Coursework 2 - Perfect Matching

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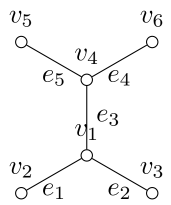
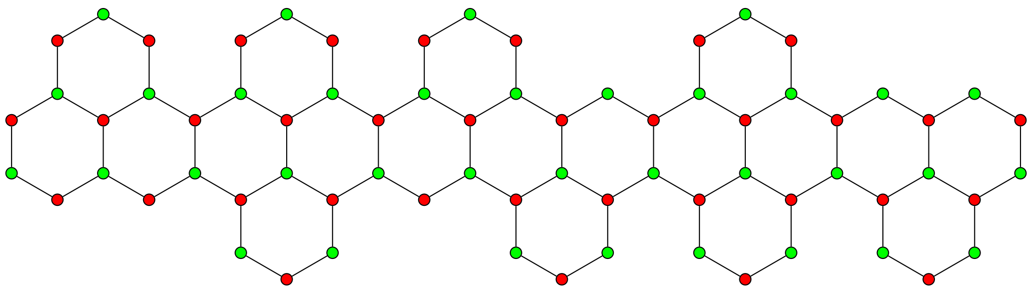
Tutorial Session: Thursday 1pm

1. In the first graph, there is no perfect match as only one of e_1, e_2 can be chosen meaning either v_2 or v_3 is M-unsaturated. Similar argument can be made for the edges e_4, e_5 and v_5, v_6.

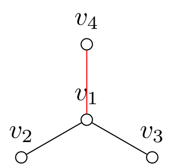




2. Prove or disprove by counterexample:

   (a) For every connected graph G and every vertex v of G there is a maximum matching M of G such that v is M-saturated.

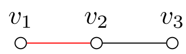
   This is false, as if every vertex v is M-saturated then the maching M is a perfect matching, which is not always true. Given a matching:



   This matching $M = \{G\}$ is a maximum one, and it has two M-unsaturated vertices, namely v_2, v_3.

(b) For every graph G without perfect matching and every vertex v of G there is a maximum matching M of G such that v is M-unsaturated.

This is false, as a connected graph will always have a maximum matching with at least one vertex that is M-saturated. Given a matching:



This matching $M = \{G\}$ is a maximum one, and it has one M-saturated vertex, namely v_1.