

Problem 3.15.5

Consider three different processors P1, P2, and P3 executing the same instruction set. P1 has a 3 GHz clock rate and a CPI of 1.5. P2 has a 2.5 GHz clock rate and a CPI of 1.0. P3 has a 4.0 GHz clock rate and has a CPI of 2.2.

- Which processor has the highest performance expressed in instructions per second?
- If the processors each execute a program in 10 seconds, find the number of cycles and the number of instructions.
- We are trying to reduce the execution time by 30% but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

- a)

CPU time = CPU clock cycles / Clock rate

CPU clock cycles = instructions * CPI

CPU time = (instructions * CPI) / Clock rate

Clock rate / CPI = instructions / CPU time

Clock rate / CPI = instructions per second

$P1 = 3\text{GHz} / 1.5 = (3 \times 10^9) / 1.5 = (2 \times 10^9)$ instructions per second

$P2 = 2.5\text{GHz} / 1.0 = (2.5 \times 10^9) / 1.0 = (2.5 \times 10^9)$ instructions per second

$P3 = 4.0\text{GHz} / 2.2 = (4 \times 10^9) / 2.2 = (1.82 \times 10^9)$ instructions per second

P2 has the highest performance because P2 carries out more instructions per second.

- b)

CPU time = CPU clock cycles / Clock rate

CPU clock cycles = CPU time x Clock rate

$$P1 = 10 \times (3 \times 10^9) = 3 \times 10^{10} \text{ cycles}$$

$$P2 = 10 \times (2.5 \times 10^9) = 2.5 \times 10^{10} \text{ cycles}$$

$$P3 = 10 \times (4 \times 10^9) = 4 \times 10^{10} \text{ cycles}$$

$$\text{CPU clock cycles} = \text{instructions} \times \text{CPI}$$

$$\text{Instructions} = \text{CPU clock cycles} / \text{CPI}$$

$$P1 = (3 \times 10^{10}) / 1.5 = 2 \times 10^{10} \text{ instructions}$$

$$P2 = (2.5 \times 10^{10}) / 1.0 = 2.5 \times 10^{10} \text{ instructions}$$

$$P3 = (4 \times 10^{10}) / 2.2 = 1.82 \times 10^{10} \text{ instructions}$$

- c)

$$\text{CPU time} = (\text{instructions} \times \text{CPI}) / \text{Clock rate}$$

$$\text{CPU time (0.7)} = (\text{instructions} \times \text{CPI (1.2)}) / \text{Clock rate}$$

$$\text{Clock rate} = (\text{instructions} \times \text{CPI (1.2)}) / \text{CPU time (0.7)}$$

$$\text{New Clock rate} = 1.71 \times \text{Old Clock Rate}$$

$$P1 = 3\text{GHz} \times 1.71 = 5.13\text{GHz}$$

$$P2 = 2.5\text{GHz} \times 1.71 = 4.27\text{GHz}$$

$$P3 = 4\text{GHz} * 1.71 = 6.84\text{GHz}$$

We multiply the old clock rate by 1.71 to get the new clock rate.

Problem 3.15.7

Consider two different implementations of the same instruction set architecture. The instructions can be divided into four classes according to their CPI (class A, B, C, and D). P1 with a clock rate of 2.5 GHz and CPIs of 1, 2, 3, and 3, and P2 with a clock rate of 3 GHz and CPIs of 2, 2, 2, and 2.

Given a program with a dynamic instruction count of 1.0E6 instructions divided into classes as follows: 10% class A, 20% class B, 50% class C, and 20% class D, which implementation is faster?

- What is the global CPI for each implementation?
- Find the clock cycles required in both cases.

