Supplemental Material: Novel technique for constraining r-process (n,γ) reaction rates

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In this supplemental material, we provide more details on the extraction procedure to obtain the level density and γ-ray strength function and a validity test of of this procedure, and also more details on the normalization of these functions for the case of ^{76}Ge.

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The extraction procedure and validity check of the obtained level density and γ-transmission coefficient.– The full description of the extraction procedure is given in Ref. [1]. An analysis of possible systematic errors related to the procedure is provided in Ref. [2]. The level density ρ and γ-transmission coefficient $\mathscr{T}$ are obtained from the experimental primary γ-ray matrix $P(E_\gamma,E_x)$ using the following expression [1]:

$$P(E_\gamma,E_x) \propto \rho(E_x-E_\gamma)\cdot\mathscr{T}(E_\gamma). \tag{1}$$

Note that Eq. (1) imposes a very strong factorization of the primary γ-ray matrix into two separable vectors, $\rho(E_x-E_\gamma)$ and $\mathscr{T}(E_\gamma)$. These vectors are determined through a global minimization:

$$\chi^2 = \frac{1}{N_{\rm free}}\sum_{E_x=E_x^{\rm min}}^{E_x^{\rm max}}\sum_{E_\gamma=E_\gamma^{\rm min}}^{E_x}\left(\frac{P_{\rm th}(E_\gamma,E_x)-P(E_\gamma,E_x)}{\Delta P(E_\gamma,E_x)}\right)^2, \tag{2}$$

where $N_{\rm free}$ is the number of degrees of freedom, and $\Delta P(E_\gamma,E_x)$ is the uncertainty matrix for the experimental first-generation γ-ray distribution. The fitted first-generation γ-ray matrix $P_{\rm th}(E,E_\gamma)$ can theoretically be approximated by

$$P_{\rm th}(E_\gamma,E_x) = \frac{\rho(E_x-E_\gamma)\mathscr{T}(E_\gamma)}{\sum_{E_\gamma=E_\gamma^{\rm min}}^{E_x}\rho(E_x-E_\gamma)\mathscr{T}(E_\gamma)}. \tag{3}$$

All vector elements $\rho(E_x-E_\gamma)$ and $\mathscr{T}(E_\gamma)$ are treated as free parameters in the iterative χ^2 minimization, and the extracted ρ and $\mathscr{T}$ functions are completely independent on the initial trial functions. The γ-transmission coefficient is assumed to be approximately independent of the initial and final excitation energy, in accordance with the generalized Brink hypothesis [3], stating that any collective decay modes have the same properties whether built on the ground state or excited states. This has been experimentally shown to be a good approximation for γ-decay in the quasi-continuum region for a wide range of nuclei [2, 4–7]. This assumption has also been investigated in detail in Ref. [2].

Some considerations must be made before extracting the level density and γ-transmission coefficient from the primary γ-ray spectra (Fig.1 of the main article). First, we see that the first-generation procedure did not give satisfactory results at low γ-ray energies (below $E_\gamma \approx 1.1$ MeV), as there are still leftovers in the primary γ-ray matrix from the γ-transitions of the 2^+ and 4^+ states with $E_x = 563$ keV and 1108 keV, respectively. This is apparent as vertical ridges for $E_\gamma = 563$ keV and 1108 keV. (We refer the reader to Ref. [2] for a thorough discussion of these methodical problems). As a consequence, we have chosen the following restrictions for the $P(E_\gamma,E_x)$ matrix: $E_\gamma^{\rm min} = 1.4$ MeV, $E_x^{\rm min} = 4.0$ MeV, and $E_x^{\rm max} = 5.9$ MeV, where we have also taken into account that the statistics is too scarce above $E_x \approx 6$ MeV to be useful for the extraction.

The quality of the extraction procedure can be inspected by comparing the primary γ-ray spectra calculated from the obtained ρ and $\mathscr{T}$ functions with the experimental ones. This is a stringent test of how well the ρ and $\mathscr{T}$ functions, which are found through a global fit of all the primary γ spectra within the fit region, can reproduce *individual* primary γ spectra of ^{76}Ge from specific narrow energy regions. This is shown in Fig. 1, where primary γ spectra for excitation-energy bins of 224 keV are compared to the results of the extracted functions. Although local deviations are seen, the data are generally very well reproduced for all excitation energies. The local deviations might be caused by large Porter-Thomas fluctuations [8], which are not taken into account in the extraction procedure.

The normalization of ρ and $\mathscr{T}$.– The global fitting to the primary γ-ray spectra uniquely defines the functional form of ρ and $\mathscr{T}$; however, the absolute value and slope cannot be determined from this procedure. Let us call the output functions from the global-fit extraction $\tilde{\rho}$ and $\tilde{\mathscr{T}}$. From these, one may construct an infinite number of other functions, which give identical fits to the $P(E_\gamma,E_x)$ matrix by [1]

$$\rho(E_x-E_\gamma) = \mathscr{A}\exp[\alpha(E_x-E_\gamma)]\,\tilde{\rho}(E_x-E_\gamma), \tag{4}$$

$$\mathscr{T}(E_\gamma) = \mathscr{B}\exp(\alpha E_\gamma)\tilde{\mathscr{T}}(E_\gamma), \tag{5}$$

where α, $\mathscr{A}$ and $\mathscr{B}$ are the transformation parameters.

