

# Computer Vision – Lecture 18

## Repetition

**09.07.2019**

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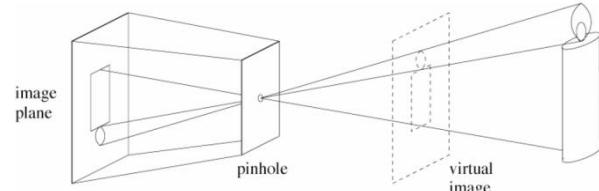
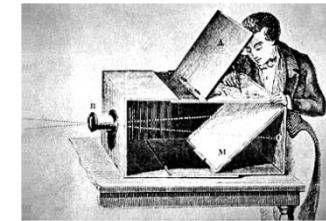
[leibe@vision.rwth-aachen.de](mailto:leibe@vision.rwth-aachen.de)

# Announcements

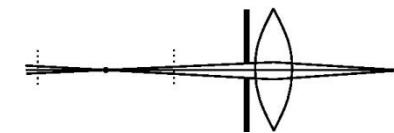
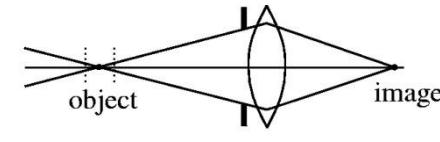
- Today, I'll summarize the most important points from the lecture.
  - It is an opportunity for you to ask questions...
  - ...or get additional explanations about certain topics.
  - So, *please do ask*.
- Today's slides are intended as an index for the lecture.
  - But they are not complete, won't be sufficient as only tool.
  - Also look at the exercises – they often explain algorithms in detail.

# Repetition

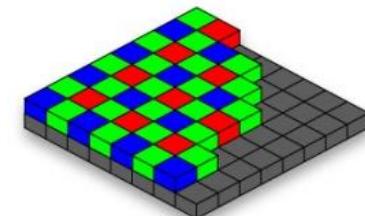
- Image Processing Basics
  - Image Formation
  - Linear Filters
  - Edge & Structure Extraction
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
- 3D Reconstruction



*Pinhole camera model*



*Lenses, focal length, aperture*

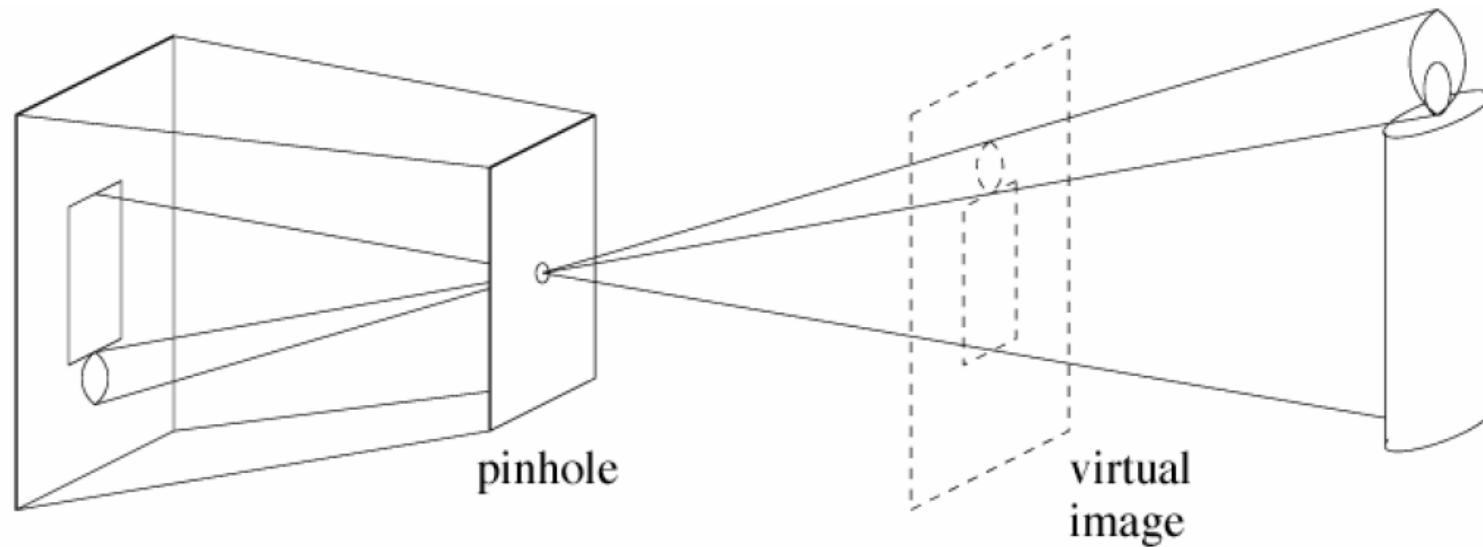


*Color sensors*

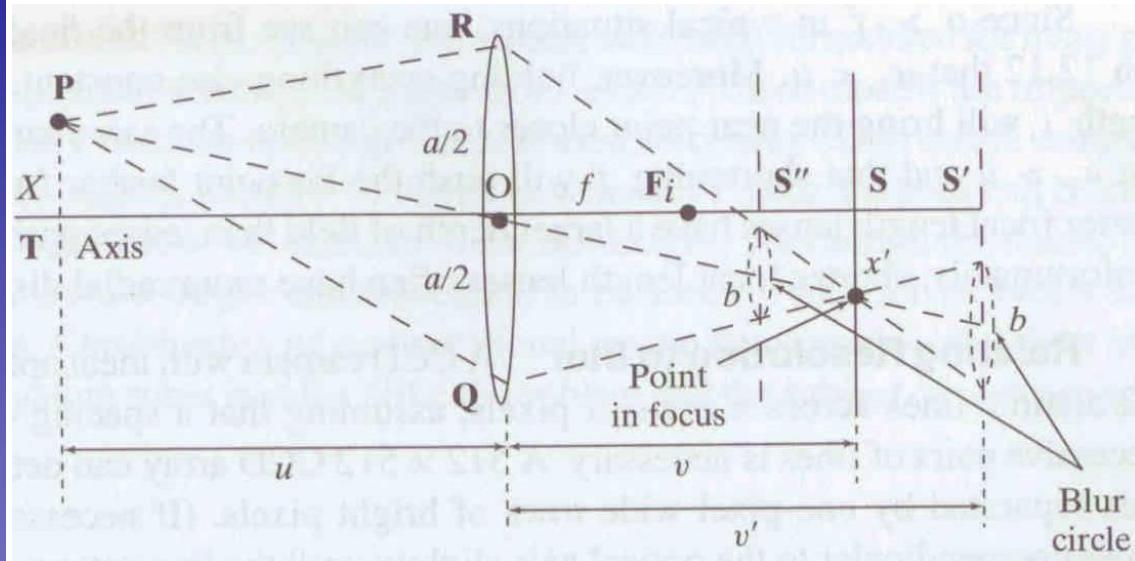
# Recap: Pinhole Camera

- (Simple) standard and abstract model today
  - Box with a small hole in it
  - Works in practice

Computer Vision Summer'19



# Recap: Focus and Depth of Field



“circles of confusion”

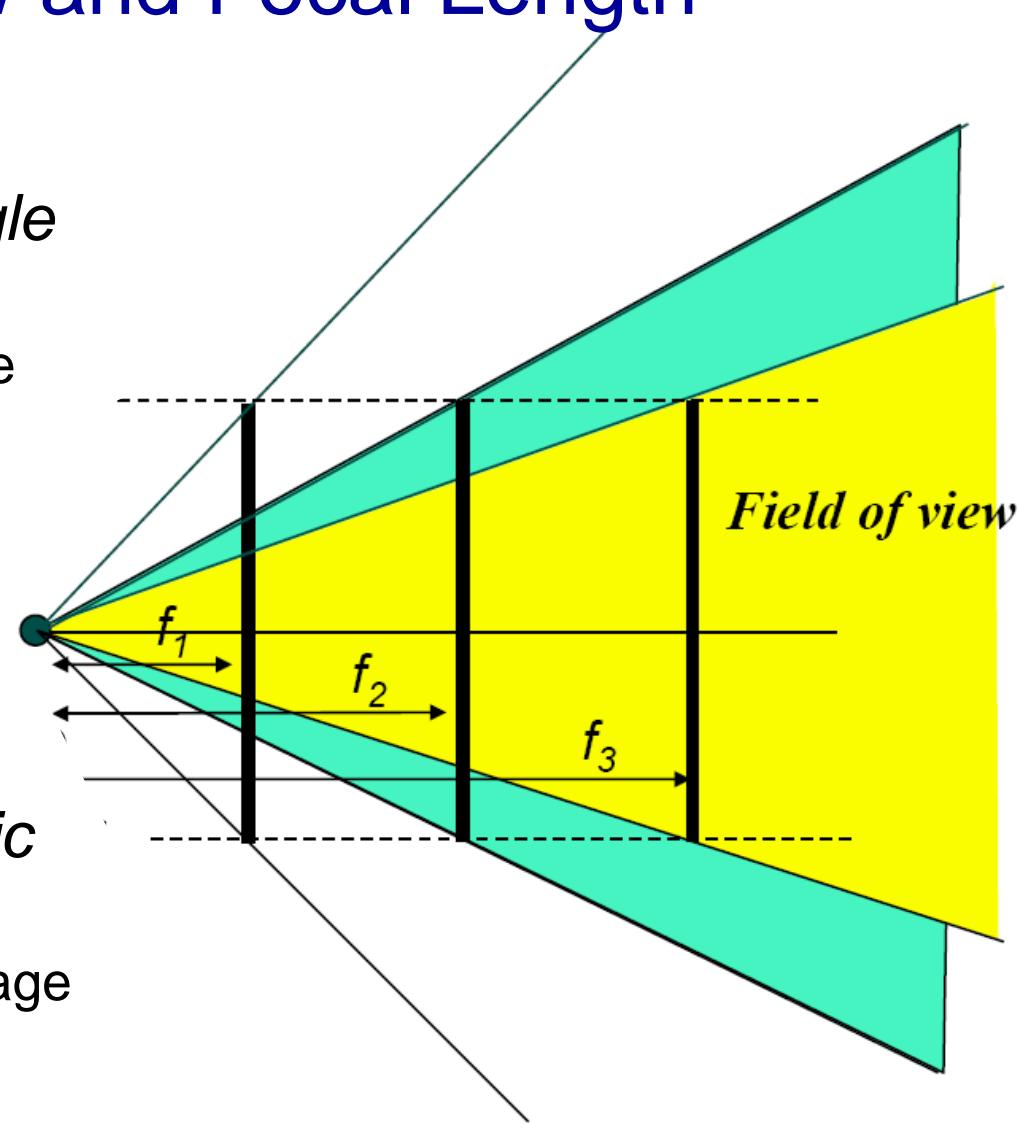
Thin lens: scene points at distinct depths come in focus at different image planes.

(Real camera lens systems have greater depth of field.)

- Depth of field: distance between image planes where blur is tolerable

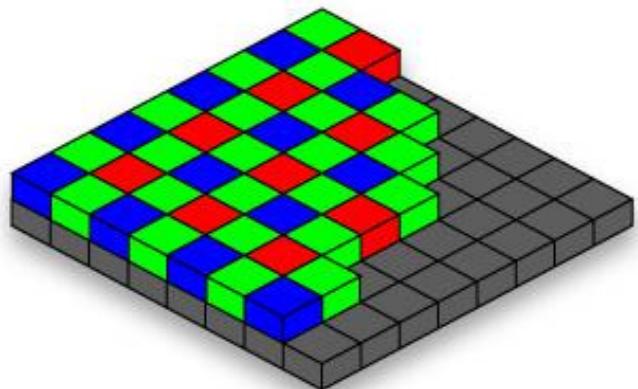
# Recap: Field of View and Focal Length

- As  $f$  gets smaller, image becomes more *wide angle*
  - More world points project onto the finite image plane
- As  $f$  gets larger, image becomes more *telescopic*
  - Smaller part of the world projects onto the finite image plane

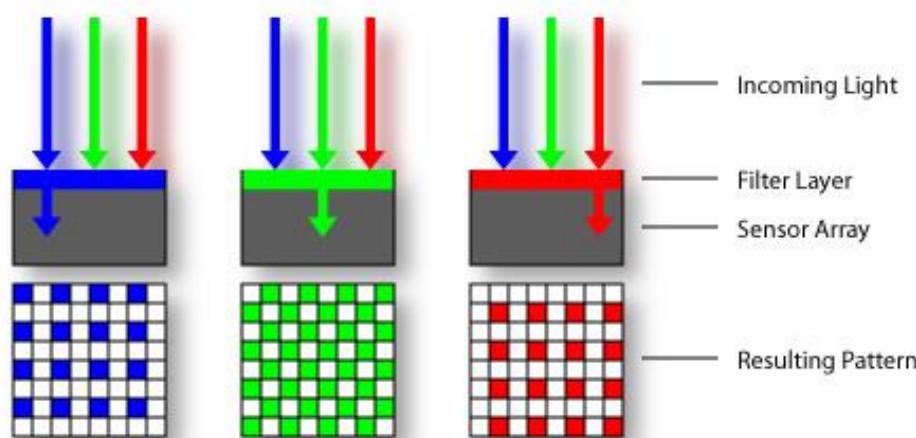
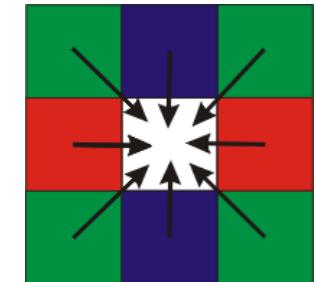


# Recap: Color Sensing in Digital Cameras

Bayer grid

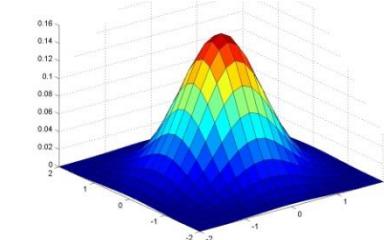


Estimate missing components from neighboring values (demosaicing)