- a)

$$1.0\text{E}6 = 1.0 * 10^6$$

CPI

	A	B	C	D
P1	1	2	3	3
P2	2	2	2	2

Instructions

	A (10%)	B (20%)	C (50%)	D (20%)
# Instructions	1.0E5	2.0E5	5.0E5	2.0E5

$$\text{Global CPI (P1)} = (1 * 1.0\text{E}5 + 2 * 2.0\text{E}5 + 3 * 5.0\text{E}5 + 3 * 2.0\text{E}5) / 1.0\text{E}6 = 2.6$$

$$\text{Global CPI (P2)} = (2 * 1.0\text{E}5 + 2 * 2.0\text{E}5 + 2 * 5.0\text{E}5 + 2 * 2.0\text{E}5) / 1.0\text{E}6 = 2.0$$

$$\text{CPU time} = \text{CPU cycles} / \text{Clock rate}$$

$$\text{CPU cycles} = \text{instructions} * \text{CPI}$$

CPU time = (instructions * CPI) / Clock rate

CPU time (P1) = $(1.0E6 * 2.6) / 2.5\text{GHz} = (1.0E6 * 2.6) / 2.5 * 10^9 = .00104\text{s}$ or 1.04 ms

CPU time (P2) = $(1.0E6 * 2.0) / 3\text{GHz} = (1.0E6 * 2.0) / 3 * 10^9 = 0.00067\text{ s}$ or 0.67ms

P2 is faster than P1

- b)

Clock cycles = instructions * CPI

Clock cycles (P1) = $1E6 * 2.6 = 2.6E6$ cycles

Clock cycles (P2) = $1E6 * 2.0 = 2.0E6$ cycles

Problem 3.15.8

Compilers can have a profound impact on the performance of an application. Assume that for a program, compiler A results in a dynamic instruction count of $1.0E9$ and has an execution time of 1.1 s, while compiler B results in a dynamic instruction count of $1.2E9$ and an execution time of 1.5 s.

- Find the average CPI for each program given that the processor has a clock cycle time of 1 ns.
- Assume the compiled programs run on two different processors. If the execution times on the two processors are the same, how much faster is the clock of the processor running compiler A's code versus the clock of the processor running compiler B's code?
- A new compiler is developed that uses only $6.0E8$ instructions and has an average CPI of 1.1. What is the speedup of using this new compiler versus using compiler A or B on the original processor?

- a)

CPU time = CPU clock cycles / Clock rate

CPU time = CPU clock cycles * Cycle time

CPU clock cycles = instructions * CPI

CPU time = (instructions * CPI) * Cycle time

CPI = CPU time / (instructions * Cycle time)

$$\text{CPI (A)} = 1.1 / (1.0\text{E}9 * 1.0\text{E}-9) = 1.1$$

$$\text{CPI (A)} = 1.5 / (1.2\text{E}9 * 1.0\text{E}-9) = 1.25$$

- b)

Clock Cycle = CPU time / (instructions * CPI)

CPU time is the same so...

(instructions * CPI) of A = 1.1E9

(instructions * CPI) of B = 1.5E9

$$1.1\text{E}9 / 1.5\text{E}9 = 1.37$$

1.37 times faster

- c)

Clock Cycles = instructions * CPI

Clock Cycle (new) = 6.0E8 * 1.1

$$\text{CPU time (A)} / (\text{CPU time (new)}) = (1.0\text{E}9 * 1.1) / (6.0\text{E}8 * 1.1) = 1.67 \text{ times faster}$$

$$\text{CPU time (B)} / (\text{CPU time (new)}) = (1.2\text{E}9 * 1.25) / (6.0\text{E}8 * 1.1) = 2.27 \text{ times faster}$$

Problem 3.15.10

Assume for arithmetic, load/store, and branch instructions, a processor has CPIs of 1, 12, and 5, respectively. Also assume that on a single processor a program requires the execution of 2.56E9 arithmetic instructions, 1.28E9 load/store instructions, and 256 million branch instructions. Assume that each processor has a 2 GHz clock frequency.

Assume that, as the program is parallelized to run over multiple cores, the number of arithmetic and load/store instructions per processor is divided by $0.7 \times p$ (where p is the

number of processors) but the number of branch instructions per processor remains the same.

