1. **EDF and Queue/Stack disciplines**

“Queue discipline” is another term for FIFO, while “Stack discipline” refers to LIFO. An EDF scheduler does not always obey the FIFO (queue) discipline, because tasks with shorter deadlines that are added later are allowed to preempt (or at least cut in front of) tasks with longer deadlines that are added earlier. EDF schedulers are also not required to obey the LIFO (stack) discipline, because a task that is added later but has a long deadline will have to wait for all previously added tasks with shorter deadlines to finish. EDF schedules are not directly related to arrival order, but instead use deadlines as the parameter for deciding execution order.

Example of how EDF does not obey FIFO (queue) discipline:

|  |  |  |  |
| --- | --- | --- | --- |
| Task | Arrival | Exec Time | Deadline |
| J­1 | 0 | 5 | 10 |
| J2 | 0 | 5 | 15 |
| J3 | 2 | 5 | 10 |

The order of these tasks will be J1, J3, J2. Even though J2 arrived before J3, it will not be scheduled until after J3 because J3’s deadline is sooner than J2’s.

Example of how EDF does not obey LIFO (stack) discipline:

|  |  |  |  |
| --- | --- | --- | --- |
| Task | Arrival | Exec Time | Deadline |
| J­1 | 0 | 5 | 11 |
| J2 | 0 | 5 | 10 |
| J3 | 2 | 5 | 15 |

The order of these tasks will be J2, J1, J3. Even though J3 arrived last, it will not be chosen as the next task to run after J2 is finished because J1’s deadline is sooner than J3’s.

1. **Subtask scheduling**

Each subtask may have to wait until there is just enough time before the deadline to run itself and all the other remaining subtasks. This moment would correspond to laxity = 0 for the subtask relative to the set of subtasks. A formula to represent the latest allowable start time could be:

for each subtask J­i.

1. **LLF scheduling**

Yes, the task set is schedulable using LLF. The diagram below shows a schedule.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **t** | 0 |  |  | 3 | 4 |  |  |  | 8 |  |  |  | 12 |  | 14 |  | 16 | 17 | 18 |  |  |  | 22 |  | 24 | 25 | 26 | 27 |  |  |  | 31 |  | 33 |  | 35 | 36 | 37 |  |  |  | 41 |  |  | 44 | 45 |  |  | 48 |  | 50 | 51 |  | 53 | 54 |  | 56 | 57 | 58 | 59 | 60 |
| **J** | E | | | B | C | | | | A | | | | E | | A | | E | B | C | | E | | C | | E | B | D | A | | | | E | | A | | E | B | C | | | | E | | | B | A | | | C | | A | E | | A | C | | A | E | B | D |  |
| Arrival | A |  |  |  |  |  |  |  |  |  | B |  |  |  |  | C |  |  |  |  | A |  |  |  |  |  |  |  |  |  | B |  |  |  |  |  |  |  |  |  | A |  |  |  |  | C |  |  |  |  | B |  |  |  |  |  |  |  |  |  |  |
| B |  |  |  |  |  |  |  |  |  | E |  |  |  |  |  |  |  |  |  | B |  |  |  |  |  |  |  |  |  | C |  |  |  |  |  |  |  |  |  | B |  |  |  |  |  |  |  |  |  | E |  |  |  |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | E |  |  |  |  |  |  |  |  |  | D |  |  |  |  |  |  |  |  |  | E |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| D |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | E |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| E |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. **Scheduling a given task set with multiple schedulers**

FCFS schedule:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **t** | | 0 | | 1 | 2 | |  |  | 5 | |  |  |  | 9 | |  |  |  |  |  | 15 | | 16 | |  |  | 19 | |  |  |  | 23 | | 24 | |  |  | 27 | |  |  |  |  |  | 33 | | 34 | | 35 | |  |  | 38 | |  |  |  | 42 | | 43 | |  |  | 46 | |  |  |  |  |  | 52 | |  |  |  | 56 | | 57 | |  |  | 60 | |
| **J** | B | | D | | | E | | | | C | | | | | A | | | | | | | B | | E | | | | C | | | | | B | | E | | | | A | | | | | | | B | | D | | E | | | | C | | | | | B | | E | | | | A | | | | | | | C | | | | | B | | E | | | |  |
| Arrival | | A | |  |  | |  |  |  | |  |  |  |  | | B |  |  |  |  | C | |  | |  |  |  | | A |  |  |  | |  | |  |  |  | |  |  | B |  |  |  | |  | |  | |  |  |  | |  | A |  |  | |  | |  | C |  | |  |  |  | B |  |  | |  |  |  |  | |  | |  |  |  | |
| B | |  |  | |  |  |  | |  |  |  |  | | E |  |  |  |  |  | |  | |  |  |  | | B |  |  |  | |  | |  |  |  | |  |  | C |  |  |  | |  | |  | |  |  |  | |  | B |  |  | |  | |  |  |  | |  |  |  | E |  |  | |  |  |  |  | |  | |  |  |  | |
| C | |  |  | |  |  |  | |  |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | | E |  |  |  | |  | |  |  |  | |  |  | D |  |  |  | |  | |  | |  |  |  | |  | E |  |  | |  | |  |  |  | |  |  |  |  |  |  | |  |  |  |  | |  | |  |  |  | |
| D | |  |  | |  |  |  | |  |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  |  |  |  | |  | |  |  |  | |  |  | E |  |  |  | |  | |  | |  |  |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |  |  |  |  | |  | |  |  |  | |
| E | |  |  | |  |  |  | |  |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |  | |  | |  |  |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |  |  |  |  | |  | |  |  |  | |

\*To break ties, a SJF policy was used.