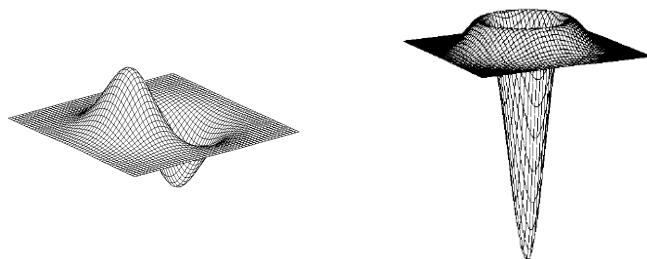


# Repetition

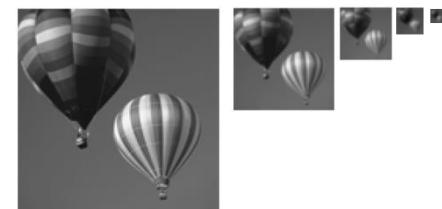
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  - **Linear Filters**
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*Gaussian Smoothing*



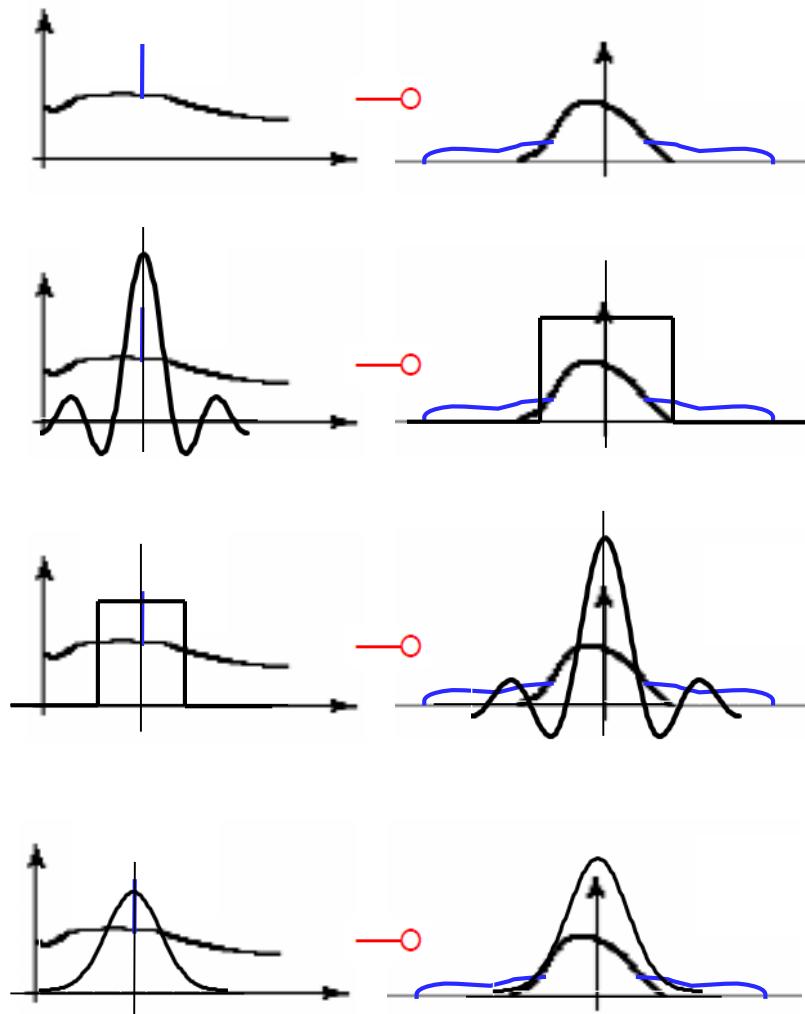
*Derivative operators*



*Gaussian/Laplacian pyramid*

# Recap: Effect of Filtering

- Noise introduces high frequencies. To remove them, we want to apply a “low-pass” filter.
- The ideal filter shape in the frequency domain would be a box. But this transfers to a spatial sinc, which has infinite spatial support.
- A compact spatial box filter transfers to a frequency sinc, which creates artifacts.
- A Gaussian has compact support in both domains. This makes it a convenient choice for a low-pass filter.

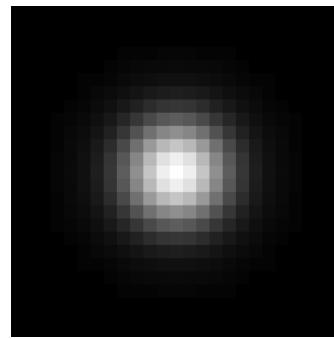
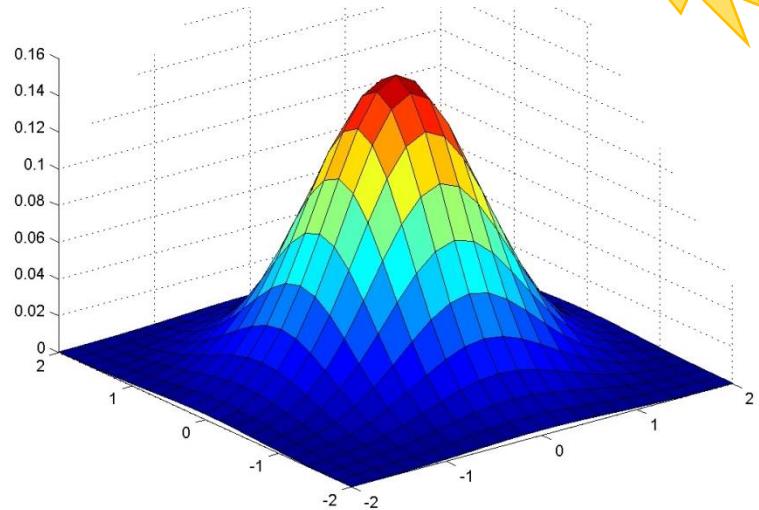


# Recap: Gaussian Smoothing

- Gaussian kernel

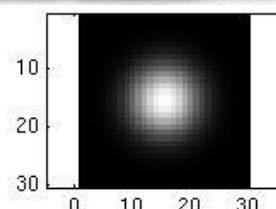
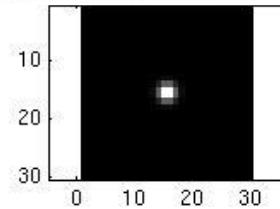
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

- Rotationally symmetric
- Weights nearby pixels more than distant ones
  - This makes sense as ‘probabilistic’ inference about the signal
- A Gaussian gives a good model of a fuzzy blob

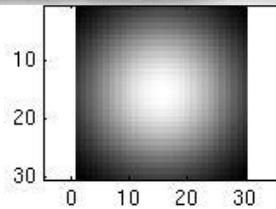


# Recap: Smoothing with a Gaussian

- Parameter  $\sigma$  is the “scale” / “width” / “spread” of the Gaussian kernel and controls the amount of smoothing.

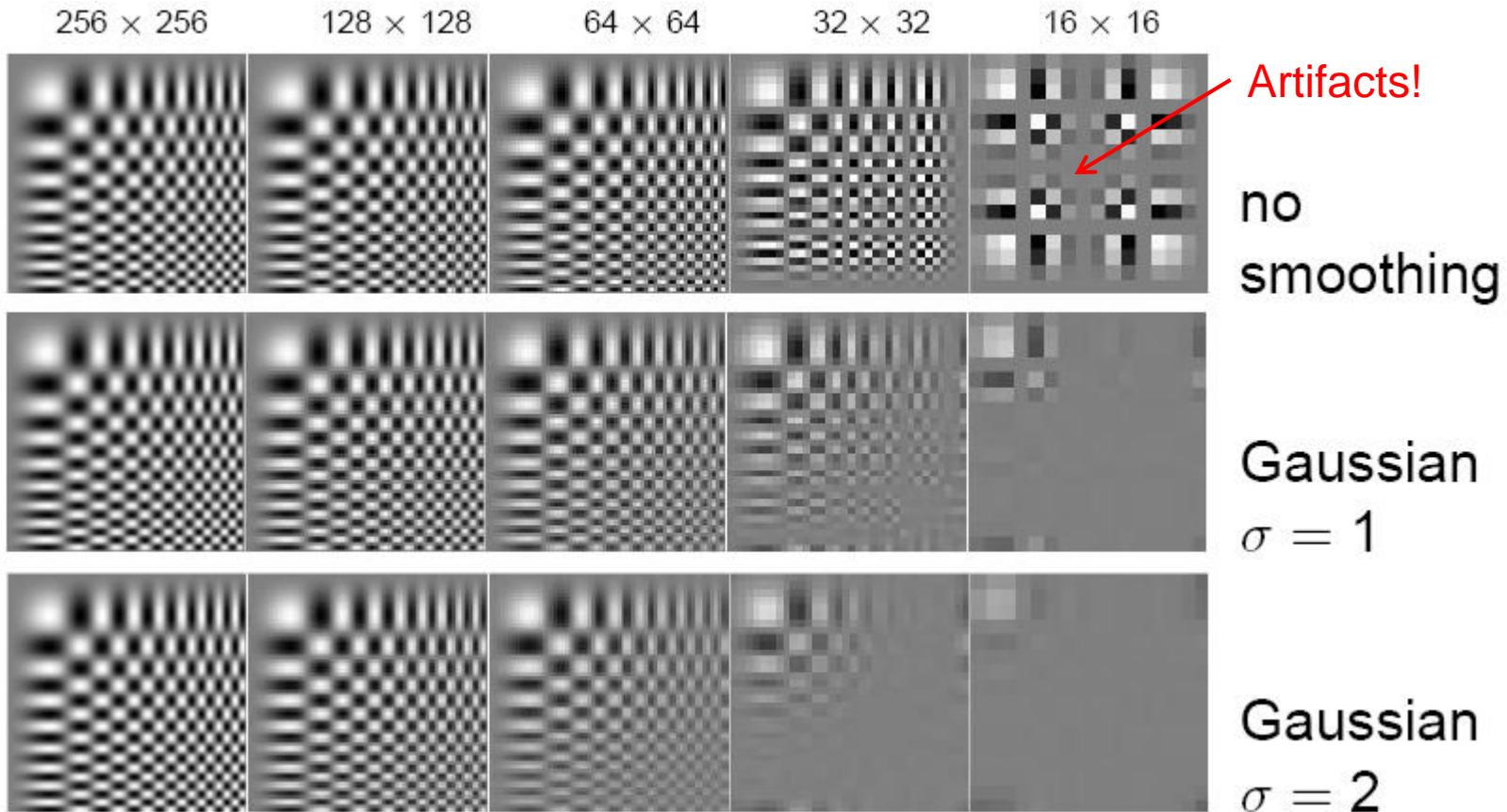


...



```
for sigma=1:3:10
    h = fspecial('gaussian', fsize, sigma);
    out = imfilter(im, h);
    imshow(out);
    pause;
end
```

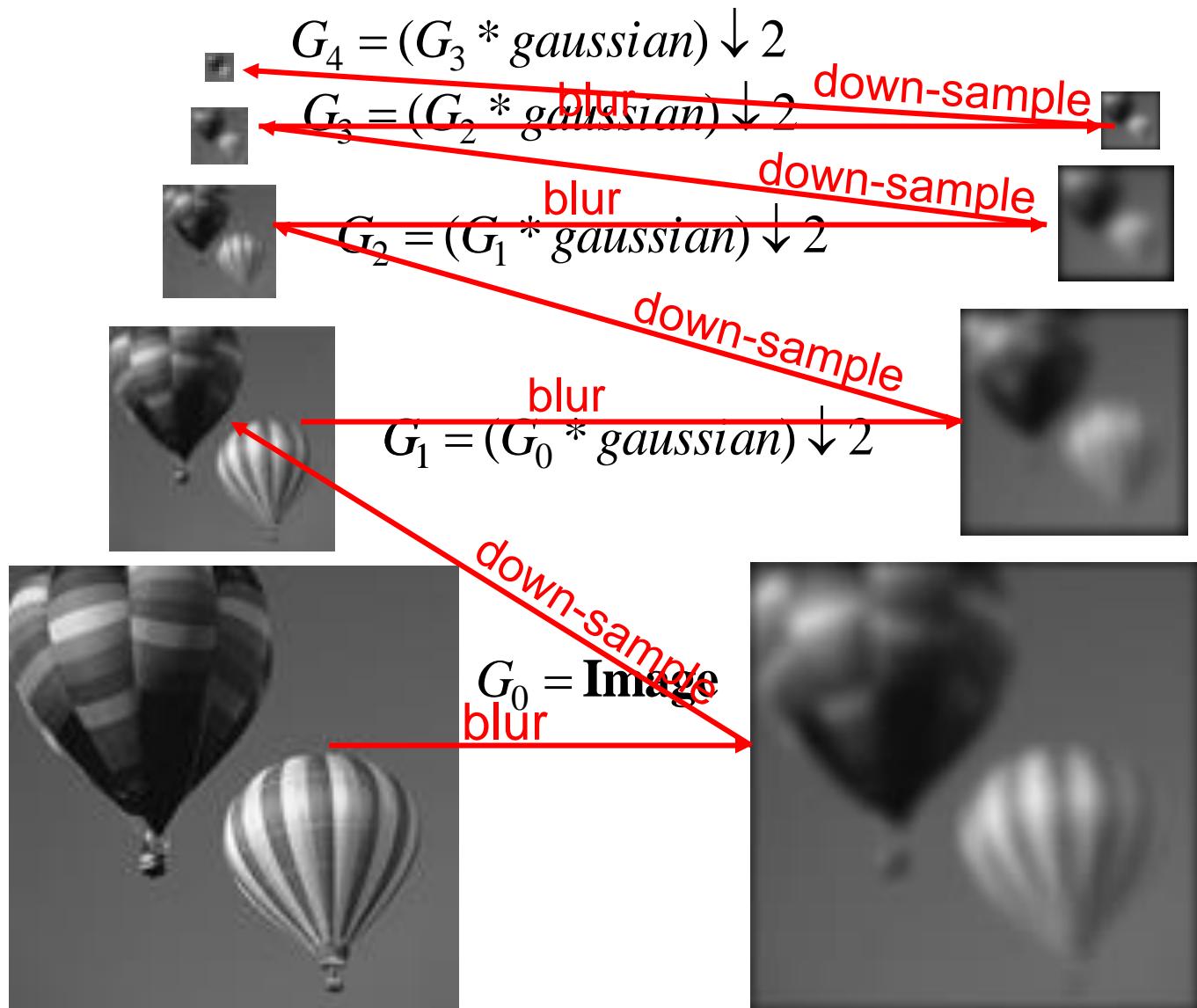
# Recap: Resampling with Prior Smoothing



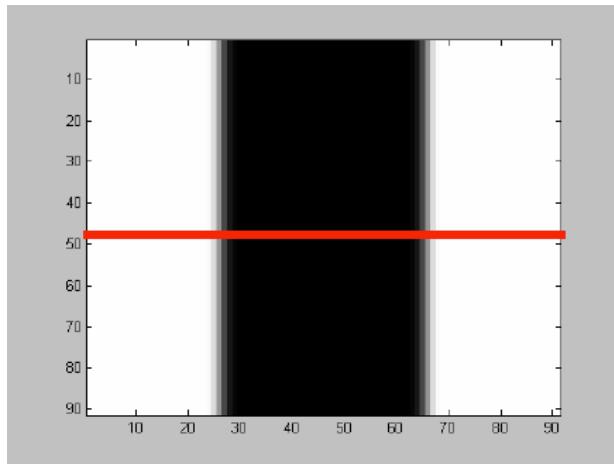
- Note: We cannot recover the high frequencies, but we can avoid artifacts by smoothing before resampling.