(a)

Find the total execution time for this program on 1, 2, 4, and 8 processors, and show the relative speedup of the 2, 4, and 8 processor result relative to the single processor result.

(b)

If the CPI of the arithmetic instructions was doubled, what would the impact be on the execution time of the program on 1, 2, 4, or 8 processors?

(c)

To what should the CPI of load/store instructions be reduced in order for a single processor to match the performance of four processors using the original CPI values?

- a)

Instructions

Arithmetic = $2.56E9$

Load/Store = $1.28E9$

Branch = $2.56E8$

CPU time = Clock cycles / Clock rate

Clock cycles = instructions * CPI

1 Processor

Clock cycles = $(2.56E9 * 1) + (1.28E9 * 12) + (2.56E8 * 5) = 1.92E10$

Time = $1.92E10 / 2.0E9 = 9.6$

2 Processors

Clock cycles = $(2.56E9 * 1) / (.7*2) + (1.28E9 * 12) / (.7*2) + (2.56E8 * 5) = 1.408E10$

Time = $1.408E10 / 2.0E9 = 7.04 \text{ sec}$

Time (1) / Time (2) = $9.6 / 7.04 = 1.36$

4 Processors

Clock cycles = $(2.56E9 * 1) / (.7*4) + (1.28E9 * 12) / (.7*4) + (2.56E8 * 5) = 7.68E9$

$$\text{Time} = 7.68\text{E}9 / 2.0\text{E}9 = 3.84 \text{ sec}$$

$$\text{Time (1)} / \text{Time (2)} = 9.6 / 3.84 = 2.5$$

8 Processors

$$\text{Clock cycles} = (2.56\text{E}9 * 1) / (.7*8) + (1.28\text{E}9 * 12) / (.7*8) + (2.56\text{E}8 * 5) = 4.48\text{E}9$$

$$\text{Time} = 4.48\text{E}9 / 2.0\text{E}9 = 2.24 \text{ sec}$$

$$\text{Time (1)} / \text{Time (2)} = 9.6 / 2.24 = 4.29$$

- b)

$$\text{CPU time} = (\text{instructions} * \text{CPI}) / \text{Clock rate}$$

1 Processor

$$\text{Clock cycles} = (2.56\text{E}9 * 2) + (1.28\text{E}9 * 12) + (2.56\text{E}8 * 5) = 2.176\text{E}10$$

$$\text{Time} = 2.176\text{E}10 / 2.0\text{E}9 = 10.88\text{sec}$$

2 Processor

$$\text{Clock cycles} = (2.56\text{E}9 * 2) / (.7*2) + (1.28\text{E}9 * 12) / (.7*2) + (2.56\text{E}8 * 5) = 1.59\text{E}10$$

$$\text{Time} = 1.59\text{E}10 / 2.0\text{E}9 = 7.95 \text{ sec}$$

4 Processor

$$\text{Clock cycles} = (2.56\text{E}9 * 2) / (.7*4) + (1.28\text{E}9 * 12) / (.7*4) + (2.56\text{E}8 * 5) = 8.59\text{E}9$$

$$\text{Time} = 8.59\text{E}9 / 2.0\text{E}9 = 4.3 \text{ sec}$$

8 Processor

$$\text{Clock cycles} = (2.56\text{E}9 * 2) / (.7*8) + (1.28\text{E}9 * 12) / (.7*8) + (2.56\text{E}8 * 5) = 4.94\text{E}9$$

$$\text{Time} = 4.94\text{E}9 / 2.0\text{E}9 = 2.47 \text{ sec}$$

- c)