EDF schedule:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **t** | | 0 | |  |  |  |  | |  |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  |  |  |  | |  | |  |  |  | |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  | 60 | |
| **J** | B | | E | | | | | C | | | | | A | | | | | | | B | | E | | | | D | | C | | | | | B | | E | | | | A | | | | | | | B | | E | | | | C | | | | | B | | E | | | | D | | C | | | | | B | | E | | | | A | | | | | | |  |
| Arrival | | A | |  |  |  |  | |  |  |  |  | |  | B |  |  |  |  | | C | |  |  |  | |  | | A |  |  |  | |  | |  |  |  | |  |  | B |  |  |  | |  | |  |  |  | |  |  | A |  | |  | |  |  | C | |  | |  |  |  | B | |  | |  |  |  | |  |  |  |  |  |  | |
| B | |  |  |  |  | |  |  |  |  | |  | E |  |  |  |  | |  | |  |  |  | |  | | B |  |  |  | |  | |  |  |  | |  |  | C |  |  |  | |  | |  |  |  | |  |  | B |  | |  | |  |  |  | |  | |  |  |  | E | |  | |  |  |  | |  |  |  |  |  |  | |
| C | |  |  |  |  | |  |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  | | E |  |  |  | |  | |  |  |  | |  |  | D |  |  |  | |  | |  |  |  | |  |  | E |  | |  | |  |  |  | |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |
| D | |  |  |  |  | |  |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  | |  |  |  |  | |  | |  |  |  | |  |  | E |  |  |  | |  | |  |  |  | |  |  |  |  | |  | |  |  |  | |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |
| E | |  |  |  |  | |  |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |  | |  |  |  | |  |  |  |  | |  | |  |  |  | |  | |  |  |  |  | |  | |  |  |  | |  |  |  |  |  |  | |

\*To break ties, a SJF policy was used. If deadlines for newly arrived tasks tied with the running task, the running task was not preempted.

RM schedule:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **t** | 0 |  |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 60 | |
| **J** | B | E | |  | |  | | C | |  | |  | |  | | A | |  | | B | | E | |  | |  | | A | | C | |  | |  | |  | | A | | \* | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  |
| Arrival | A |  |  | |  | |  | |  | |  | |  | |  | |  | | B | |  | |  | |  | |  | | C | |  | |  | |  | |  | | A | |  | |  | |  | |  | |  | |  | |  | |  | |  | | B | |  | |  | |  | |  | |  | |  | |  | |  | |  | | A | |  | |  | |  | |  | | C | |  | |  | |  | |  | | B | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| B |  |  | |  | |  | |  | |  | |  | |  | |  | | E | |  | |  | |  | |  | |  | |  | |  | |  | |  | | B | |  | |  | |  | |  | |  | |  | |  | |  | |  | | C | |  | |  | |  | |  | |  | |  | |  | |  | |  | | B | |  | |  | |  | |  | |  | |  | |  | |  | |  | | E | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| C |  |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | | E | |  | |  | |  | |  | |  | |  | |  | |  | |  | | D | |  | |  | |  | |  | |  | |  | |  | |  | |  | | E | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| D |  |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | | E | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| E |  |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |

This task set is not RM-schedulable. Task A misses its deadline at time = 20.

1. **Scheduling with precedence constraints**

Revise the deadlines to integrate information about precedence.

* Map out scheduling blocks over a timespan = LCM of periods of all tasks.

LCM(42, 42, 14) = 42

* Assign an absolute deadline to each block.

d1,1 = 42; d1,2 = 42; d1,3 = 42; d2,1 = 12; d2,2 = 12; d2,3 = 26; d2,4 = 26; d2,5 = 40; d2,6 = 40; d3,1 = 42

* Topologically order the blocks and then reverse it.
  + Draw a precedence graph.