# Recap: The Gaussian Pyramid

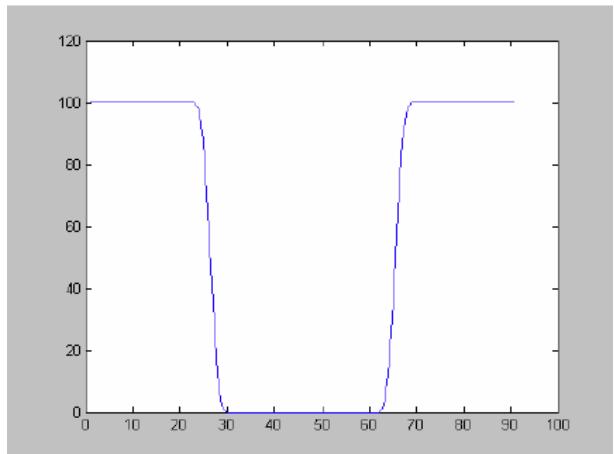
Low resolution



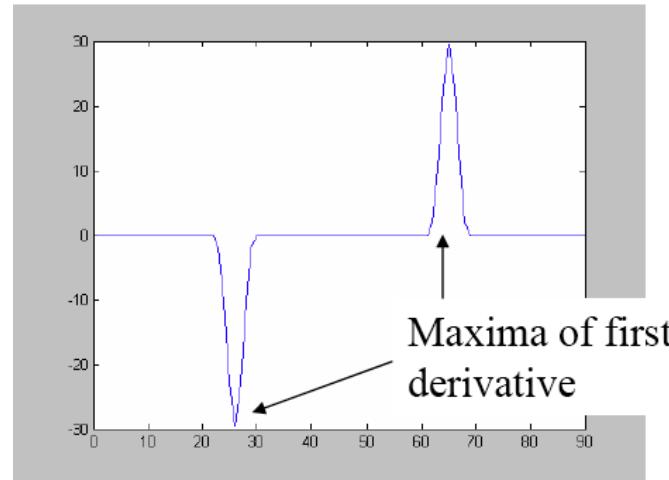
# Recap: Derivatives and Edges...



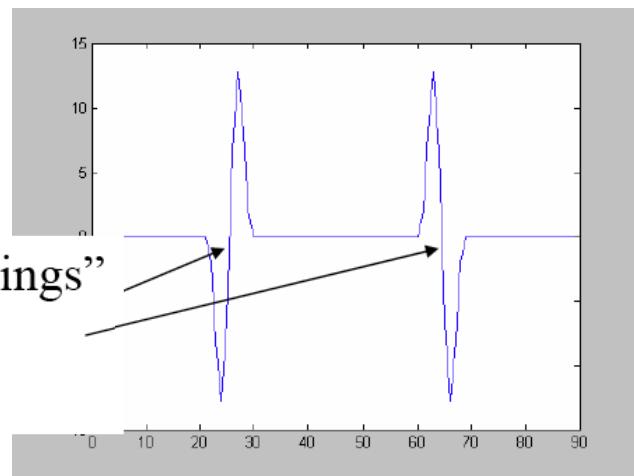
1st derivative



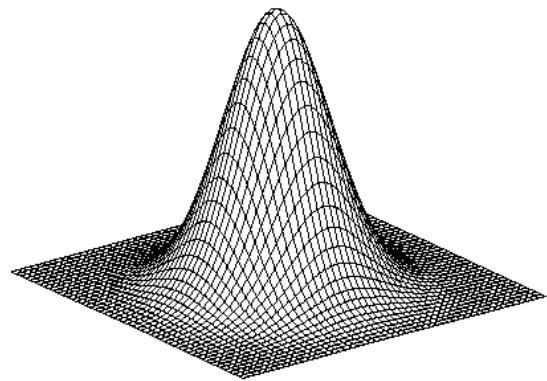
2nd derivative



“zero crossings”  
of second  
derivative

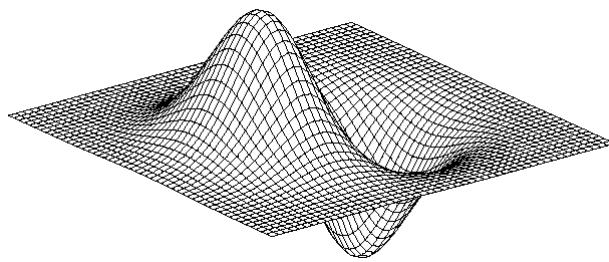
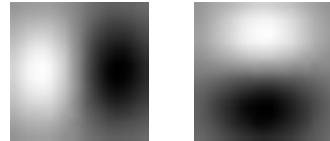


# Recap: 2D Edge Detection Filters



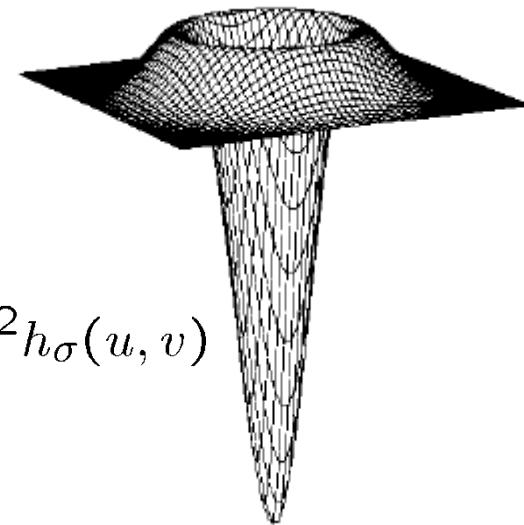
Gaussian

$$h_\sigma(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



Derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u, v)$$



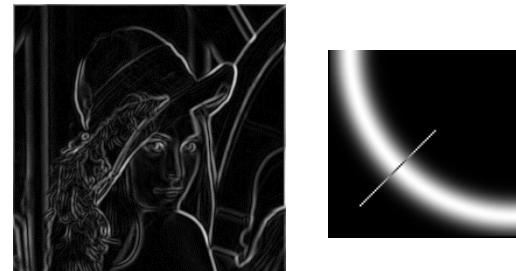
Laplacian of Gaussian

- $\nabla^2$  is the Laplacian operator:

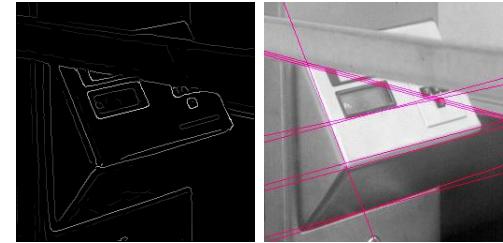
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

# Repetition

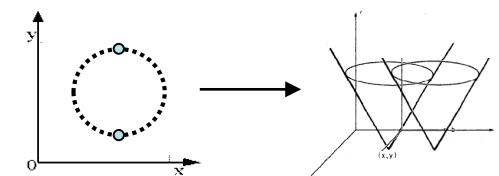
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*Canny edge detector*



*Hough transform for lines*



*Hough transform for circles*

# Recap: Canny Edge Detector

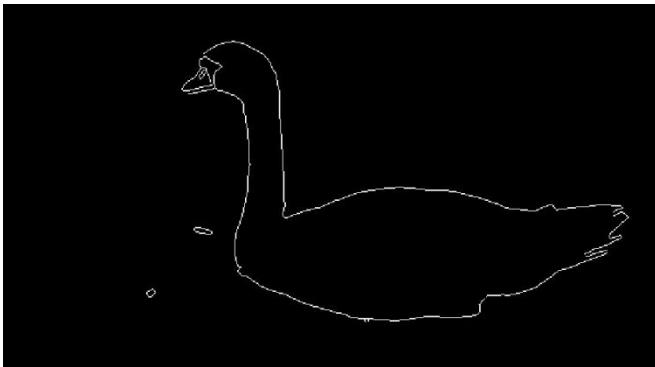
1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
  - Thin multi-pixel wide “ridges” down to single pixel width
4. Linking and thresholding (hysteresis):
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB:

```
>> edge(image, 'canny');  
>> help edge
```



# Recap: Edges vs. Boundaries



Edges useful signal to indicate occluding boundaries, shape.

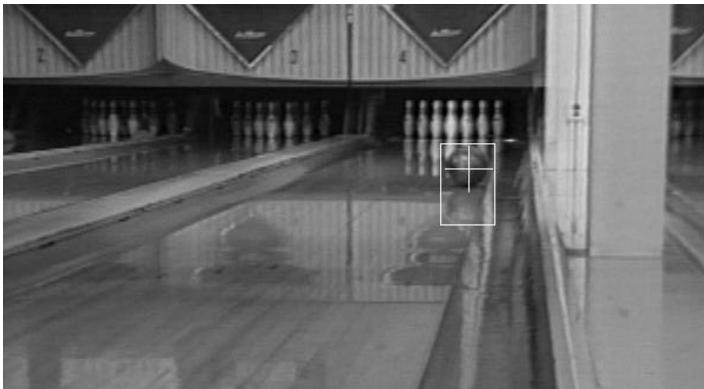
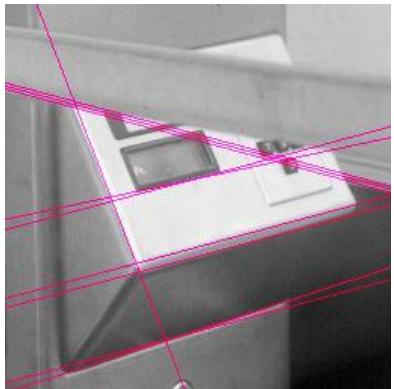
Here the raw edge output is not so bad...

Slide credit: Kristen Grauman



...but quite often boundaries of interest are fragmented, and we have extra “clutter” edge points.

# Recap: Fitting and Hough Transform



Given a model of interest, we can overcome some of the missing and noisy edges using fitting techniques.



With voting methods like the Hough transform, detected points vote on possible model parameters.

# Recap: Hough Transform

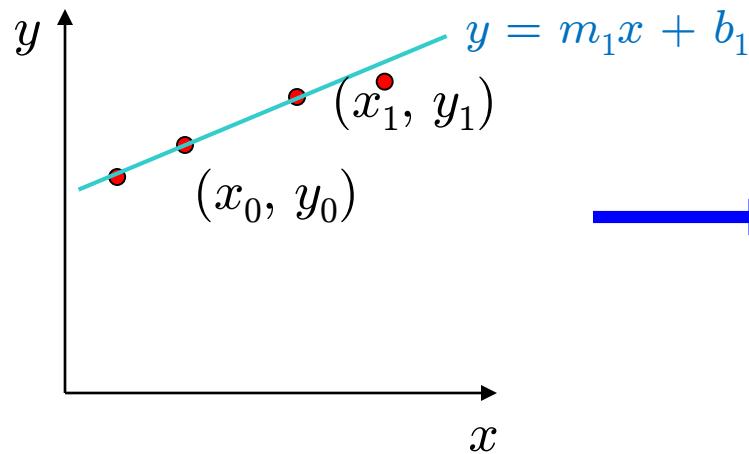
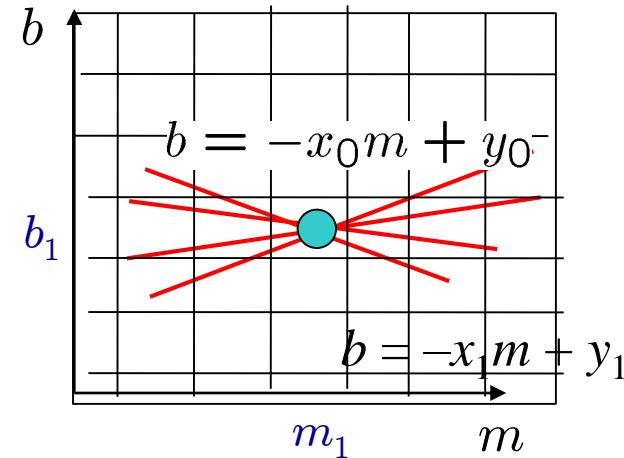


Image space

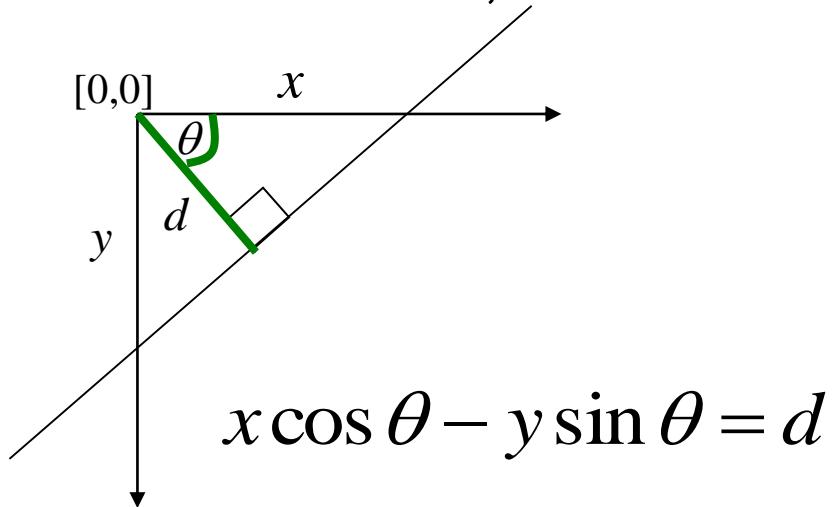


Hough (parameter) space

- How can we use this to find the most likely parameters ( $m, b$ ) for the most prominent line in the image space?
  - Let each edge point in image space *vote* for a set of possible parameters in Hough space
  - Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

