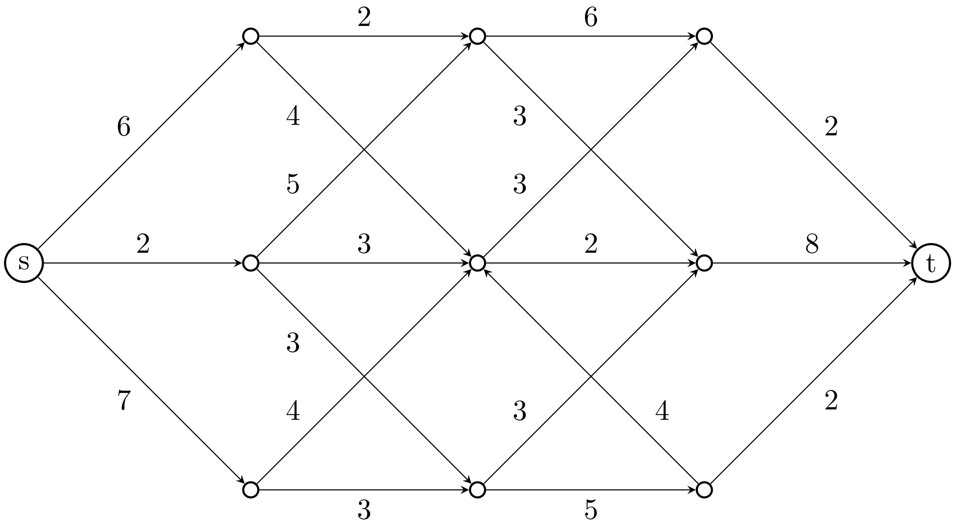
Graph Algorithms and Complexity Theory - Coursework 1

Oskar Mampe

Semester 1 Session 2019–2020

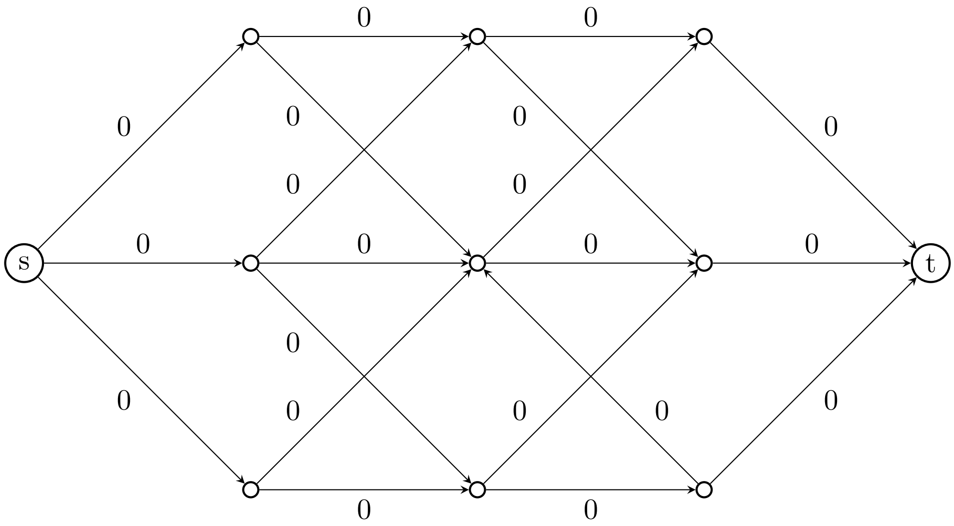
The following minimal cut and maximum flow can be calculated by doing the following. Residual capacities and an augmenting path indicated by bold edges.

