriously skewed from our population mean.

There is one surefire way to mitigate the risk of having a skewed sample mean — take a larger set of samples. The sample mean of a larger sample set will more closely approximate the population mean. This phenomenon, known as the Central Limit Theorem, states that if we have a large enough sample size, all of our sample means will be sufficiently close to the population mean.

Later, we'll learn how to put numeric values on "large enough" and "sufficiently close".

24 Hypothesis Tests

When observing differences in data, a data analyst understands the possibility that these differences could be the result of random chance.

Suppose we want to know if men are more likely to sign up for a given programming class than women. We invite 100 men and 100 women to this class. After one week, 34 women sign up, and 39 men sign up. More men than women signed up, but is this a "real" difference?

We have taken sample means from two different populations, men and women. We want to know if the difference that we observe in these sample means reflects a difference in the population means. To formally answer this question, we need to re-frame it in terms of probability:

"What is the probability that men and women have the same level of interest in this class and that the difference we observed is just chance?"

In other words, "If we gave the same invitation to every person in the world, would more men still sign up?"

A more formal version is: "What is the probability that the two population means are the same and that the difference we observed in the sample means is just chance?"

These statements are all ways of expressing a null hypothesis. A null hypothesis is a statement that the observed difference is the result of chance.

Hypothesis testing is a mathematical way of determining whether we can be confident that the null hypothesis is false. Different situations will require different types of hypothesis testing, which we will learn about in the next lesson.

25 Type I or Type II

When we rely on automated processes to make our decisions for us, we need to be aware of how this automation can lead to mistakes. Computer programs are as fallible as the humans who design them. As humans capable of programming, the responsibility is on us to understand what can go wrong and what we can do to contain these foreseeable problems.

In statistical hypothesis testing, we concern ourselves primarily with two types of error. The first kind of error, known as a Type I error, is finding a correlation between things that are not related. This error is sometimes called a "false positive" and occurs when the null hypothesis is rejected even though it is true.

For example, let's say you conduct an A/B test for an online store and conclude that interface B is significantly better than interface A at directing traffic to a checkout page. You have rejected the null hypothesis that there is no difference between the two interfaces. If, in reality, your results were due to the groups you happened to pick, and there is actually no significant difference between interface A and interface B in the greater population, you have been the victim of a false positive.

The second kind of error, a Type II error, is failing to find a correlation between things that are actually related. This error is referred to as a "false negative" and occurs when the null hypothesis is accepted even though it is false.

For example, with the A/B test situation, let's say that after the test, you concluded that there was no significant difference between interface A and interface B. If there actually is a difference in the population as a whole, your test has resulted in a false negative.

26 P-Values

We have discussed how a hypothesis test is used to determine the validity of a null hypothesis. A hypothesis test provides a numerical answer, called a p-value, that helps us decide how confident we can be in the result. In this context, a p-value is the probability that we yield the observed statistics under the assumption that the null hypothesis is true.

A p-value of 0.05 would mean that there is a 5

Before conducting a hypothesis test, we determine the necessary threshold we would need before concluding that the results are significant. A higher p-value is more likely to give a false positive so if we want to be very sure that the result is not due to just chance, we will select a very small p-value.

It is important that we choose the significance level before we perform our statistical hypothesis tests to yield a p-value. If we wait until after we see the results, we might pick our threshold such that we get the result we want to see. For instance, if we're trying to publish our results, we might set a

significance level that makes our results seem statistically significant. Choosing our significance level in advance helps keep us honest.

Generally, we want a p-value of less than 0.05, meaning that there is less than a 5

27 Examples

Suppose we were exploring the relationship between local honey and allergies. Which of these would be a statement of the null hypothesis? Local honey has no effect on allergies, any relationship between consuming local honey and allergic outbreaks is due to chance. Correct! The null hypothesis states that any difference observed within sample means is coincidental.

Which of the following describes a Type II error? A survey on preferred ice cream flavors not establishing a clear favorite when the majority of people prefer chocolate.

Which of these describes a sample mean? The mean of a subset of our population which is hopefully, but not necessarily, representative of the overall average.

What is a p-value? In a hypothesis test, a p-value is the probability that the null hypothesis is true.

Which of the following hypothesis tests would be used to compare two sets of numerical data?

Chi Square x 1 Sample T-Test x ANOVA x 2 Sample T-Test

Analysis of variance is used to determine if three or more numerical samples come from the same population.

Which of these is an accurate statement of the Central Limit Theorem? For a large enough sample size, our sample mean will be sufficiently close to the population mean.

What is a statistical hypothesis test? A way of quantifying the truth of a statement.

ANOVA is a type of hypothesis test, but does not cover all types of hypothesis test.

28 Key Points

Let's take a second and review. In this lesson, you learned the basics of the NumPy package. Here are some key points:

Arrays are a special type of list that allows us to store values in an organized manner. An array can be created by either defining it directly using np.array() or by importing a CSV using np.genfromtxt('file.csv', delimiter=','). An operation (such as addition) can be performed on every element in an array by simply

performing it on the array itself. Elements can be selected from arrays using their index and array locations, both of which start at 0. Logical operations can be used to create new, more focused arrays out of larger arrays.

The next lesson will explore how to analyze these arrays and use means, medians, and standard deviations to tell a story. But first, practice what you've learned by working through the following checkpoints.