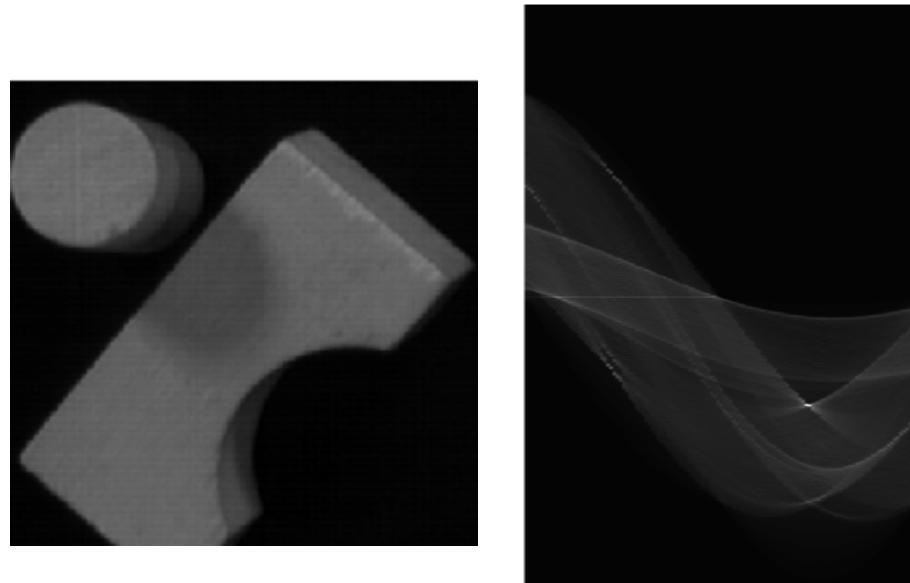
# Recap: Hough Transf. Polar Parametrization

- Usual  $(m, b)$  parameter space problematic: can take on infinite values, undefined for vertical lines.



$d$  : perpendicular distance from line to origin

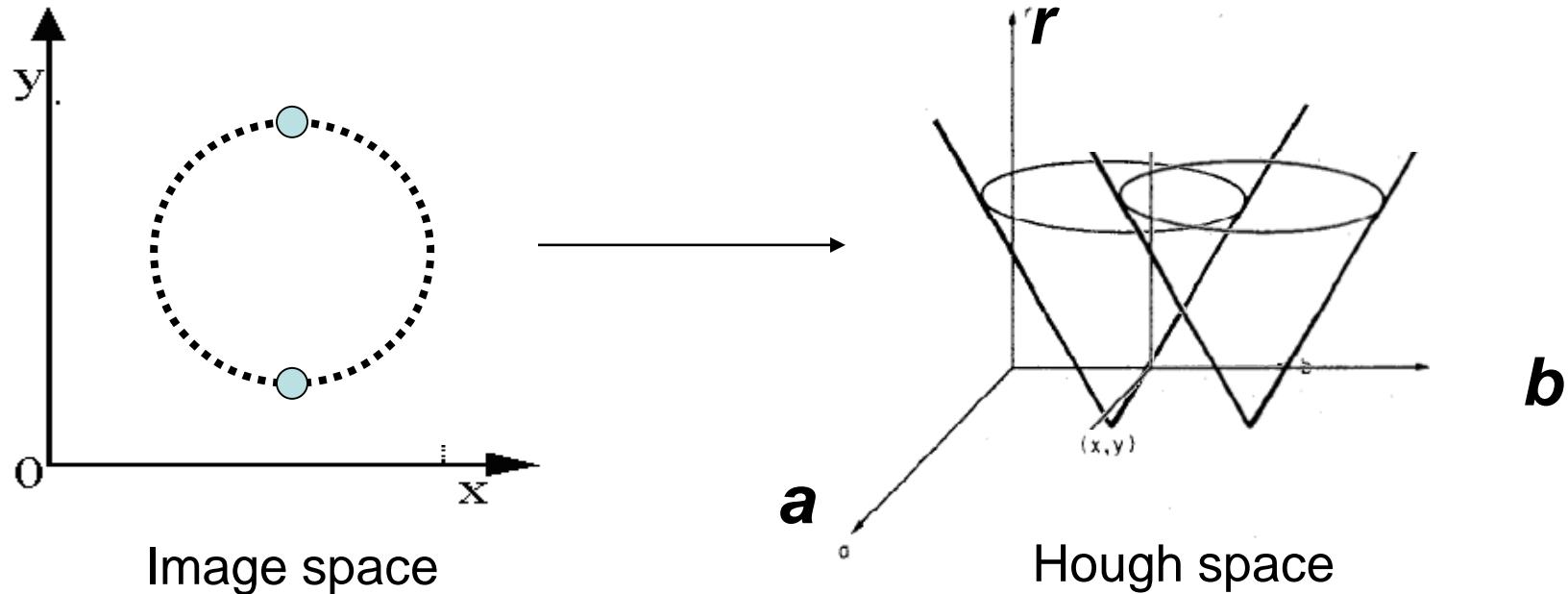
$\theta$  : angle the perpendicular makes with the x-axis



see  
Exercise 1.5!

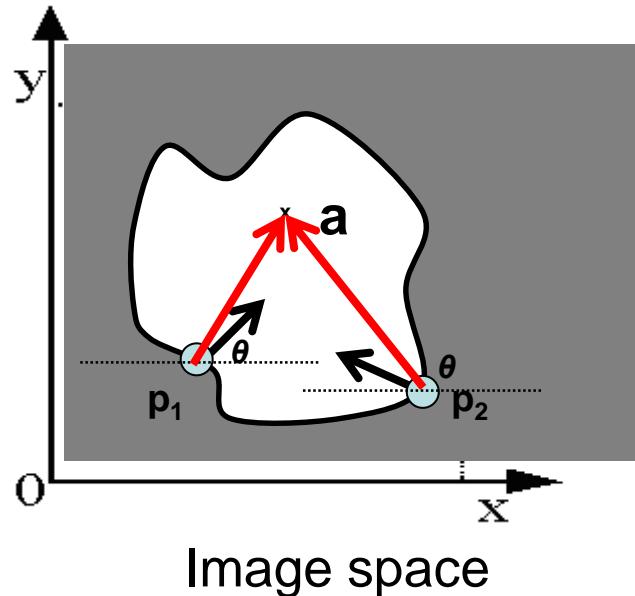
# Recap: Hough Transform for Circles

- Circle: center  $(a, b)$  and radius  $r$   
$$(x_i - a)^2 + (y_i - b)^2 = r^2$$
- For an unknown radius  $r$ , unknown gradient direction



# Recap: Generalized Hough Transform

- What if want to detect arbitrary shapes defined by boundary points and a reference point?



At each boundary point,  
compute displacement vector:

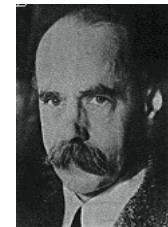
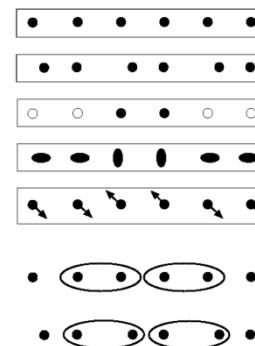
$$\mathbf{r} = \mathbf{a} - \mathbf{p}_i$$

For a given model shape:  
store these vectors in a table  
indexed by gradient  
orientation  $\theta$ .

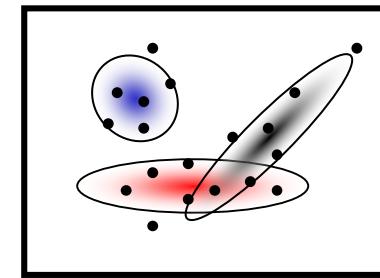
D.H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980.

# Repetition

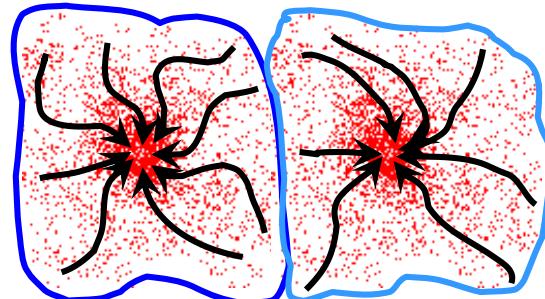
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Gestalt factors



K-Means & EM clustering



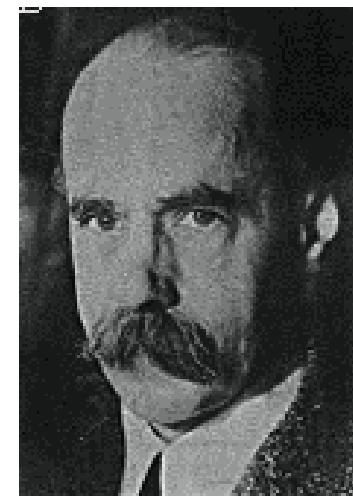
Mean-shift clustering

# Recap: Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

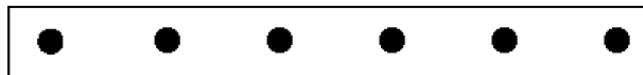
*"I stand at the window and see a house, trees, sky.  
Theoretically I might say there were 327 brightnesses  
and nuances of colour. Do I have "327"? No. I have sky,  
house, and trees."*

Max Wertheimer  
(1880-1943)



Untersuchungen zur Lehre von der Gestalt,  
*Psychologische Forschung*, Vol. 4, pp. 301-350, 1923  
<http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm>

# Recap: Gestalt Factors



Not grouped



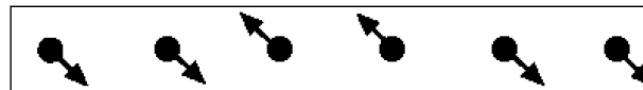
Proximity



Similarity



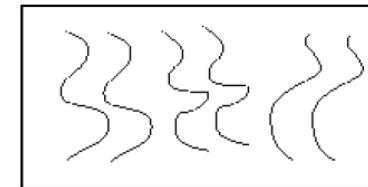
Similarity



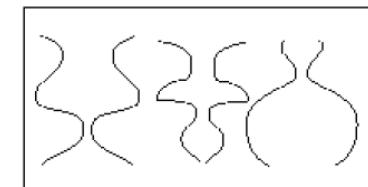
Common Fate



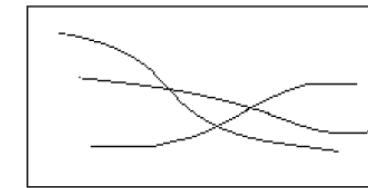
Common Region



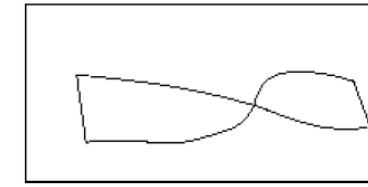
Parallelism



Symmetry



Continuity

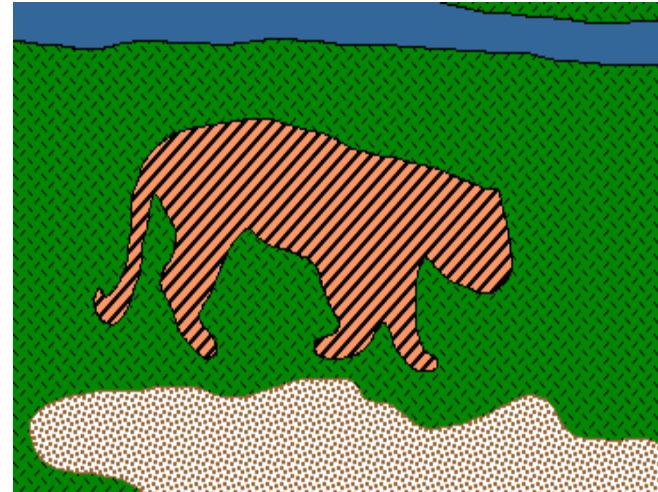


Closure

- These factors make intuitive sense, but are very difficult to translate into algorithms.