Original Network:

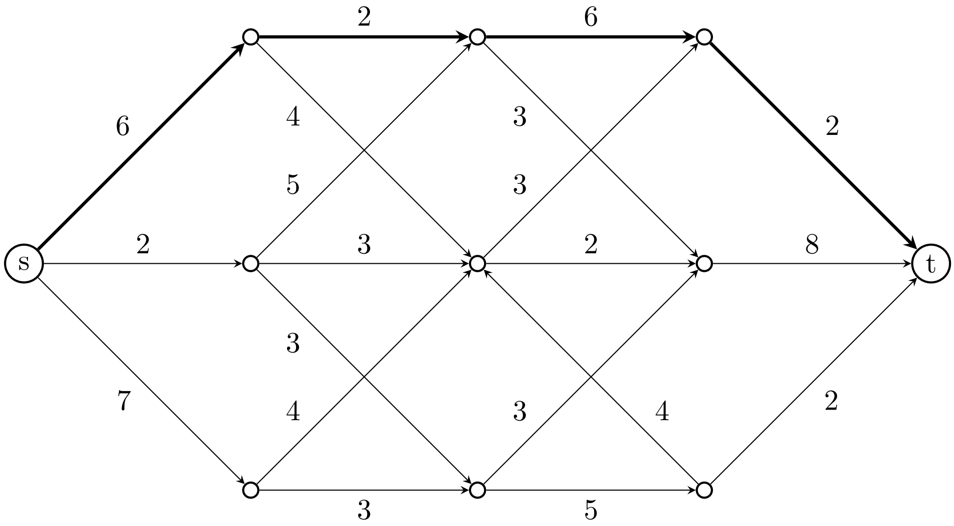


First Pass

Original Network:

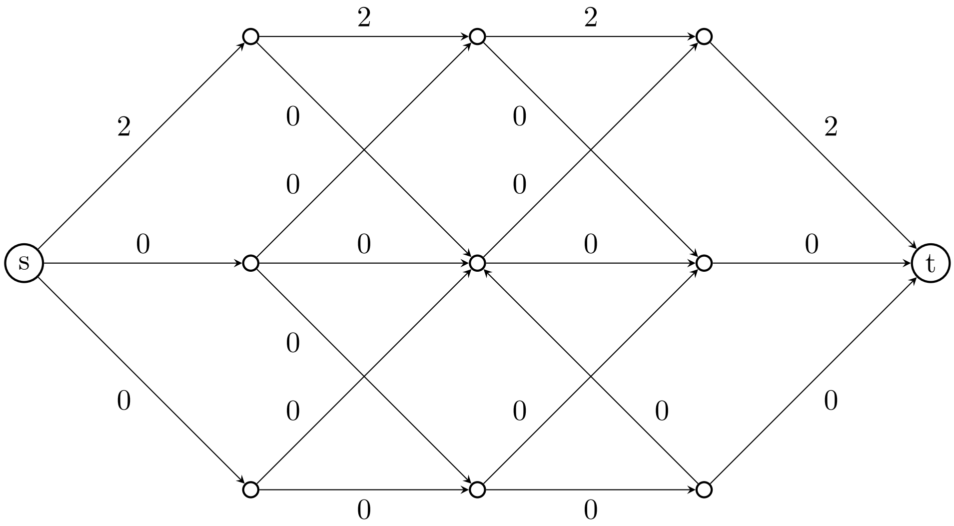


Residual Network:

