Parameterizations of level density for fission calculations

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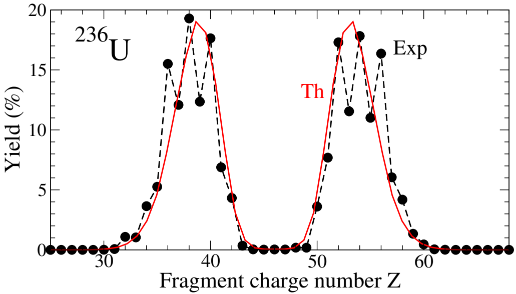
Motivation

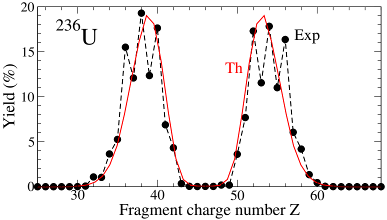
- Random walk on 5D potential energy surface remarkably successful in describing fission-fragment distributions,



Calculation using model of J. Randrup, P. Möller,
PRL 106, 132503 (2011); PRC 84, 034613 (2011).

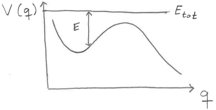
- Level density, ρ, plays role in controlling the diffusion in the random walk, and affects fragment distribution.

Motivation



- Classical model for shape degrees of freedom **q**, Langevin + assumption of strong damping. [E.g. J. Randrup, P. Möller, A. J. Sierk, PRC 84, 034613 (2011)]

- Random walk,
  - Drift term $-\frac{\partial V}{\partial \mathbf{q}}\Delta t$, 5D potential energy surface $V(\mathbf{q})$.
  - Stochastic term $\sqrt{2T\Delta t}\xi$,
- Temperature $\frac{1}{T} = \frac{\partial}{\partial E}\ln[\rho(E)]$, $E = E_{\mathrm{tot}} - V(\mathbf{q})$.
- Currently ρ from simple parameterization (Ignatyuk).



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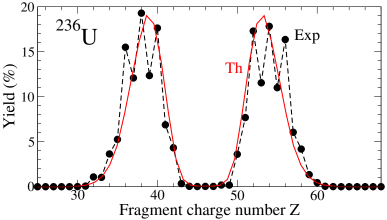

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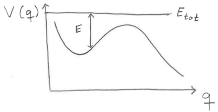
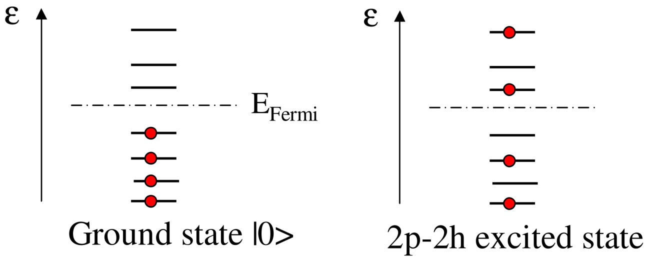


Goal: Include realistic level densities at each deformation

Combinatorial model for level density

- Many-body level density by combinatorial model [H. Uhrenholt *et al* NPA 913, 127 (2013)]
- Deformed mean field, same as used in fission pot. surface [P. Moller *et al*, At. Data Nucl. Data Tables 59, 185 (1995)]
- Realistic level density, but very time consuming to calculate at high energies.



Fermi-gas relations for level density

- Relations for non-interacting Fermi gas,

$$\rho(E) \sim e^{2\sqrt{aE}}.$$

- Level density parameter a, related to single-particle level density at Fermi level, g_0,

$$a = \frac{\pi^2}{6} g_0 = \frac{A}{e_0}.$$

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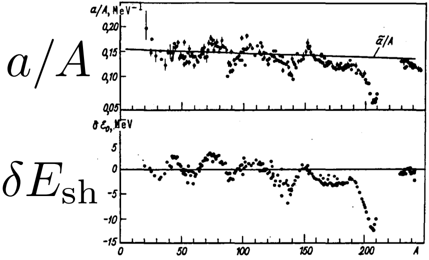
- Correlation between a-parameter and shell-correction energy $\delta E_{\rm sh}$,



Describe by modified a-parameter,

$$a(E) = \tilde{a}\left[1 + (1 - e^{-E/E_{\mathbf{damp}}})\frac{\delta E_{\rm sh}}{E}\right].$$

A. V. Ignatyuk,
IAEA-INDC(CCP)-233/L (1985)

Behavior of Ignatyuk correction

Fermi gas level density,

$$\rho(E) \sim e^{2\sqrt{aE}},$$

$$a(E) = \tilde{a}\left[1 + (1 - e^{-E/E_{\mathrm{damp}}})\frac{\delta E_{\mathrm{sh}}}{E}\right].$$

Excitation energy E,

$$E = E_{\mathrm{tot}} - V(\vec{q}) = E_{\mathrm{tot}} - V_{\mathrm{LD}}(\vec{q}) - \delta E_{\mathrm{sh}}.$$

Low energy limit,

$$a(E)E \to \tilde{a}\left(1 + \frac{\delta E_{sh}}{E_{\mathrm{damp}}}\right)E, \quad E/E_{damp} \to 0.$$

FG expression with modified $\tilde{a}$ parameter.
High energy limit,

$$a(E)E \to \tilde{a}(E + \delta E_{\mathrm{sh}}) = \tilde{a}(E_{\mathrm{tot}} - V_{\mathrm{LD}}(\vec{q})), \quad E/E_{\mathrm{damp}} \to \infty.$$

FG level density for liquid drop.