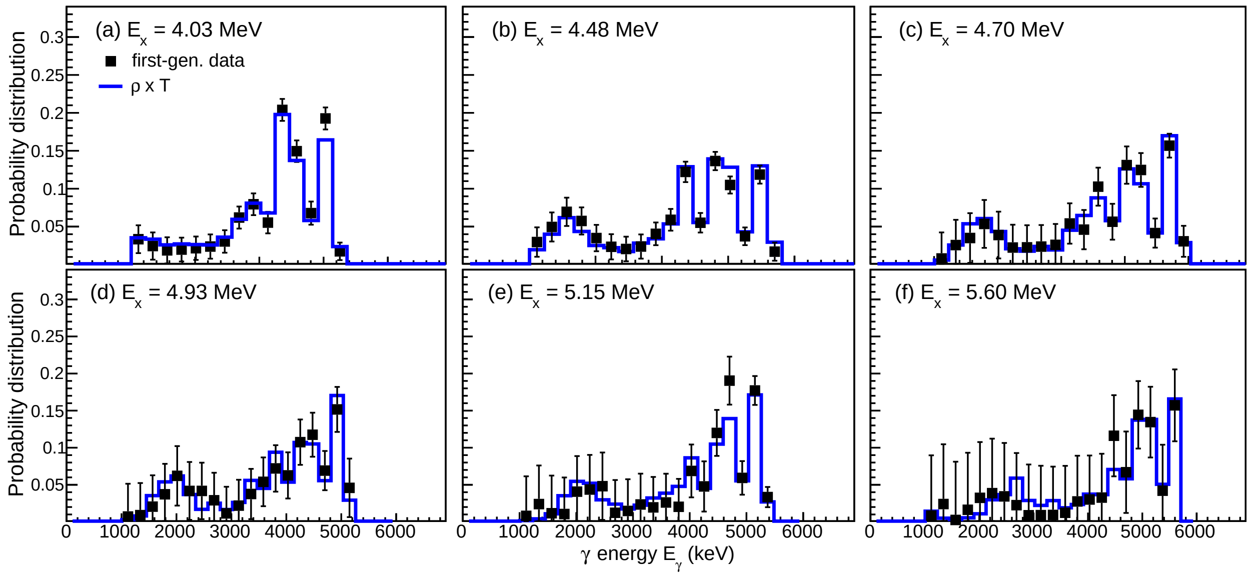

FIG. 1: (Color online) Test of the extraction procedure to obtain the level density and γ-transmission coefficient. The experimental distributions of primary γ-rays for various excitation energies in the range $E_x = 4.02-5.59$ MeV (data points in panels (a)$-$(f)) are compared to the product $\rho(E_x-E_\gamma)\cdot\mathscr{T}(E_\gamma)$ (blue line) of the extracted level density and γ-transmission coefficient. Excitation-energy bins are 224 keV wide.

To normalize the level density, i.e. to determine α and $\mathscr{A}$, two anchor points are needed. At low excitation energies, known, discrete levels [9] are used. At the neutron separation energy S_n, data from neutron-resonance experiments (e.g. Refs. [10, 11]) have been applied to calculate the total level density at S_n, $\rho(S_n)$, for all spins and parities [1]; $\rho(S_n)$ is then used as a second anchor point for the normalization. However, in the case of ^{76}Ge, there is no information from neutron-resonance experiments available. Therefore, other measures must be taken to normalize the level density.

We have estimated $\rho(S_n)$ with two different approaches:

1. Applying the most recent systematics from von Egidy and Bucurescu [12] for the spin cutoff parameter to calculate $\rho(S_n)$ for the Ge isotopes with known s-wave neutron-resonance spacing D_0, and infer the ^{76}Ge anchor point $\rho_1(S_n)$: *norm-1*. This is done in the same way as in Refs. [13, 14].

2. Using recent microscopic calculations of Goriely, Hilaire and Koning [15] within the combinatorial-plus-Hartree-Fock-Bogoliubov (c.+HFB) approach, *norm-2*.

These two options for the level density represent two extremes: norm-1 implies a narrow spin distribution centered at low spin-quantum numbers, and will in general give low values for the level density for excitation energies below the neutron separation energy. On the other hand, norm-2 displays a broad spin distribution and gives a high level density in the considered E_x region. Therefore, we take norm-1 as the lower limit and norm-2 as the upper limit. More details are given in the following.