# Recap: Image Segmentation

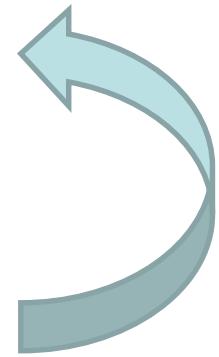
- Goal: identify groups of pixels that go together



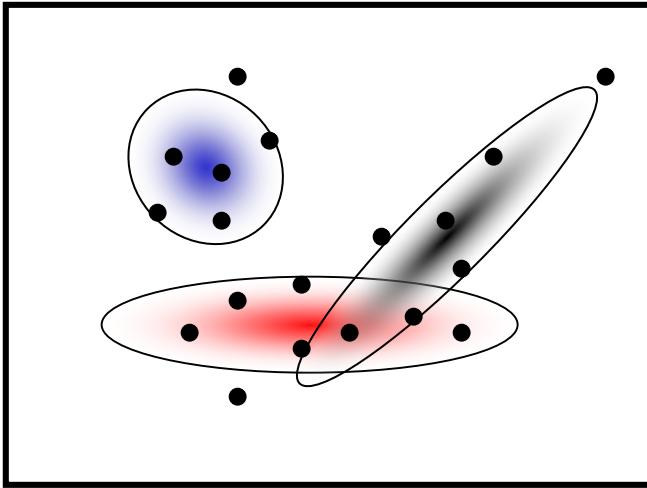
# Recap: K-Means Clustering

- Basic idea: randomly initialize the  $k$  cluster centers, and iterate between the two following steps
  1. Randomly initialize the cluster centers,  $c_1, \dots, c_K$
  2. Given cluster centers, determine points in each cluster
    - For each point  $p$ , find the closest  $c_i$ . Put  $p$  into cluster  $i$
  3. Given points in each cluster, solve for  $c_i$ 
    - Set  $c_i$  to be the mean of points in cluster  $i$
  4. If  $c_i$  have changed, repeat Step 2
- Properties
  - Will always converge to *some* solution
  - Can be a “local minimum”
    - Does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points } p \text{ in cluster } i} \|p - c_i\|^2$$

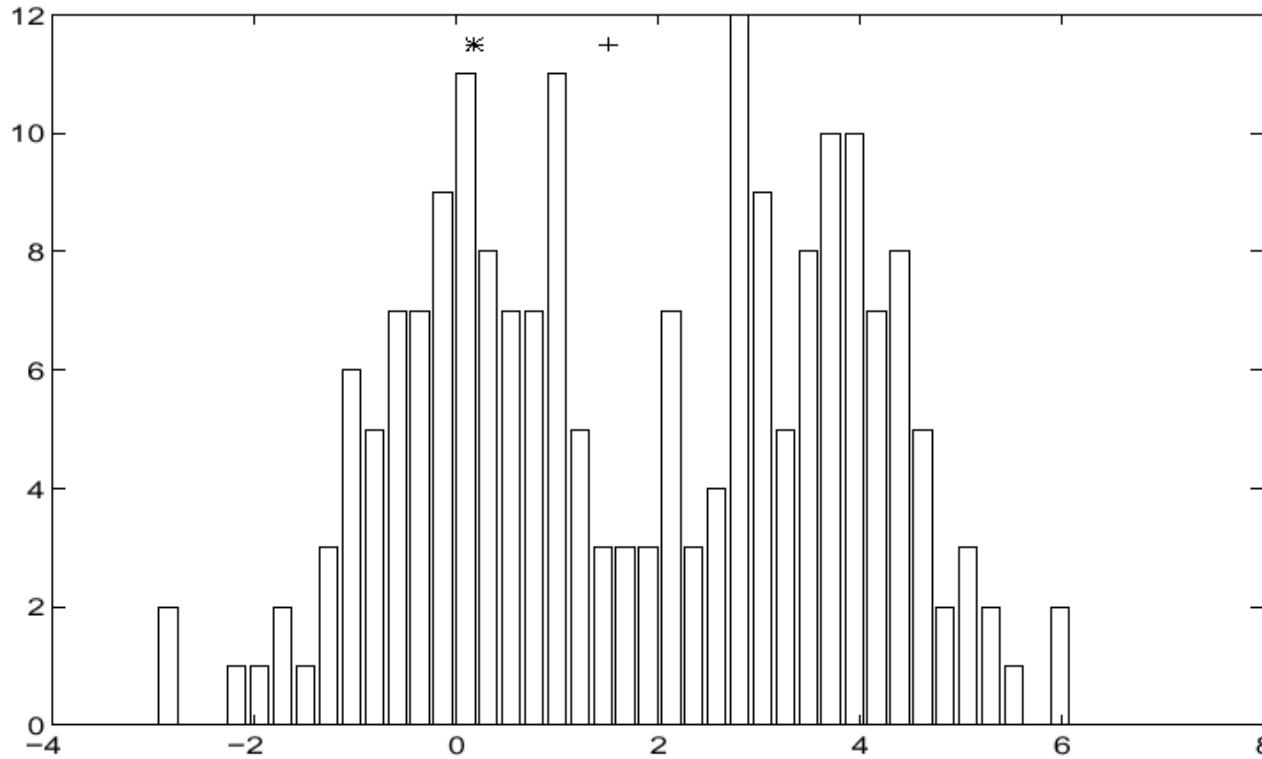


# Recap: Expectation Maximization (EM)



- Goal
  - Find blob parameters  $\theta$  that maximize the likelihood function:
$$p(\text{data}|\theta) = \prod_{n=1}^N p(\mathbf{x}_n|\theta)$$
- Approach:
  1. E-step: given current guess of blobs, compute ownership of each point
  2. M-step: given ownership probabilities, update blobs to maximize likelihood function
  3. Repeat until convergence

# Recap: Mean-Shift Algorithm

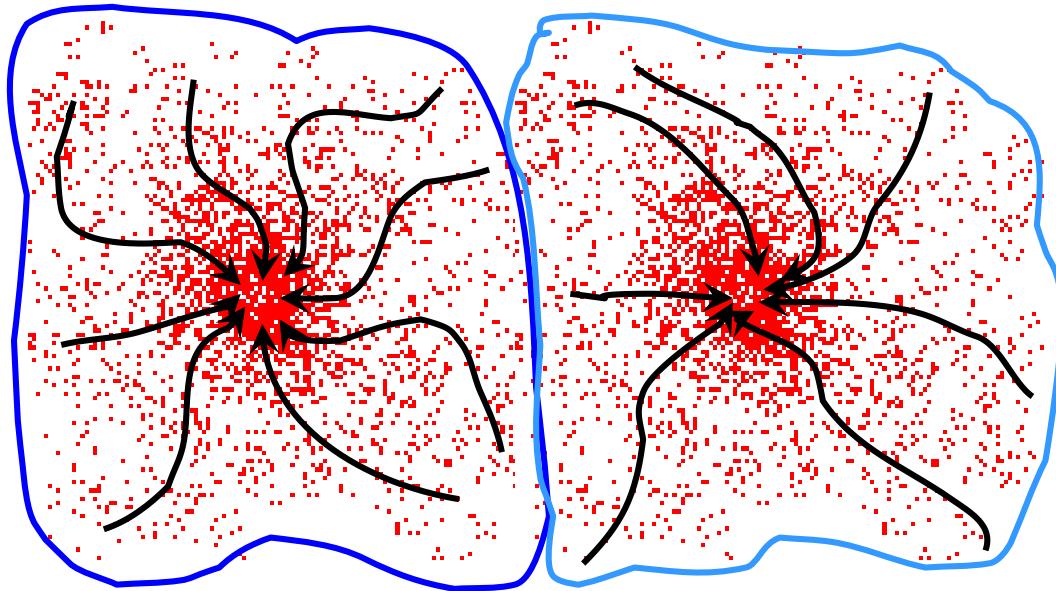


- Iterative Mode Search
  1. Initialize random seed, and window  $W$
  2. Calculate center of gravity (the “mean”) of  $W$ :
  3. Shift the search window to the mean
  4. Repeat Step 2 until convergence

$$\sum_{x \in W} x H(x)$$

# Recap: Mean-Shift Clustering

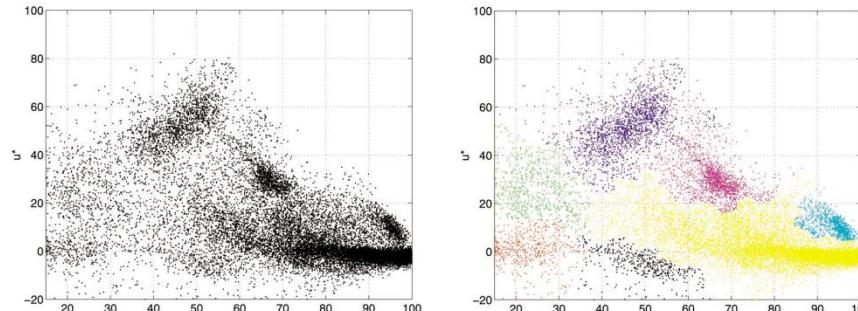
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



# Recap: Mean-Shift Segmentation

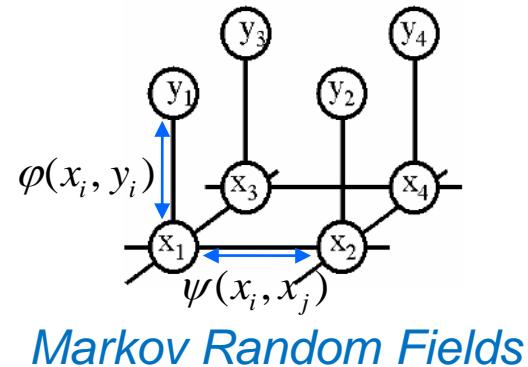
**see  
Exercise 2.1!**

- Find features (color, gradients, texture, etc)
  - Initialize windows at individual pixel locations
  - Perform mean shift for each window until convergence
  - Merge windows that end up near the same “peak” or mode

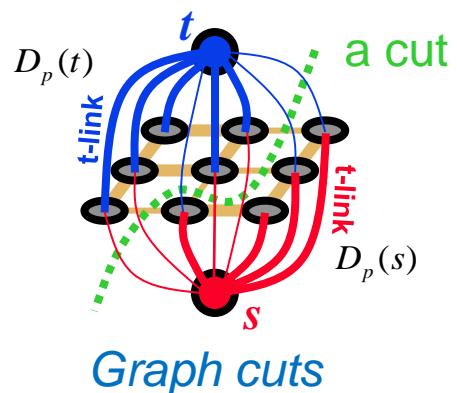


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- 3D Reconstruction



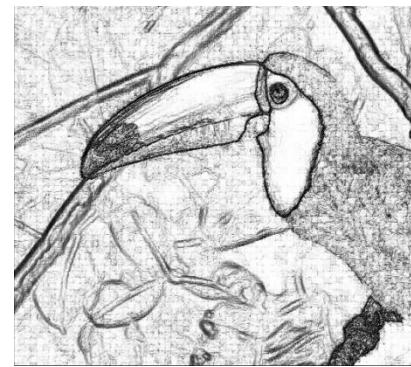
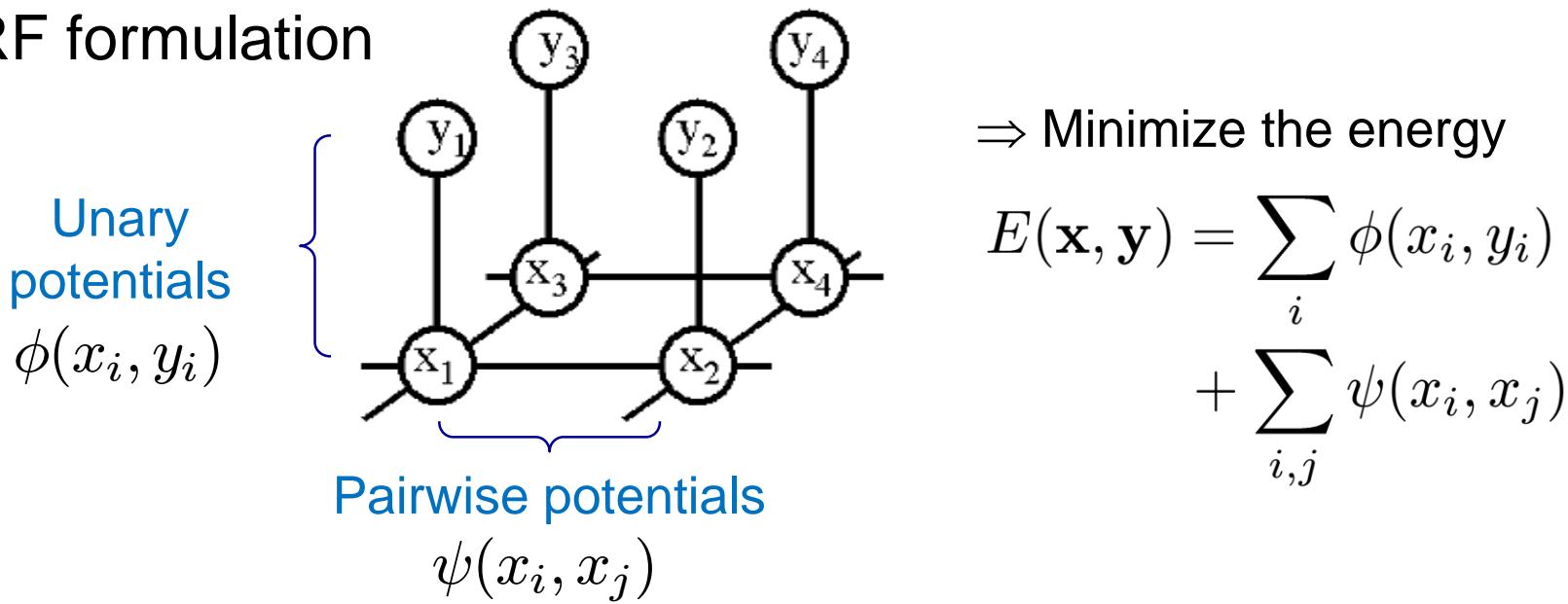
Markov Random Fields



Graph cuts

# Recap: MRFs for Image Segmentation

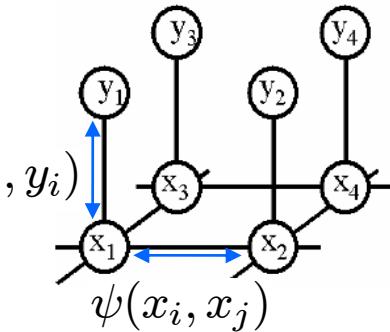
- MRF formulation



# Recap: Energy Formulation

- Energy function

$$E(\mathbf{x}, \mathbf{y}) = \underbrace{\sum_i \phi(x_i, y_i)}_{\text{Unary potentials}} + \underbrace{\sum_{i,j} \psi(x_i, x_j)}_{\text{Pairwise potentials}}$$

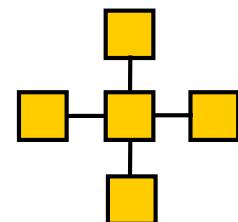


- Unary potentials  $\phi$

- Encode local information about the given pixel/patch
- How likely is a pixel/patch to belong to a certain class (e.g. foreground/background)?