4 Processors

$$\text{Clock cycles} = (2.56\text{E}9 * 1) / (.7*4) + (1.28\text{E}9 * 12) / (.7*4) + (2.56\text{E}8 * 5) = 7.68\text{E}9$$

$$\text{Time} = 7.68\text{E}9 / 2.0\text{E}9 = 3.84 \text{ sec}$$

1 Processor

$$3.84 \text{ sec} = [(2.56\text{E}9 * 1) + (1.28\text{E}9 * x) + (2.56\text{E}8 * 5)] / 2\text{E}9 \text{ GHz}$$

$$7.68\text{E}9 = (2.56\text{E}9 * 1) + (1.28\text{E}9 * x) + (2.56\text{E}8 * 5)$$

$$7.68\text{E}9 = 3.84\text{E}9 + 1.28\text{E}9x$$

$$3.84\text{E}9 = 1.28\text{E}9x$$

$$3 = x$$

$$3/12 = .25$$

You have to reduce the CPI of load/store by 25%

Problem 3.15.12

The results of the SPEC CPU2006 bzip2 benchmark running on an AMD Barcelona has an instruction count of $2.389\text{E}12$, an execution time of 750 s, and a reference time of 9650 s.

(a)

Find the CPI if the clock cycle time is 0.333 ns.

(b)

Find the SPECratio.

(c)

Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% without affecting the CPI.

(d)

Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% and the CPI is increased by 5%.

(e)

Find the change in the SPECratio for this change.

(f)

Suppose that we are developing a new version of the AMD Barcelona processor with a 4 GHz clock rate. We have added some additional instructions to the instruction set in such a way that the number of instructions has been reduced by 15%. The execution time is reduced to 700 s and the new SPECratio is 13.7. Find the new CPI.

(g)

This CPI value is larger than obtained in 1.11.1 as the clock rate was increased from 3 GHz

to 4 GHz. Determine whether the increase in the CPI is similar to that of the clock rate. If they are dissimilar, why?

(h)

By how much has the CPU time been reduced?

(i)

For a second benchmark, libquantum, assume an execution time of 960 ns, CPI of 1.61, and clock rate of 3 GHz. If the execution time is reduced by an additional 10% without affecting to the CPI and with a clock rate of 4 GHz, determine the number of instructions.

(j)

Determine the clock rate required to give a further 10% reduction in CPU time while maintaining the number of instructions and with the CPI unchanged.

(k)

Determine the clock rate if the CPI is reduced by 15% and the CPU time by 20% while the number of instructions is unchanged.

- a)

Instruction = 2.389×10^{12}

CPU time = 750 sec

Reference time = 9650 sec

Clock cycle time = $.333 \text{ ns} = 3.33 \times 10^{-10}$

CPU time = (instructions * CPI) * Cycle time

$750 = (2.389 \times 10^{12} * \text{CPI}) * 3.33 \times 10^{-10}$

$2.23 \times 10^{12} = 2.389 \times 10^{12} * \text{CPI}$

$0.94 = \text{CPI}$

- b)

SPECratio = ref time / exe time

$\text{SPECratio} = 9650 / 750 = 12.8$

- c)

$$\text{CPU time} = (\text{instructions} * (1.10) * \text{CPI}) * \text{cycle time}$$

$$\text{CPU time would also increase by 10\%} = 825 \text{ sec}$$

- d)

$$\text{CPU time} = (\text{instructions} * (1.10) * \text{CPI} * (1.05)) * \text{cycle time}$$

$$\text{CPU time would increase by 15.5\%} = 866.25 \text{ sec}$$

- e)

$$\text{SPECratio} = 9650 / 866.25$$

$$= 11.14 \text{ sec}$$

- f)

$$\text{Clock rate} = 4\text{GHz} = 4\text{E9 cycles/sec}$$

$$\text{Instructions} = 2.389\text{E12} * .85 = 2.03\text{E12}$$

$$\text{Execution time} = 700\text{s}$$

$$\text{SPECratio} = 13.7$$

$$\text{CPI} = ?$$

$$\text{CPU time} = (\text{instructions} * \text{CPI}) / \text{Clock rate}$$

$$700 = (2.03\text{E12} * \text{CPI}) / 4\text{E9}$$

$$1.38 = \text{CPI}$$

- g)

If the clock rate increases, the CPI will also increase. If we look at the equation $\text{CPU time} = (\text{instructions} * \text{CPI}) / \text{Clock rate}$, if Clock rate increases, CPU time will also increase. Thus, the greater the CPU time, the greater the CPI.