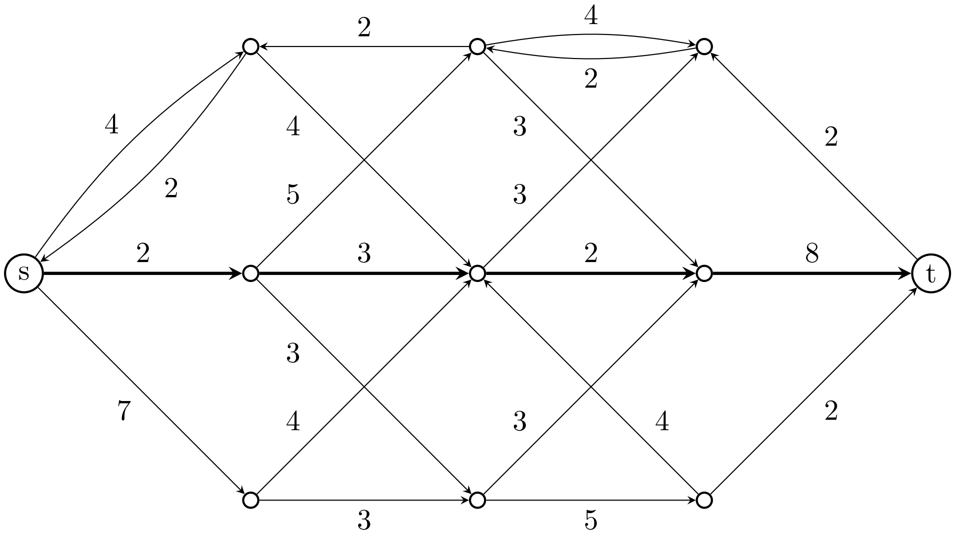


Second Pass

Original Network:

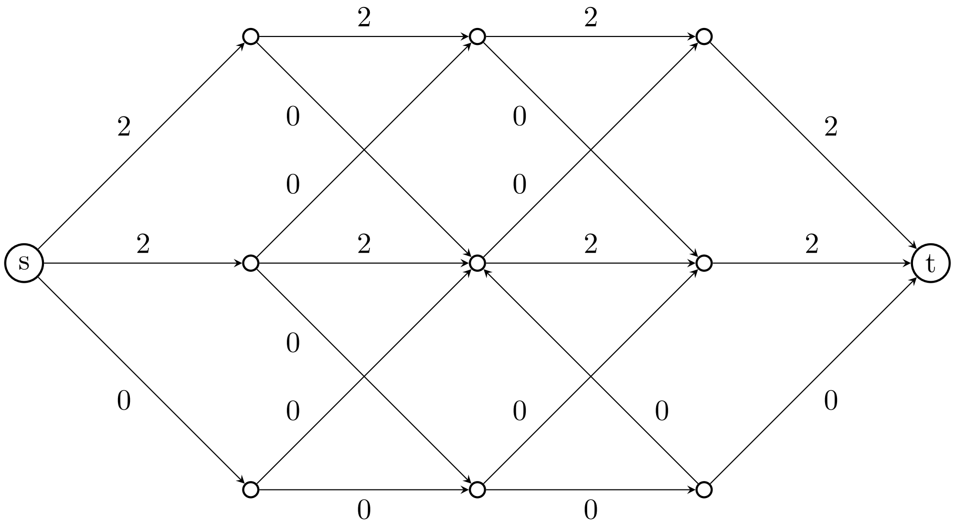


Residual Network:

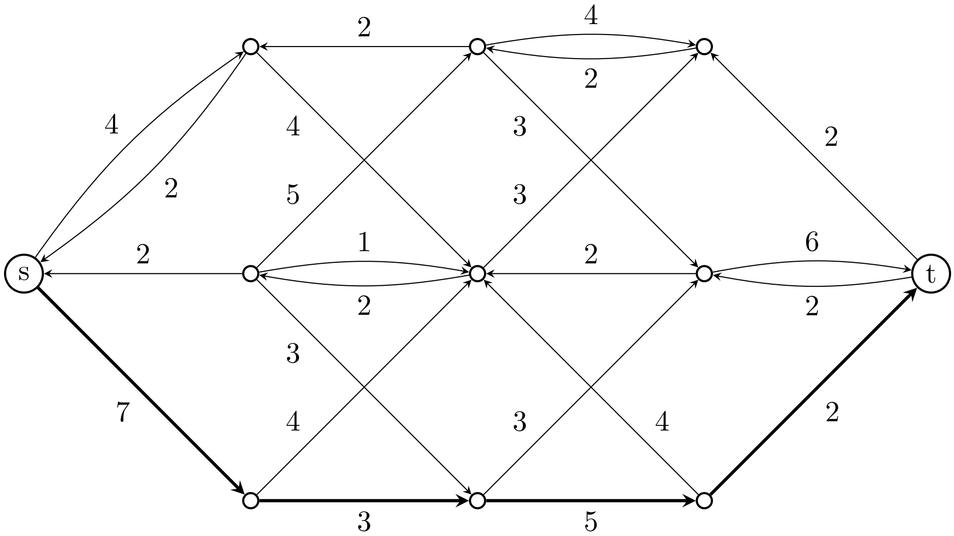


Third Pass

Original Network:

