

Taking the vector $\vec{v} = (2, 1, 4)$ and angle $\theta_0 = 22.5$.

$$q_0 = (\cos(\theta_0), \sin(\theta_0) \times |\vec{v}|)$$

$$||\vec{v}|| = \sqrt{2^2 + 1^2 + 4^2} = \sqrt{21} = 4.58258 \text{ 5.d.p}$$

$$|\vec{v}| = \frac{\vec{v}}{||\vec{v}||} = \frac{(2, 1, 4)}{4.58258} = (0.43644, 0.21822, 0.87287)$$

$$\cos(\theta_0) = 0.92388$$

$$\sin(\theta_0) = 0.38268$$

$$q_0 = (0.92388, 0.38268 \times 0.43644, 0.38268 \times 0.21822, 0.38268 \times 0.87287)$$

$$q_0 = (0.92388, 0.16702, 0.08351, 0.33403)$$

Taking the vector $\vec{w} = (0, 1, 1)$ and angle $\theta_{100} = -61.5$.

$$q_{100} = (\cos(\theta_{100}), \sin(\theta_{100}) \times |\vec{w}|)$$

$$||w|| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2} = 1.41421$$

$$|w| = \frac{|w|}{||w||} = \frac{(0, 1, 1)}{1.41421} = (0, 0.70711, 0.70711)$$

$$\cos(\theta_{100}) = 0.47716$$

$$\sin(\theta_{100}) = -0.87882$$

$$q_{100} = (0.47716, 0, -0.87882 \times 0.70711, -0.87882 \times 0.70711)$$

$$q_{100} = (0.47716, 0, -0.62142, -0.62142)$$

$$q_0^* = 0.92388 - 0.16702i - 0.08351j - 0.33403k$$

$$q_{100} = 0.47716 + 0i - 0.62142j - 0.62142k$$

$$q_{\Delta} = q_0^* \times q_{100}$$

$$\begin{aligned} q_{\Delta} &= 0.44084 + 0i - 0.57412j - 0.57412k \\ &\quad - 0.07970i - 0i^2 + 0.10379ij + 0.10379ik \\ &\quad - 0.03985j - 0ji + 0.05189j^2 + 0.05189jk \\ &\quad - 0.15939k - 0ki + 0.20757kj + 0.20757k^2 \\ &= 0.44084 - 0.57412j - 0.57412k - 0.07970i \\ &\quad + 0.10379k - 0.10379j - 0.03985j - 0.05189 \\ &\quad + 0.05189i - 0.15939k - 0.20757i - 0.20757 \end{aligned}$$

$$q_{\Delta} = 0.18138 - 0.23538i - 0.71776j - 0.62972k$$

$$(q_{\delta})^{100} = q_{\Delta}$$

$$\theta_{\delta} = \frac{\theta_{\Delta}}{100}$$

$$\theta_{\Delta} = \arccos(0.18138) = 79.54985$$

$$\theta_{\delta} = 0.79550$$

$$\sin(79.54985) = 0.98341$$

$$\begin{aligned} \vec{v} &= \left(\frac{0.23538}{0.98341}, \frac{0.71776}{0.98341}, \frac{0.62972}{0.98341} \right) \\ \vec{v} &= (0.23935, 0.72987, 0.64034) \end{aligned}$$

$$\cos(0.79550) = 0.99990$$

$$\sin(0.79550) = 0.01388$$

$$\vec{v} = (0.99990, 0.01388 \times 0.23935, 0.01388 \times 0.72987, 0.01388 \times 0.64034)$$

$$\vec{v} = (0.99990, 0.00332, 0.01013, 0.00889)$$