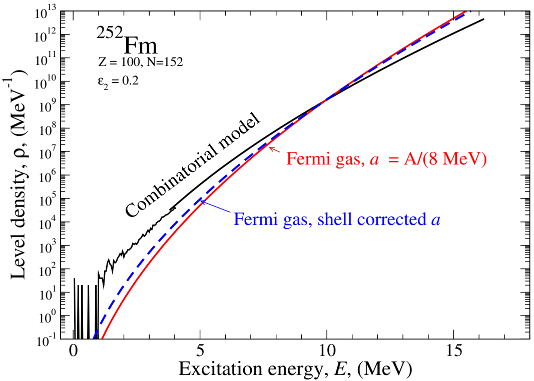
Microscopic level dens compared to Fermi Gas

Fermi-gas formula,

$$\rho(E) \propto e^{2\sqrt{aE}},$$

with parameters used in previous fission calculations: $a = A/8$ MeV and,

$$a(E) = \frac{A}{8\ \text{MeV}}\left[1 + \left(1 - e^{-E/(18.5\ \text{MeV})}\right)\frac{\delta E_{sh}}{E}\right],$$



Fermi-gas model breaks down at low energies.
(slightly improved if formula with prefactors is used)

Including realistic level densities

- Fission will pass through regions with both negative (minima) and positive (saddle) δE_{shell}.
- Can one parametrization fit the microscopic level density for different deformations /different δE_{shell}?

Fit Fermi-Gas relation to level densities at different deformations in the nucleus being studied.

$$\rho_{\text{FG}} = C e^{2\sqrt{a(E^*)E^*}},$$

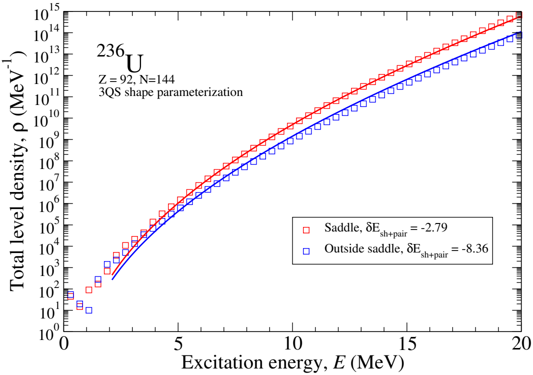
with Ignatiuk's form for δE_{sh} dependence,

$$a(E) = \frac{A}{e_0}\left[1 + (1 - e^{-E/E_{\text{damp}}})\frac{\delta E_{sh}}{E}\right].$$

Pairing treated with backshift $\Delta = 12/\sqrt{A}$.
Free parameters C, e_0, E_{damp}.
Fitting range $E \geq 10$ MeV.

Including realistic level densities

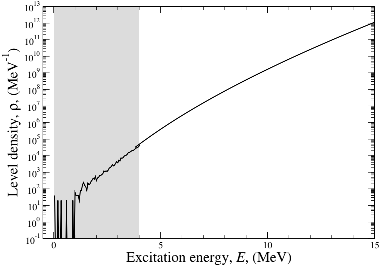
Parameterize microscopic level dens

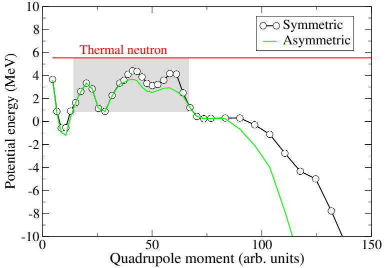


- Fit $\log(\rho_{\rm FG})$ to log of microscopic level density for 11 deformations. Obtained parameters, $e_0 = 12.2, \quad E_{\rm damp} = 57.6$.
- OK fit for all different deformations at $E \gtrsim 5$ MeV, parameterization captures effect of shell correction.
- $\Rightarrow$ For high E, can include realistic level densities in fission calculation without expensive microscopic calculation at each deformation.
- For low E, microscopic level density needed!

Summary

- Aim: Calculation of fission-fragment distribution with microscopic level density consistent with potential energy model.
- Can include effect of realistic level densities using locally fitted formula when excitation energy is $\gtrsim 5$ MeV.
- Level density at low excitation energies, relevant for diffusion in the saddle point region, differ greatly from Fermi gas formula.





Outlook:

- Strategy: Combine microscopic results for low energy with fit at higher energies.
- Best parameterization? Quantify errors.