- h)

$$700 \text{ sec} / 750 \text{ sec} = .93$$

$$1 - 0.93 = .067 = 6.7\%$$

- i)

$$\text{CPU time} = (\text{instructions} * \text{CPI}) / \text{clock rate}$$

$$9.6\text{E-}7 = (\text{instructions} * 1.61) / 3\text{E}9$$

$$\text{Instructions} = 1788.82$$

$$9.6\text{E-}7 * .9 = (\text{instructions} * 1.61) / 4\text{E}9$$

$$\text{Instructions} = 2146.58$$

- j)

$$8.64\text{E-}7 * .9 = (2146.58 * 1.61) / \text{clock rate}$$

$$\text{Clock rate} = 4.44\text{GHz}$$

- k)

$$9.6\text{E-}7 * .8 = (2146.58 * 1.61 * (.85)) / \text{clock rate}$$

$$\text{Clock rate } 3.8 \text{ GHz}$$

Problem 3.15.15

Assume a program requires the execution of 50×10^6 FP instructions, 110×10^6 INT instructions, 80×10^6 L/S instructions, and 16×10^6 branch instructions. The CPI for each type of instruction is 1, 1, 4, and 2, respectively. Assume that the processor has a 2 GHz clock rate.

(a)

By how much must we improve the CPI of FP instructions if we want the program to run two times faster?

(b)

By how much must we improve the CPI of L/S instructions if we want the program to run two times faster?

(c)

By how much is the execution time of the program improved if the CPI of INT and FP instructions is reduced by 40% and the CPI of L/S and Branch is reduced by 30%?

- a)

Original Program:

$$\text{CPU time} = [(50\text{E}6 * 1) + (110\text{E}6 * 1) + (80\text{E}6 * 4) + (16\text{E}6 * 2)] / 2\text{E}9$$

$$\text{CPU time} = .256 \text{ sec}$$

$$0.256 / 2 = [(50\text{E}6 * x) + (110\text{E}6 * 1) + (80\text{E}6 * 4) + (16\text{E}6 * 2)] / 2\text{E}9$$

$$0.128 = [(50\text{E}6 * x) + (110\text{E}6 * 1) + (80\text{E}6 * 4) + (16\text{E}6 * 2)] / 2\text{E}9$$

$$2.56\text{E}8 = [(50\text{E}6 * x) + (110\text{E}6 * 1) + (80\text{E}6 * 4) + (16\text{E}6 * 2)]$$

$$-2.06\text{E}8 = 50\text{E}6 * x$$

$$-4.12 = x = \text{CPI}$$

CPI is negative, not possible

- b)

$$0.128 = [(50\text{E}6 * 1) + (110\text{E}6 * 1) + (80\text{E}6 * x) + (16\text{E}6 * 2)] / 2\text{E}9$$

$$2.56\text{E}8 = [(50\text{E}6 * 1) + (110\text{E}6 * 1) + (80\text{E}6 * x) + (16\text{E}6 * 2)]$$

$$6.4\text{E}7 = 80\text{E}6 * x$$

$$0.8 = x = \text{CPI}$$

- c)

$$\text{INT} = 1 * 0.6 = 0.6$$

$$\text{FP} = 1 * 0.6 = 0.6$$

$$L/S = 4 * 0.7 = 2.8$$

$$\text{Branch} = 2 * 0.7 = 1.4$$

$$\text{CPU time} = [(50\text{E}6 * 0.6) + (110\text{E}6 * 0.6) + (80\text{E}6 * 2.8) + (16\text{E}6 * 1.4)] / 2\text{E}9$$

$$\text{CPU time} = .1712$$

$$0.1712 / 0.256 / = .66875$$

$$1 - 0.66875 = .33125$$

The execution time is improved by about 33%.