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Leiden University

Absence of Phase Transitions and Preservation of Gibbs Property Under Renormalization

Oskar Vavtar

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Contents

1 Introduction (probably better name)

I assume this chapter will include a brief introduction to Gibbs measures/thermodynamical formalism (possibly including definitions of Ising and Potts model), as well as the theory of fuzzy Gibbs measures, results from Berghout's thesis.

2 (Non-)Gibbsianity of fuzzy Potts model

This chapter aims to introduce the fuzzy Potts model and provide an alternative, independent proof of Häggström's theorem [reference] about its (non-)Gibbsianity, using the results due to Berghout and Verbitskiy [reference]. Moreover, it introduces the notion of random cluster representations, a powerful tool in the theory of Potts model, which is used in the proof.

2.1 Fuzzy Potts model

In this introductory section of the chapter, we define the fuzzy Potts model and state the celebrated result about its (non-)Gibbsianity, due to Häggström [reference]. Moreover, we explain the strategy and structure of our alternative proof of (part of) the said result.

Consider the Potts model with spin space $\{1,\ldots,q\}$, $q \geq 3$ integer, on lattice $\mathbb{L}$, say $\mathbb{L} = \mathbb{Z}^d$, which defines a model on $\Omega = \{1,\ldots,q\}^{\mathbb{L}}$. The *fuzzy Potts model* is defined by considering some integer $1 < s < q$, so that the spin space is $\{1,\ldots,s\}$ and the whole model defined on $\Sigma = \{1,\ldots,s\}^{\mathbb{L}}$. Moreover, we consider a vector $\mathrm{r} = (r_1,\ldots,r_s) \in \mathbb{N}^s$, such that $r_1 + \ldots + r_s = q$ and define a fuzzy transformation $\pi_{\mathrm{r}} : \{1,\ldots,q\} \to \{1,\ldots,s\}$ by putting

$$\pi_{\mathrm{r}}(a) := \begin{cases} 1: & 1 \leq a \leq r_1, \\ 2: & r_1 + 1 \leq a \leq r_1 + r_2 \\ \ldots & \\ n: & \sum_{i=1}^{n-1} r_i + 1 \leq a \leq \sum_{i=1}^{n} r_i, \\ \ldots & \\ s: & \sum_{i=1}^{s-1} r_i + 1 \leq a \leq q, \end{cases}$$

i.e., $\pi_a = n$ iff $a \in (\sum_{i=1}^{n-1} r_i, \sum_{i=1}^{n} r_i] \cap \mathbb{N}$, $n \in \{1,\ldots,s\}$. In other words, the entire fuzzy map $\pi = \pi_{\mathrm{r}}$ is encoded by a single s-vector r.

Fixing $q \geq 2$, $\beta \geq 0$ and writing $\mu_{q,\beta}^{\mathbb{Z}^d,\#}$ for the Gibbs measure of the Potts model on $\{1,\ldots,q\}^{\mathbb{Z}^d}$ for boundary condition $\# \in \{0,\ldots,q\}$ with inverse temperature β, the fuzzy transformation π_{r} induces the fuzzy Gibbs measure

$$\nu_{q,\beta,\mathrm{r}}^{\mathbb{Z}^d,\#} := \mu_{q,\beta}^{\mathbb{Z}^d,\#} \circ \pi_{\mathrm{r}}^{-1}.$$

Of great interest is the potential Gibbsianity of such measure. Something about the Häggström's result blahblahblah. Recall that for $q \geq 3$ and $d \geq 2$, there exists $\beta_c(d,q)$ such that for each $\beta < \beta_c(d,q)$, $\mu_{q,\beta}^{\mathbb{Z}^d,0} = \ldots = \mu_{q,\beta}^{\mathbb{Z}^d,q}$, i.e., there is a unique Gibbs measure of the Potts model on $\{1,\ldots,q\}^{\mathbb{Z}^d}$ with inverse temperature β.

Theorem 2.1 (Häggström, 2003, [reference]). Let $d \geq 2$, $q \geq 3$, $\# \in \{0,\ldots,q\}$ and $\mathrm{r} = (r_1,\ldots,r_s)$ with $1 < s < q$, $r_1 + \ldots + r_s = q$. Consider a fuzzy Gibbs measure $\nu_{q,\beta,\mathrm{r}}^{\mathbb{Z}^d,\#} = \mu_{q,\beta}^{\mathbb{Z}^d,\#} \circ \pi_{\mathrm{r}}^{-1}$.

(i) For each $\beta < \beta_c(d, \min_{1\leq i\leq s} r_i)$, the measure $\nu_{q,\beta,\mathrm{r}}^{\mathbb{Z}^d,\#}$ is a Gibbs measure.

(ii) The non-Gibbs part.