- Pairwise potentials  $\psi$

- Encode neighborhood information
- How different is a pixel/patch's label from that of its neighbor? (e.g. based on intensity/color/texturedifference, edges)

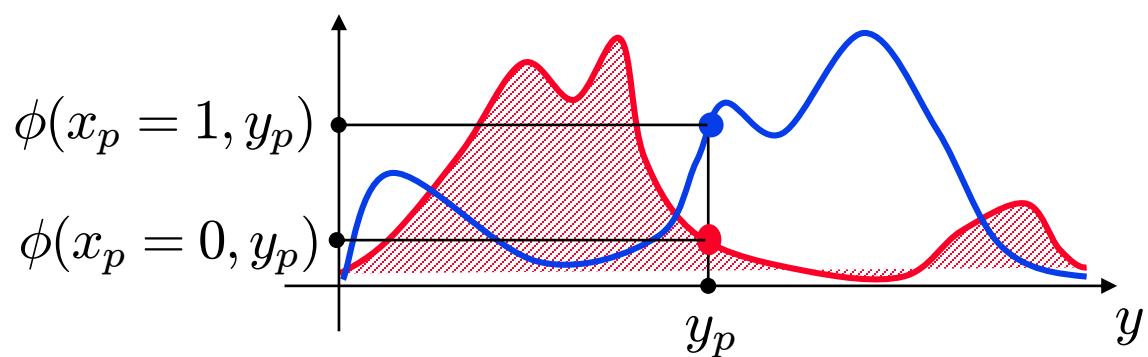


# Recap: How to Set the Potentials?

- Unary potentials
  - E.g. color model, modeled with a **Mixture of Gaussians**

$$\phi(x_i, y_i; \theta_\phi) = \log \sum_k \theta_\phi(x_i, k) p(k|x_i) \mathcal{N}(y_i; \bar{y}_k, \Sigma_k)$$

⇒ Learn color distributions for each label



# Recap: How to Set the Potentials?

- Pairwise potentials

- Potts Model

$$\psi(x_i, x_j; \theta_\psi) = \theta_\psi \delta(x_i \neq x_j)$$

- Simplest discontinuity preserving model.
    - Discontinuities between any pair of labels are penalized equally.
    - Useful when labels are unordered or number of labels is small.

- Extension: “Contrast sensitive Potts model”

$$\psi(x_i, x_j, g_{ij}(y); \theta_\psi) = \theta_\psi g_{ij}(y) \delta(x_i \neq x_j)$$

where

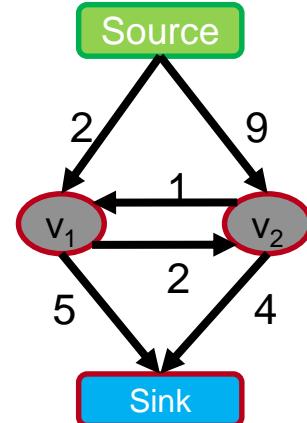
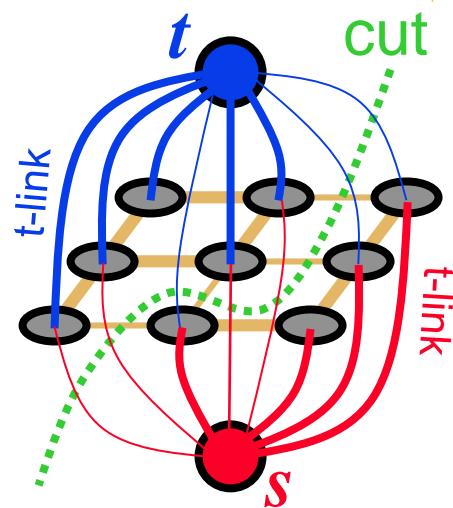
$$g_{ij}(y) = e^{-\beta \|y_i - y_j\|^2} \quad \beta = 2 / \text{avg}(\|y_i - y_j\|^2)$$

⇒ Discourages label changes except in places where there is also a large change in the observations.

# Recap: Graph-Cuts Energy Minimization

see  
Exercise 2.2!

- Solve an equivalent graph cut problem
  1. Introduce extra nodes: source and sink
  2. Weight connections to source/sink (t-links) by  $\phi(x_i = s)$  and  $\phi(x_i = t)$ , respectively.
  3. Weight connections between nodes (n-links) by  $\psi(x_i, x_j)$ .
  4. Find the minimum cost cut that separates source from sink.  
⇒ Solution is equivalent to minimum of the energy.
- s-t Mincut can be solved efficiently
  - Dual to the well-known max flow problem
  - Very efficient algorithms available for regular grid graphs (1-2 MPixels/s)
  - Globally optimal result for 2-class problems

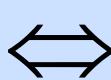


# Recap: When Can s-t Graph Cuts Be Applied?

$$E(L) = \sum_p \underset{\text{t-links}}{\text{Unary potentials}} E_p(L_p) + \sum_{pq \in N} \underset{\text{n-links}}{\text{Pairwise potentials}} E(L_p, L_q)$$
$$L_p \in \{s, t\}$$

- s-t graph cuts can only globally minimize **binary energies** that are **submodular**.  
[Boros & Hummer, 2002, Kolmogorov & Zabih, 2004]

$E(L)$  can be minimized  
by s-t graph cuts

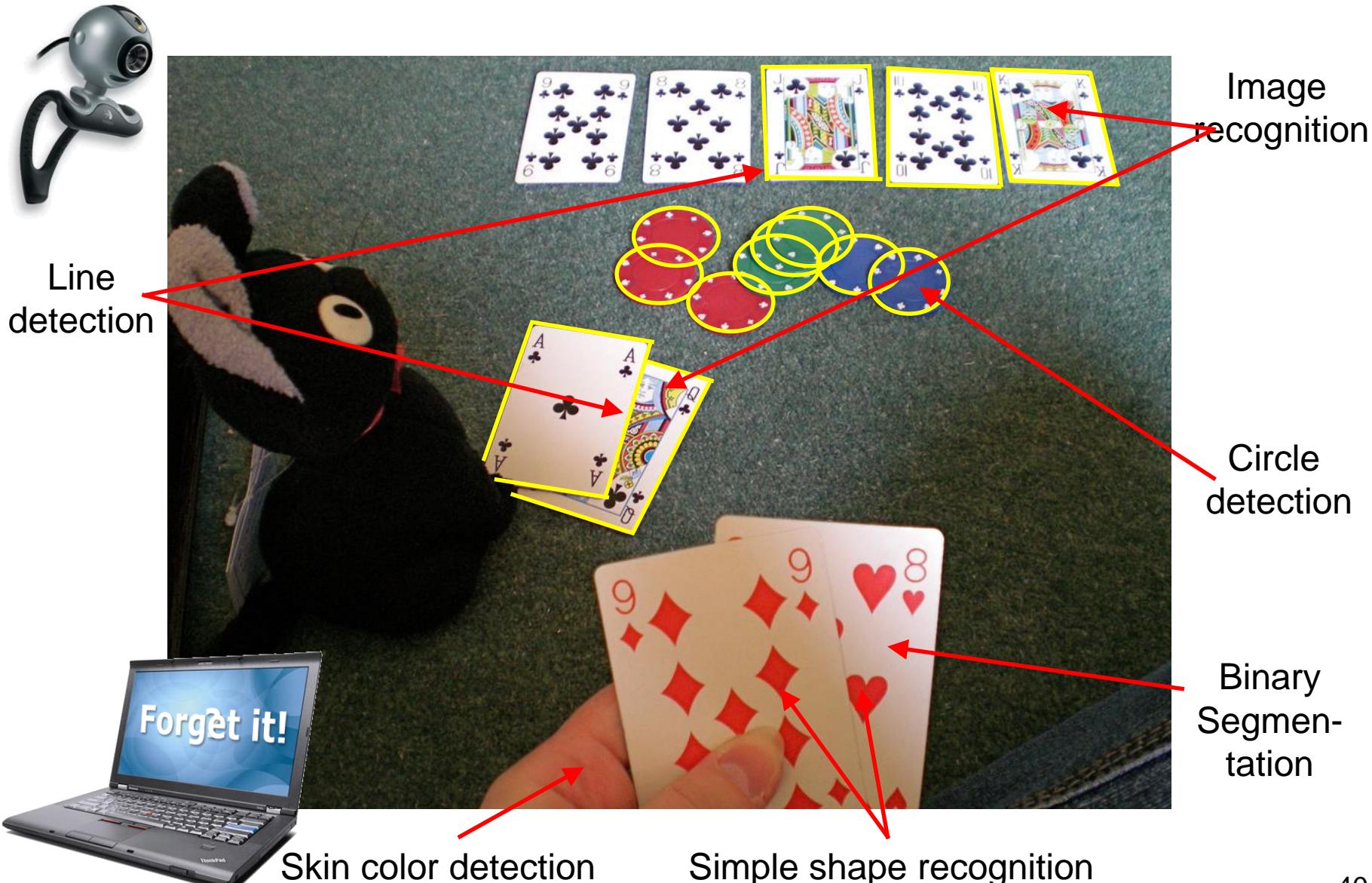


$$E(s, s) + E(t, t) \leq E(s, t) + E(t, s)$$

Submodularity (“convexity”)

- Submodularity is the discrete equivalent to convexity.
  - Implies that every local energy minimum is a global minimum.  
⇒ Solution will be globally optimal.

# First Applications Take Up Shape...

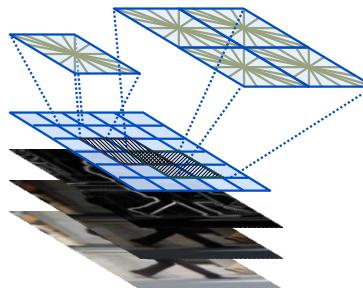
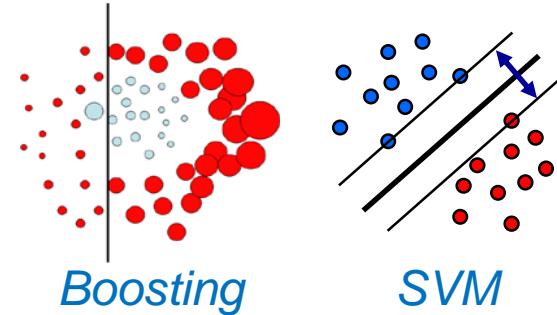


# Repetition

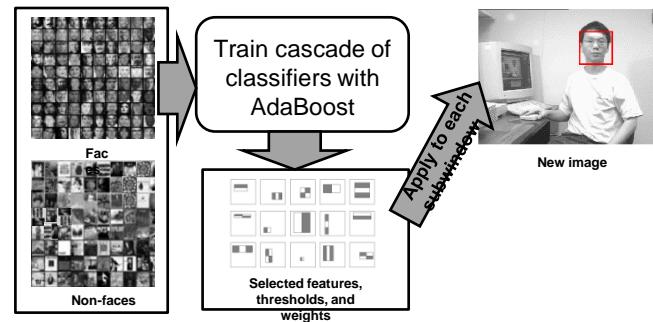
- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
  - Sliding Window based Object Detection
- Local Features & Matching
- Deep Learning
- 3D Reconstruction



*Sliding window principle*



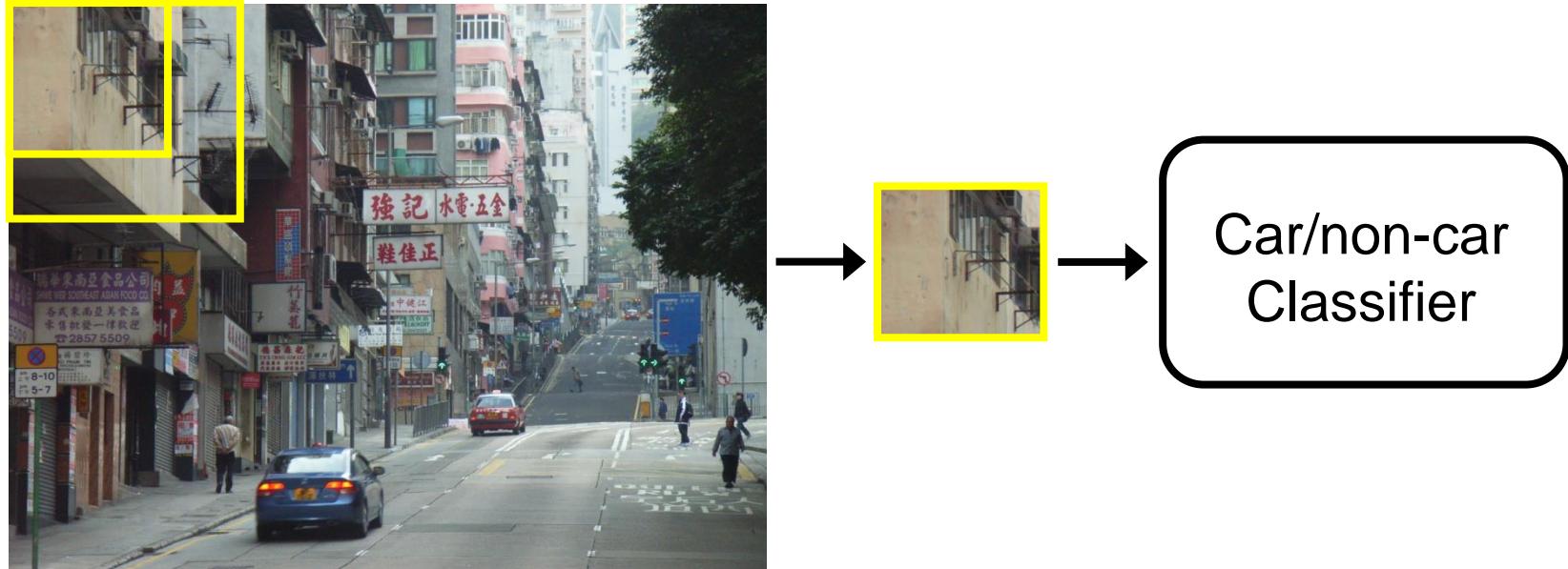
*HOG detector*



*Viola-Jones face detector*

# Recap: Sliding-Window Object Detection

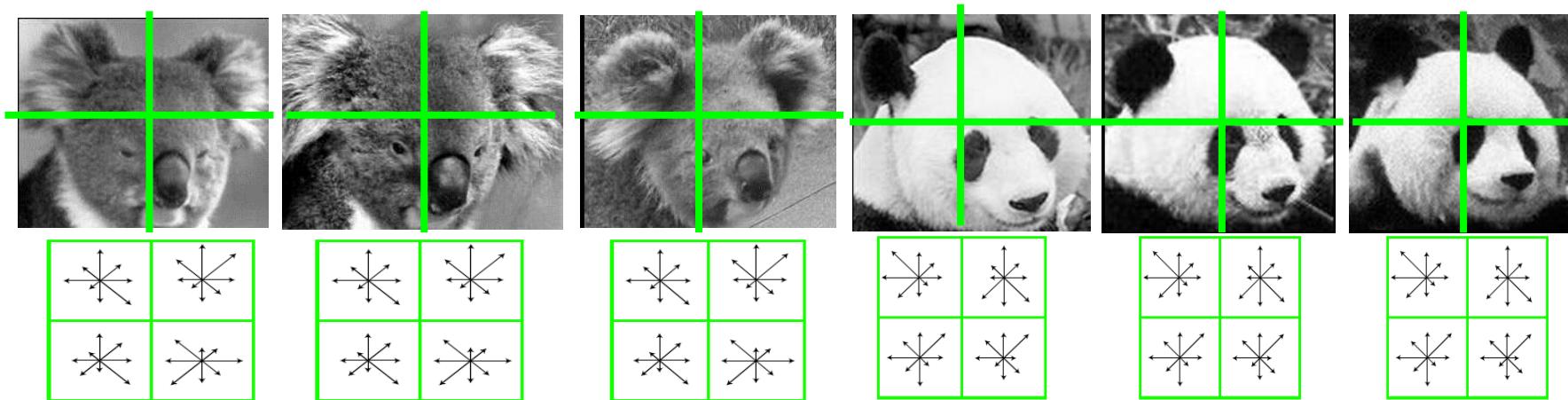
- If object may be in a cluttered scene, slide a window around looking for it.