1

2

3

1

2

3

6

4

5

1

T1

T2

T3

* + A forward topological could be: (T3,1 T2,­1 T1,­1 T2,­2 T1,­2 T2,­3 T1,­3 T2,­4 T2,­5 T2,­6)
  + Reversing that = (T2,­6 T2,­5 T2,­4 T1,­3 T2,­3 T1,­2 T2,­2 T1,­1 T2,­1 T3,1)
* Work through the reverse topological order and adjust deadlines for blocks with outgoing edges in the precedence graph.
  + T2,­6 has no outgoing edges. d’2,6 = d2,6 = **40**.
  + T2,­5 has an edge to T2,­6. d’2,5 = min(d2,5, d’2,6 – c2,6) = min(40, 40 – 4) = min(40, 36) = **36**.
  + T2,­4 has an edge to T2,­5. d’2,4 = min(d2,4, d’2,5 – c2,5) = min(26, 36 – 2) = min(26, 34) = **26**.
  + T1,­3 has an edge to T2,­6. d’1,3 = min(d1,3, d’2,6 – c2,6) = min(42, 36 – 4) = min(42, 32) = **32**.
  + T2,­3 has edges to T1,­3 and T2,4. d’2,3 = min(d2,3, d’2,4 – c2,4, d’1,3 – c1,3)   
    = min(26, 26 – 4, 42 – 6) = min(26, 22, 36) = **22**.
  + T1,­2 has edges to T1,­3 and T2,4. d’1,2 = min(d1,2, d’1,3 – c1,3, d’2,4 – c2,4)   
    = min(42, 32 – 6, 26 – 4) = min(42, 26, 22) = **22**.
  + T2,­2 has an edge to T2,­3. d’2,2 = min(d2,2, d’2,3 – c2,3) = min(12, 26 – 2) = min(12, 24) = **12**.
  + T1,­1 has edges to T1,2 and T2,2. d’1,1 = min(d1,1, d’1,2 – c1,2, d’2,2 – c2,2)   
    = min(42, 22 – 6, 12 – 4) = min(42, 16, 8) = **8**.
  + T2,­1 has edges to T1,2 and T2,2. d’2,1 = min(d2,1, d’1,2 – c1,2, d’2,2 – c2,2)   
    = min(12, 22 – 6, 12 – 4) = min(12, 16, 8) = **8**.
  + T3,­1 has no outgoing edges. d’3,1 = d3,1 = **42**.
* We can schedule the adjusted task blocks with EDF. Original arrival times are respected.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **t** | 0 |  |  |  | 4 |  | 6 |  |  |  | 10 |  |  |  |  |  | | 16 |  | | 18 |  |  |  | | 22 |  |  |  |  |  | | 28 |  | | 30 |  |  |  | | 34 |  |  |  |  |  |  |  | 42 | |  | |
| **J** | T1,1 | | | | T2,1 | | T2,2 | | | | T1,2 | | | | | | T2,3 | | | T2,4 | | | | | T1,3 | | | | | | | T2,5 | | | T2,6 | | | | | T3,1 | | | | | | | | |  |  | |  |

1. **Task set density**

The density of the task system is defined as the sum of the densities of all the tasks in the system. Suppose there is a system of independent preemptable tasks whose density is <= 1 but it is not schedulable on a uniprocessor. Not being schedulable means that there is at least one task which misses its deadline in any order that tasks are scheduled. Finish this one later!

1. **RM and EDF schedulability tests**

Since all 3 systems have 3 tasks, the expression from Schedulability Test 2 = 3 \* (21/3 – 1) = 0.78

System (a):   
U = (4 / 10) + (4 / 16) + (4 / 20) = 0.4 + 0.25 + 0.2 = 0.85  
Yes, system (a) is EDF schedulable by Schedulability Test 4.

U > 0.78, so we need to try Schedulability Test 3 to check if the system is RM schedulable.

* J1 is RM schedulable because c1 = 4 <= 10 = p1.
* J2 is RM schedulable because c1 + c2 = 8 <= 10 = p1.
* J3 is RM schedulable because 2\*c1 + c2 + c3 = 16 <= 16 = p2.

So yes, system (a) is RM schedulable.

System (b):  
U = (2 / 5) + (3 / 10) + (3 / 12) = 0.4 + 0.3 + 0.25 = 0.95  
Yes, system (b) is EDF schedulable by Schedulability Test 4.

* J1 is RM schedulable because c1 = 2 <= 5 = p1.
* J2 is RM schedulable because c1 + c2 = 5 <= 5 = p1.
* J3 is RM schedulable because 2\*c1 + c2 + c3 = 10 <= 10 = p2.

So yes, system (b) is RM schedulable.

System (c):  
U = (3 / 7) + (5 / 9) + (3 / 15) = 0.43 + 0.45 + 0.2 = 1.08  
No, system (c) is not EDF schedulable, by Schedulability Test 4.

System (c) is not RM schedulable either, since EDF is an optimal scheduler.

1. **Constructing task set**

This problem is equivalent to creating a task set that is RM schedulable, since EDF and LLF are optimal and can schedule any task set that is schedulable by another method.

J1: c = 2; p = 4  
J2: c = 2; p = 6  
J3: c = 2; p = 12

3\*c1 + 2\*c2 + c3 = 6 + 4 + 2 = 12 <= 12 = p3 so the task set is RM schedulable.

1. **Non-simple schedulable utilization (RM schedulability test 3)**

The task set shown in problem (8) does not pass schedulability test 2 because its utilization is (2/4) + (2/6) + (2/12) = 0.50 + 0.33 + 0.17 = 1.00, which is greater than the utilization for a set of 3 tasks = 0.78 (already calculated in problem (7)). But it is RM schedulable, as shown in problem (8).

1. **Multiprocessor scheduling**

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1. **RM and EDF equivalence**

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1. **EDF loss of optimal**

…

1. **RM schedulability test 2/3**

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