For norm-1, a spin distribution of the form [16]

$$g(E_x,J) \simeq \frac{2J+1}{2\sigma^2(E_x)}\exp\left[-(J+1/2)^2/2\sigma^2(E_x)\right] \qquad (6)$$

is applied, and the spin cutoff parameter is given by [12] $\sigma^2(E_x) = 0.391A^{0.675}(E_x-0.5Pa')^{0.312}$, where Pa' is the deuteron pairing energy determined with the prescription of Ref. [12]. The total level density is calculated from the experimental D_0 values and the spin cutoff parameter $\sigma(E_x)$ according to Ref. [1], assuming an equal parity distribution at S_n. The resulting systematics is shown in Fig. 2a. The spin cutoff parameter at S_n used for ^{76}Ge is $\sigma(S_n) = 3.73$, and we estimate $\rho_1(S_n) = 4.70\cdot10^4$ MeV^{-1}, indicating an s-wave spacing of $D_{0,1} = 323$ eV. Further, for norm-1, we have interpolated the ^{76}Ge data and $\rho_1(S_n)$ with the constant-temperature (CT) model [16, 17], which describes very well measured level densities for a wide range of nuclei and excitation energies [14, 18, 19]. This is given as $\rho(E_x) = 1/T\exp[(E_x-E_0)/T]$, where E_0 is an energy shift and T is the constant nuclear temperature. For ^{76}Ge, we have used $T = 0.92$ MeV and $E_0 = -0.39$ MeV, in excellent agreement with the systematics of Ge isotopes from Ref. [12].

For norm-2, we have applied an excitation-energy shift $\delta = -0.69$ MeV to get a reasonably good agreement with known, discrete levels at low excitation energies. We obtain $\rho_2(S_n) = 7.07\cdot10^4$ MeV^{-1}, and an s-wave spacing of $D_{0,2} = 315$ eV. Note that in the calculations of Ref. [15], no assumptions are made neither on the general shape of the spin distribution, nor on the parity distribution.

To normalize the γ-transmission coefficient, information

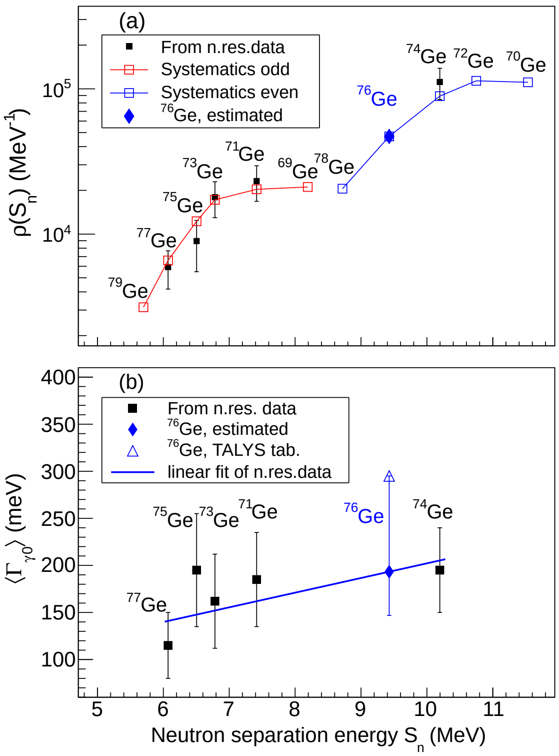

FIG. 2: (Color online) (a) Estimating $\rho_1(S_n)$ for ^{76}Ge (blue diamond) from systematics of Ge isotopes. The semi-experimental $\rho(Sn)$ values (small black squares) are calculated with D_0 data taken from Ref. [10]. The systematics values are given as red and blue open squares for the odd and even Ge isotopes, respectively, and are multiplied with a common factor of 1.2 to get them within the error bars; (b) estimating $\langle\Gamma_{\gamma 0}\rangle$ for ^{76}Ge, based on a linear fit of the known values for the Ge isotopes and including the tabulated TALYS estimate [20] as an upper limit.

on the average, total radiative width $\langle\Gamma_{\gamma 0}\rangle$ is needed to determine the scaling factor $\mathscr{B}$. Based on systematics for the Ge isotopes using neutron-resonance data from Ref. [10], we estimate $\langle\Gamma_{\gamma 0}\rangle = 193^{+102}_{-46}$ meV with the upper limit taken from the tabulated values in TALYS [20], see Fig. 2b. Further, the slope of the γSF is determined from a reduced upper anchor point $\rho(S_n)$ for the level density (similar to the case in Ref. [7]), as the β-decay of ^{76}Ga will dominantly populate states with spin quantum numbers $J = 1,2,3$, taking the ground-state spin/parity of ^{76}Ga to be 2^- [21]. With the assumption that the β-decay of ^{76}Ga populates the $J = 1,2,3$ states with an approximately equal cross section, we get the reduced $\rho_1(S_n, J = 1,2,3) = 1.90 \cdot 10^4$ MeV^{-1} and $\rho_2(S_n, J = 1,2,3) = 2.04 \cdot 10^4$ MeV^{-1} for norm-1 and norm-2, respectively. The transmission coefficient $\mathscr{T}(E_\gamma)$ is then normalized accordingly, as explained in detail in previous publications [2, 22].

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