- Essentially, this is a brute-force approach with many local decisions.

# Recap: Gradient-based Representations

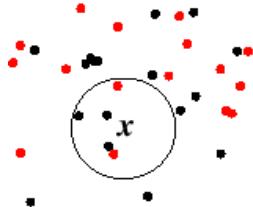
- Consider edges, contours, and (oriented) intensity gradients



- Summarize local distribution of gradients with histogram
  - Locally orderless: offers invariance to small shifts and rotations
  - Contrast-normalization: try to correct for variable illumination

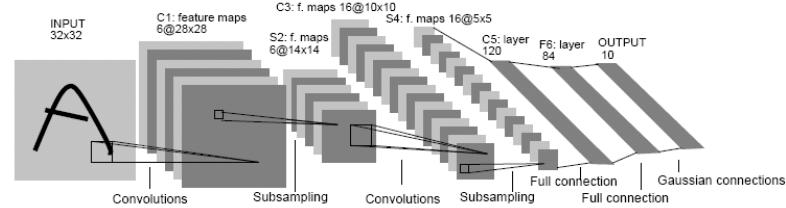
# Classifier Construction: Many Choices...

## Nearest Neighbor



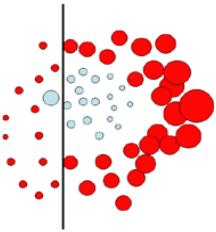
Berg, Berg, Malik 2005,  
Chum, Zisserman 2007,  
Boiman, Shechtman, Irani 2008, ...

## Neural networks



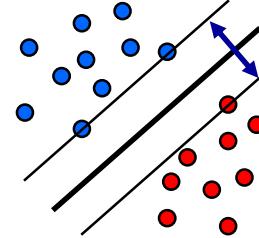
LeCun, Bottou, Bengio, Haffner 1998  
Rowley, Baluja, Kanade 1998  
...

## Boosting



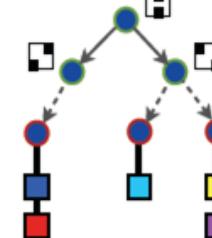
Viola, Jones 2001,  
Torralba et al. 2004,  
Opelt et al. 2006,  
Benenson 2012, ...

## Support Vector Machines



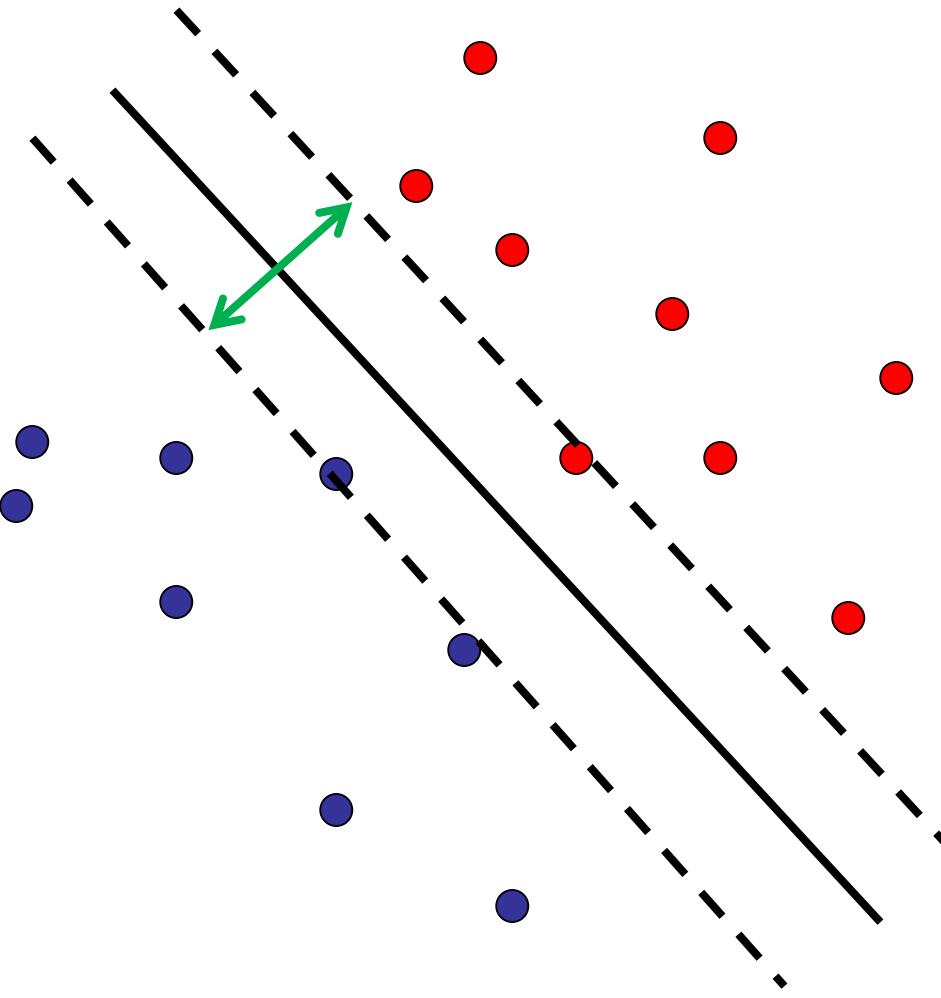
Vapnik, Schölkopf 1995,  
Papageorgiou, Poggio '01,  
Dalal, Triggs 2005,  
Vedaldi, Zisserman 2012

## Randomized Forests



Amit, Geman 1997,  
Breiman 2001,  
Lepetit, Fua 2006,  
Gall, Lempitsky 2009, ...

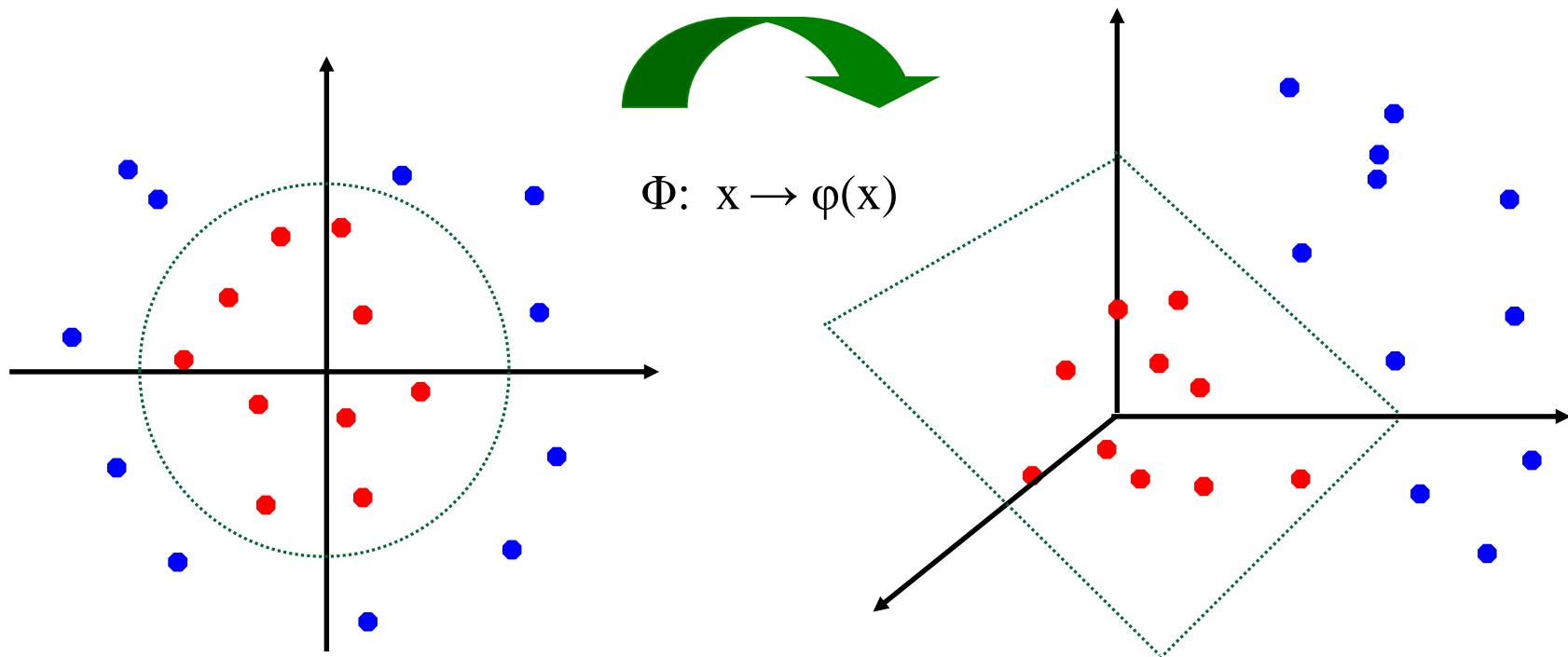
# Recap: Support Vector Machines (SVMs)



- Discriminative classifier based on *optimal separating hyperplane* (i.e. line for 2D case)
- Maximize the *margin* between the positive and negative training examples

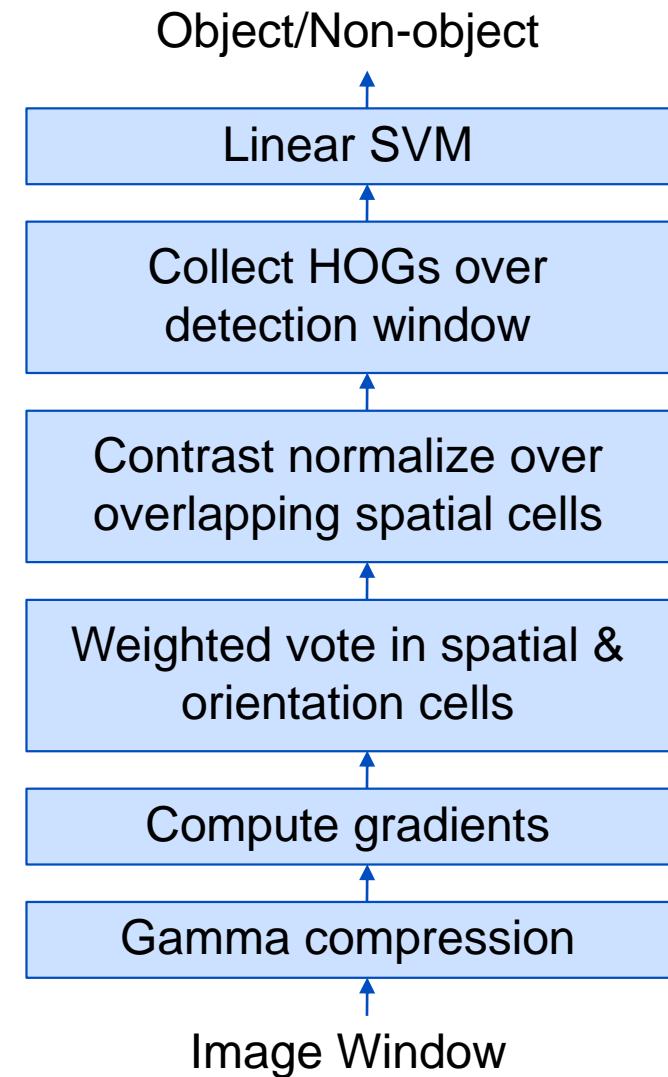
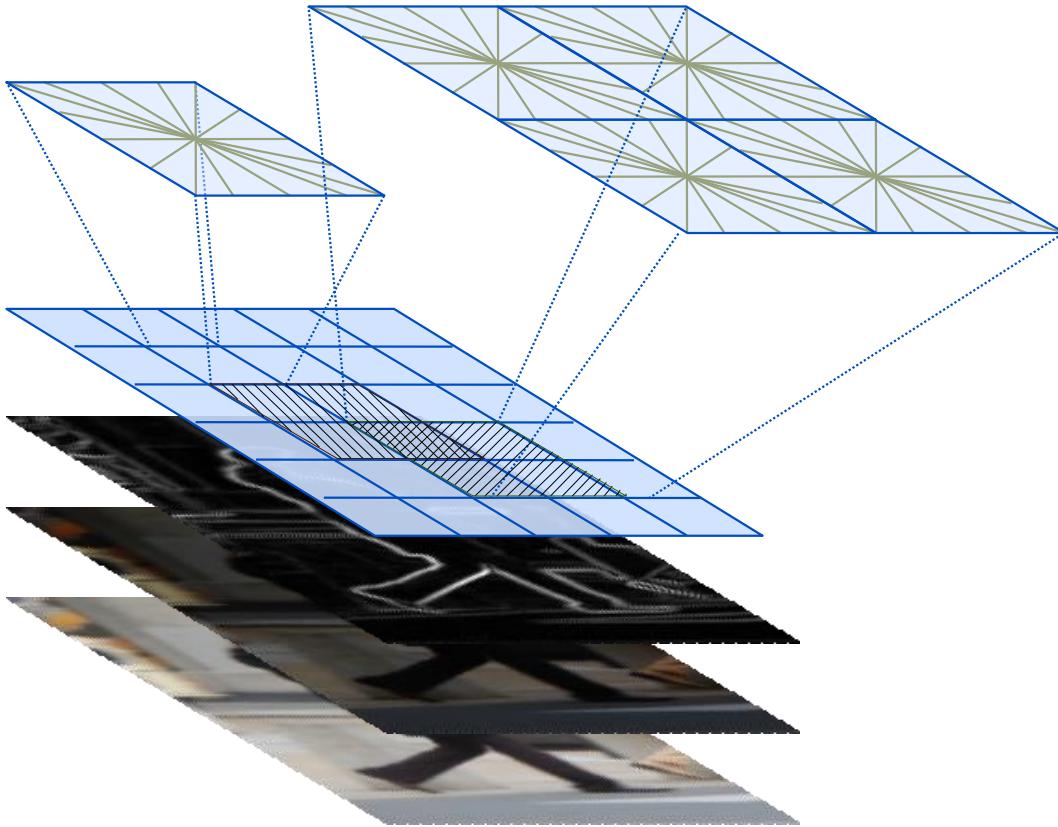
# Recap: Non-Linear SVMs