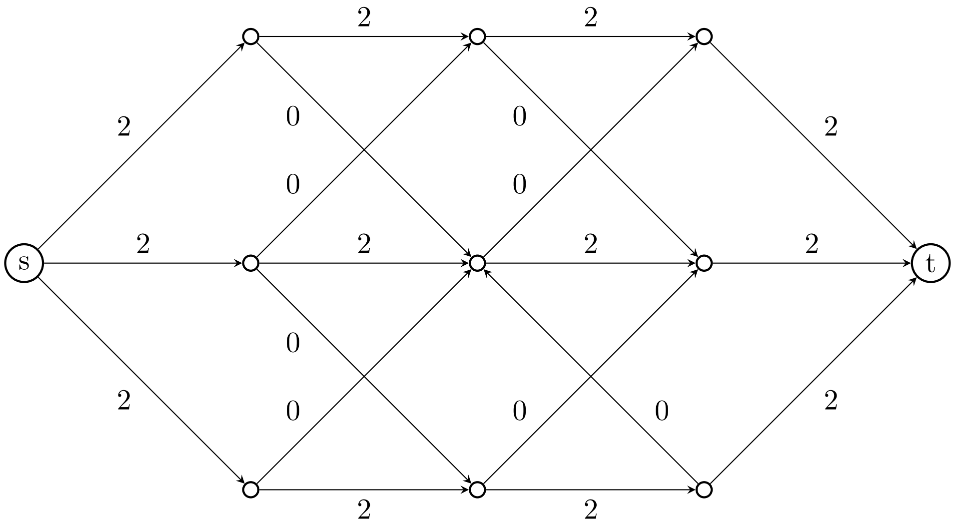


Residual Network:

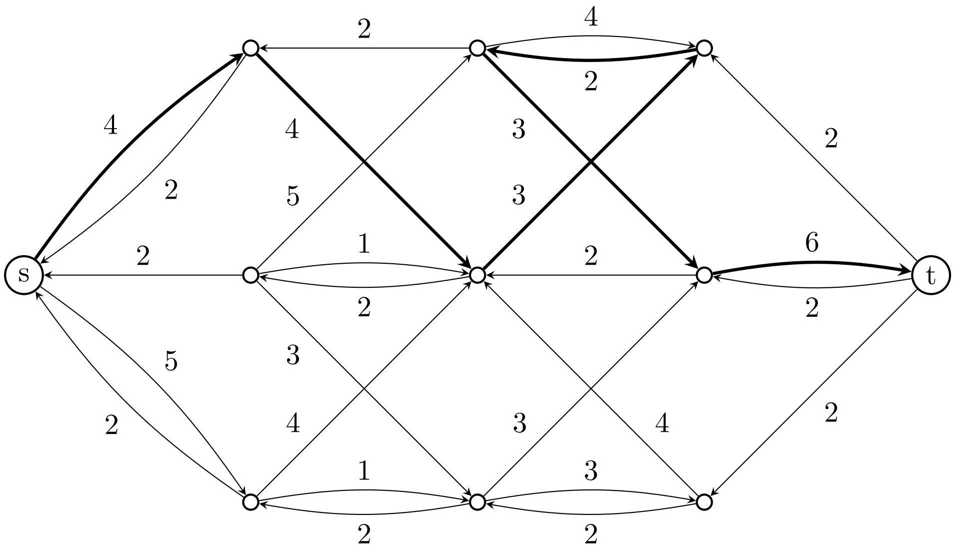


Fourth Pass

Original Network:

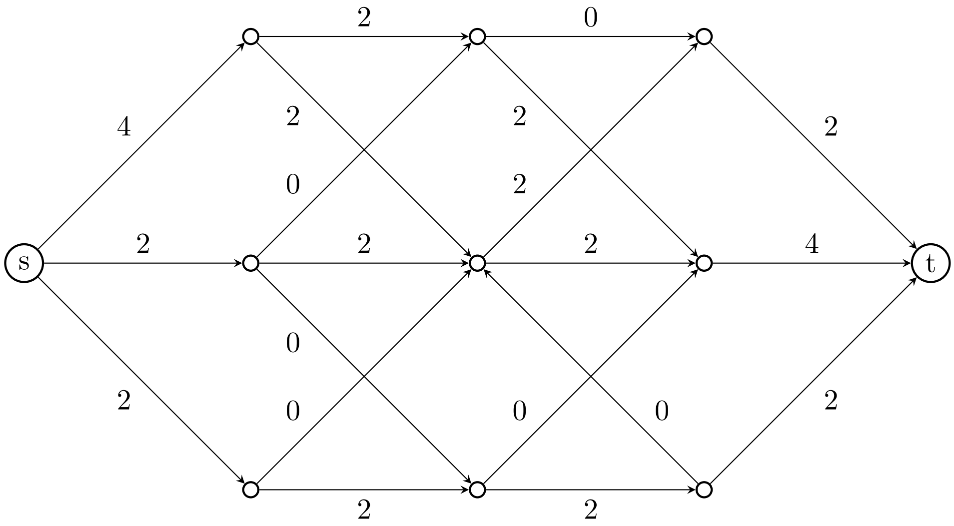


Residual Network:

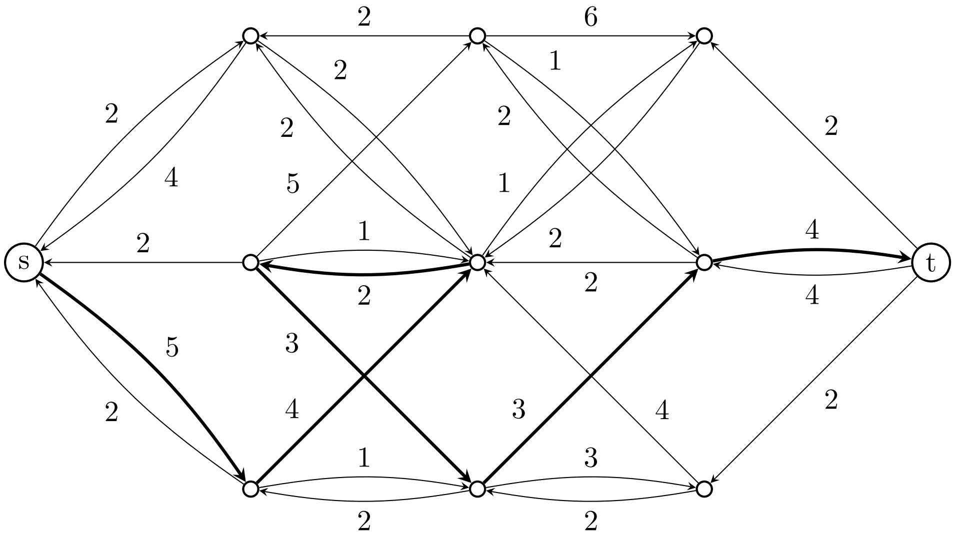


Fifth Pass

Original Network:

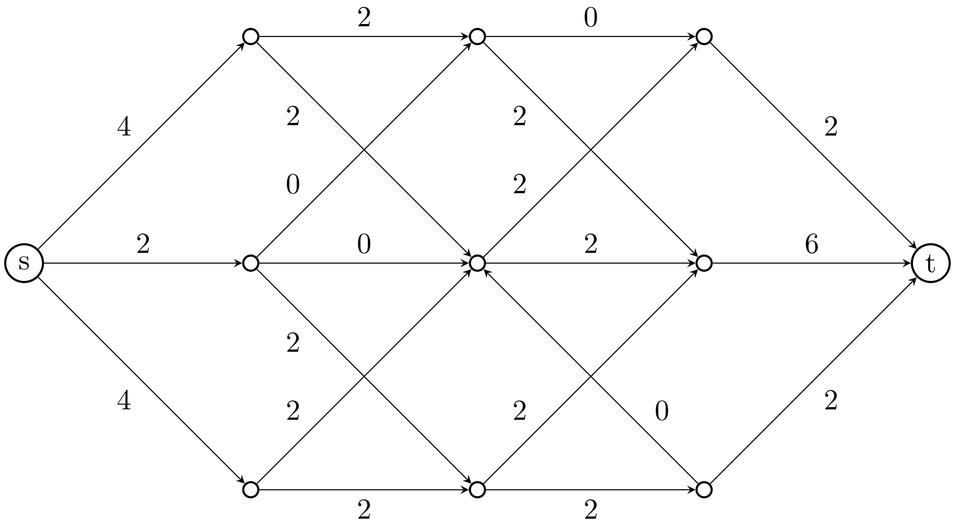


Residual Network:

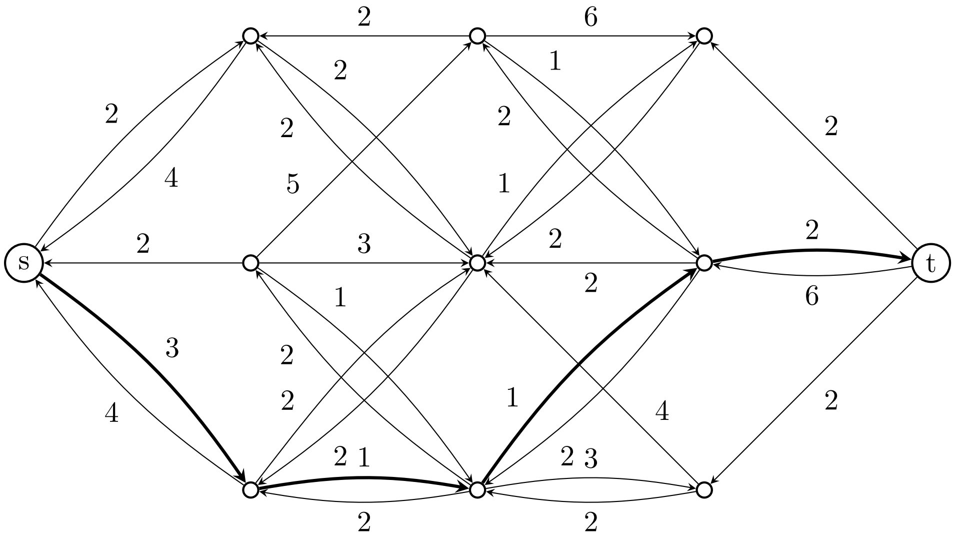


Sixth Pass

Original Network:

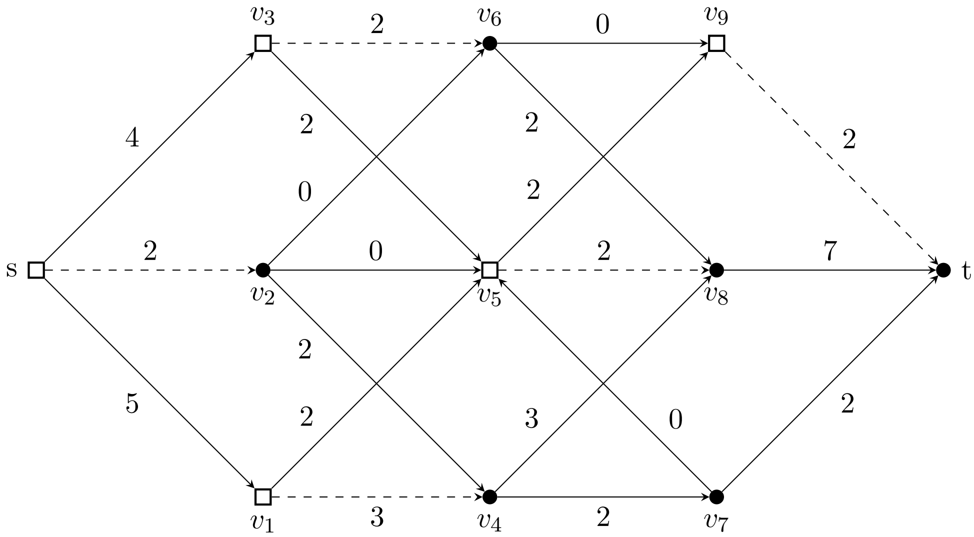


Residual Network:

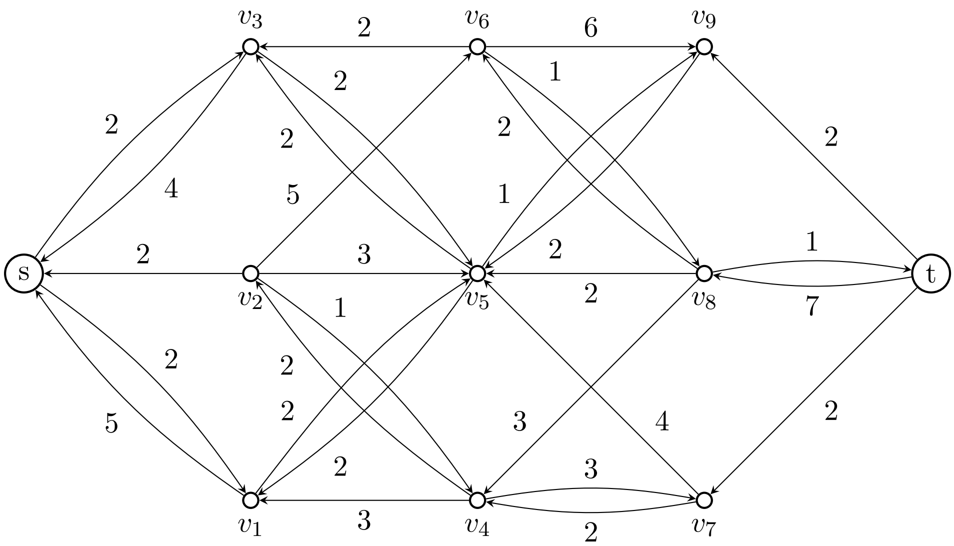


Seventh Pass

Original Network:



Residual Network:



1. The maximum flow has value 11. Formally

$$|f| = f(\{s\},\ V\backslash\{s\}) = \sum_{\substack{(a,b)\ \in \\ <\{s\},\ V\backslash\{s\}>}} f(a,b) - \sum_{\substack{(b,a)\ \in \\ <\{s\},\ V\backslash\{s\}>}} f(b,a) = 2 + 2 + 7 = 11$$

2. A minimal cut is indicated by the dashed lines on the original network. However, more formally, $< \{s, v_1, v_3, v_5, v_9\}, \{t, v_2, v_4, v_6, v_7, v_8\} >$ is the minimal cut, the edges being $(s, v_2), (v_1, v_4), (v_3, v_6), (v_9, t), (v_5, v_8)$. Finally the cut's capacity is 11, as

$$c(A, B) = \sum_{(a,b)\ \in\ <A,B>} c(a, b) = 2 + 2 + 2 + 2 + 3 = 11$$