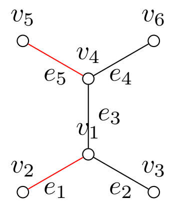
Coursework 2 - Perfect Matching

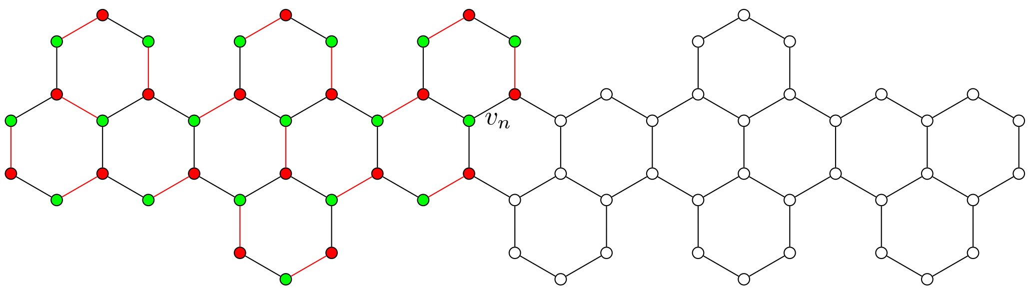
Oskar Mampe

Tutorial Session: Thursday 1pm

1. In the first graph, there is no perfect match as only one of e_1, e_2 can be chosen meaning either v_2 or v_3 is M-unsaturated. Similar argument can be made for the edges e_4, e_5 and v_5, v_6.



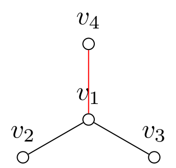
Further, this second graph also does not have a perfect match. According to Hall's Theorem, $|N(S)| \geq |S|$. However, when checking for this equality, if you count the green vertices as $|S| = 18$ and red nodes as $|N(S)| = 17$, the size of $|S|$ is larger. Therefore, at this point in the algorithm, there will always be one node M-unsaturated, which was v_n in this case.



2. Prove or disprove by counterexample:

   (a) For every connected graph G and every vertex v of G there is a maximum matching M of G such that v is M-saturated.

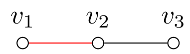
   This is false, as if every vertex v is M-saturated then the maching M is a perfect matching, which is not always true. Given a matching:



This matching $M = \{G\}$ is a maximum one, and it has two M-unsaturated vertices, namely v_2, v_3.

(b) For every graph G without perfect matching and every vertex v of G there is a maximum matching M of G such that v is M-unsaturated.

This is false, as a connected graph will always have a maximum matching with at least two vertices that is M-saturated. Given a matching:



This matching $M = \{G\}$ is a maximum one, and it has two M-saturated vertices, namely v_1, v_2.