- General idea: The original input space can be mapped to some higher-dimensional feature space where the training set is separable:



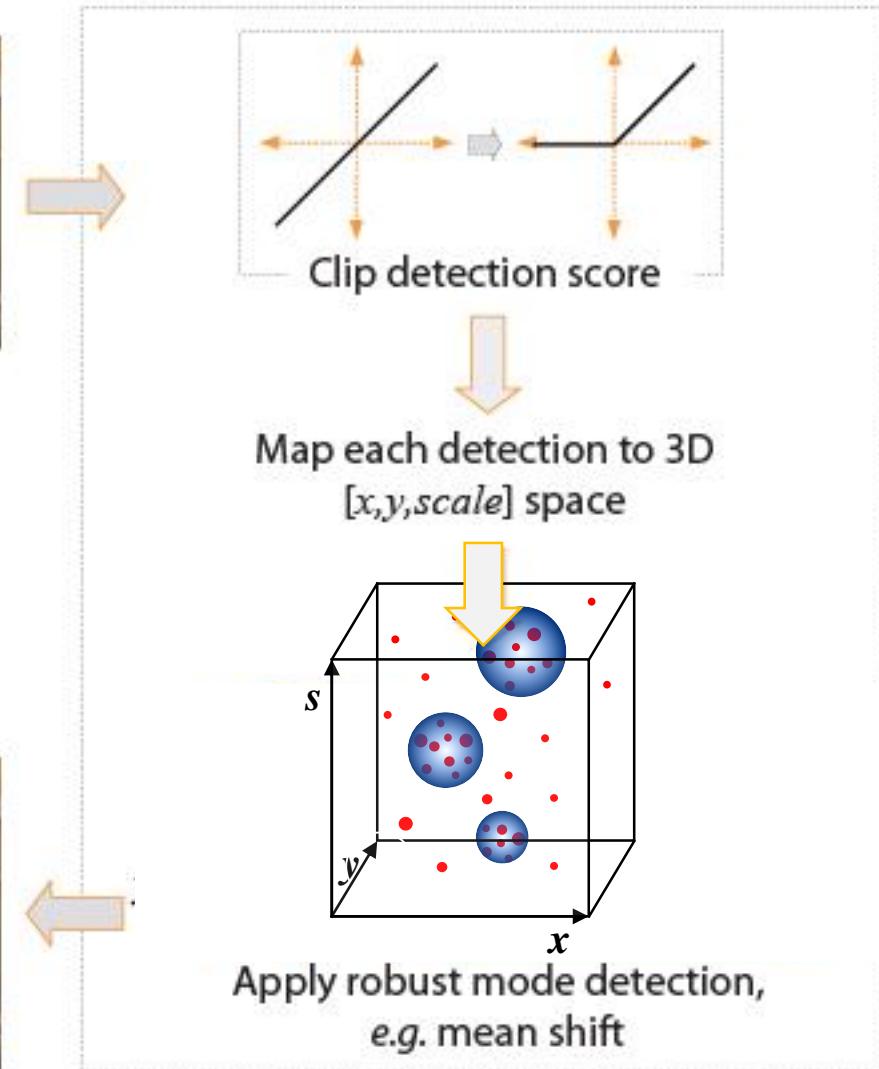
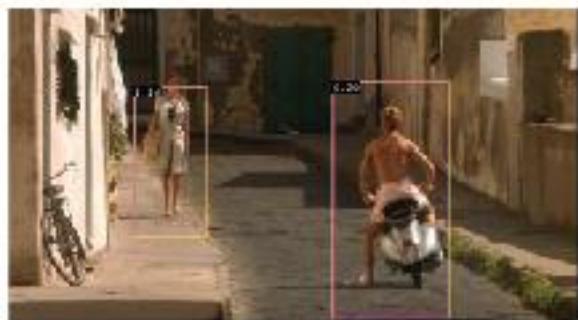
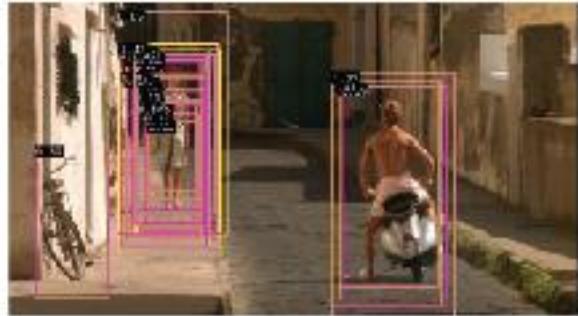
# Recap: HOG Descriptor Processing Chain

- SVM Classification
  - Typically using a linear SVM



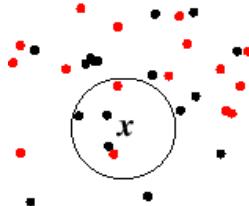
see  
Exercise 2.3!

# Recap: Non-Maximum Suppression



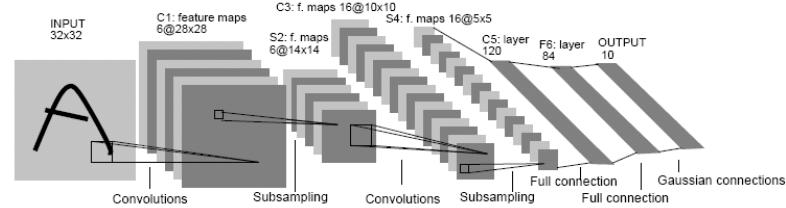
# Classifier Construction: Many Choices...

## Nearest Neighbor



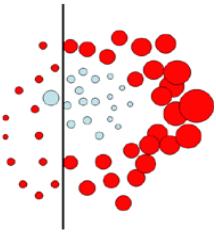
Shakhnarovich, Viola, Darrell 2003  
Berg, Berg, Malik 2005,  
Boiman, Shechtman, Irani 2008, ...

## Neural networks



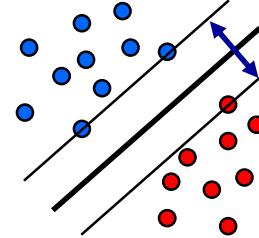
LeCun, Bottou, Bengio, Haffner 1998  
Rowley, Baluja, Kanade 1998  
...

## Boosting



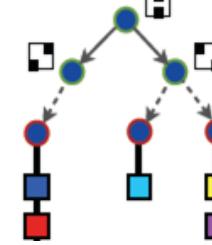
Viola, Jones 2001,  
Torralba et al. 2004,  
Opelt et al. 2006,  
Benenson 2012, ...

## Support Vector Machines



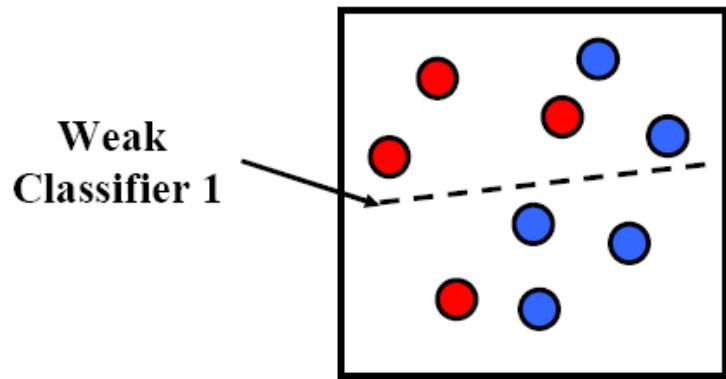
Vapnik, Schölkopf 1995,  
Papageorgiou, Poggio '01,  
Dalal, Triggs 2005,  
Vedaldi, Zisserman 2012

## Randomized Forests

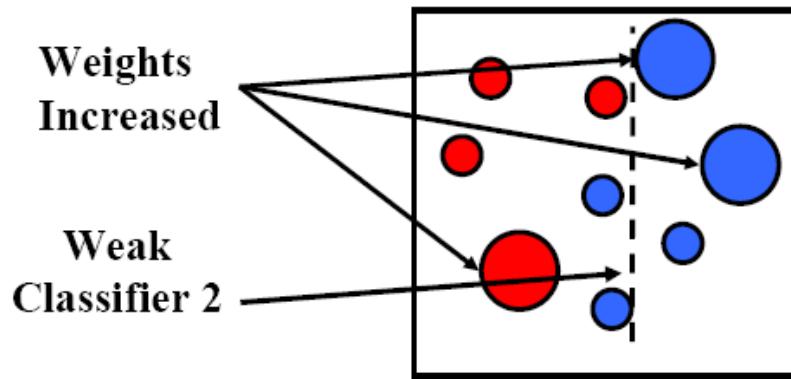


Amit, Geman 1997,  
Breiman 2001,  
Lepetit, Fua 2006,  
Gall, Lempitsky 2009, ...

# Recap: AdaBoost

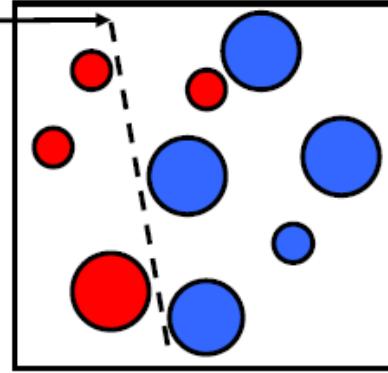


Weak  
Classifier 1



Weights  
Increased  
Weak  
Classifier 2

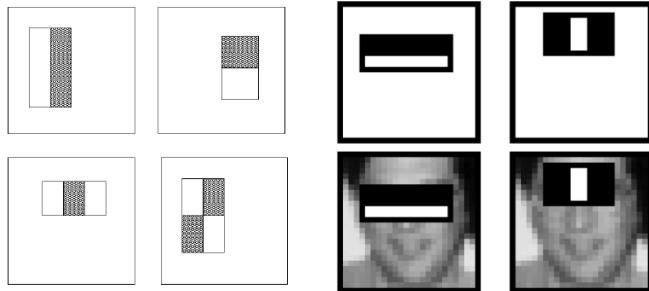
Weak  
classifier 3



Final classifier is  
combination of the  
weak classifiers

# Recap: Viola-Jones Face Detection

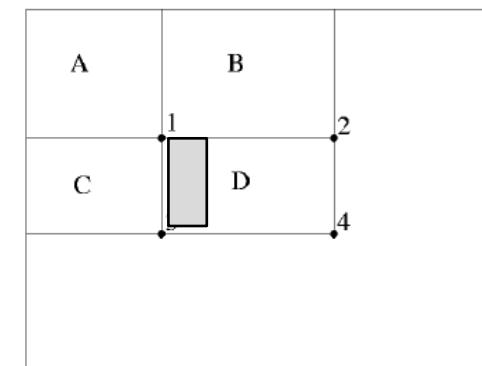
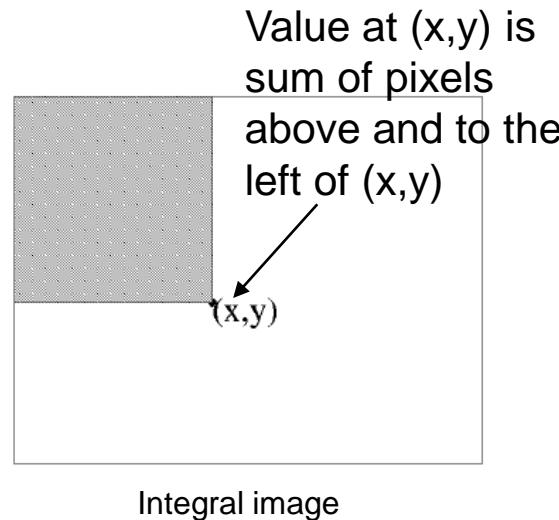
“Rectangular” filters



Feature output is difference between adjacent regions

Efficiently computable with integral image: any sum can be computed in constant time

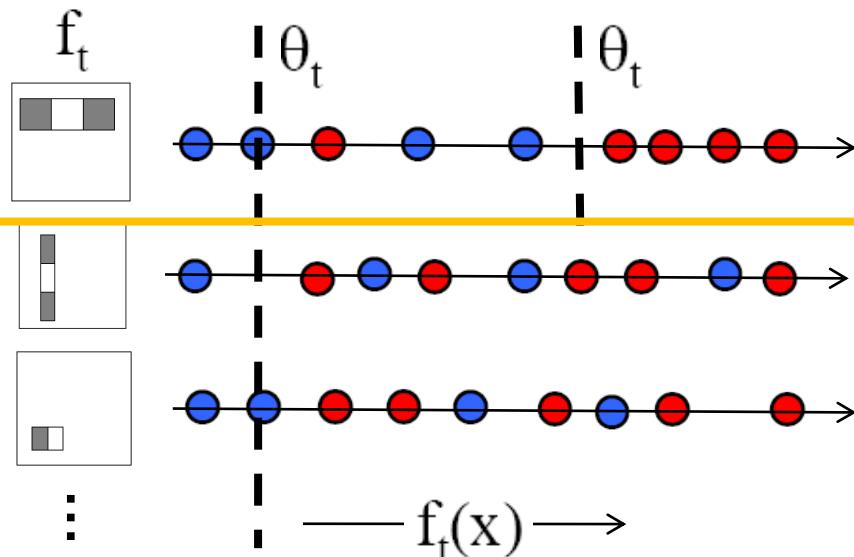
Avoid scaling images → scale features directly for same cost



$$\begin{aligned}D &= 1 + 4 - (2 + 3) \\&= A + (A + B + C + D) - (A + C + A + B) \\&= D\end{aligned}$$

# Recap: AdaBoost Feature+Classifier Selection

- Want to select the single rectangle feature and threshold that best separates **positive** (faces) and **negative** (non-faces) training examples, in terms of *weighted* error.



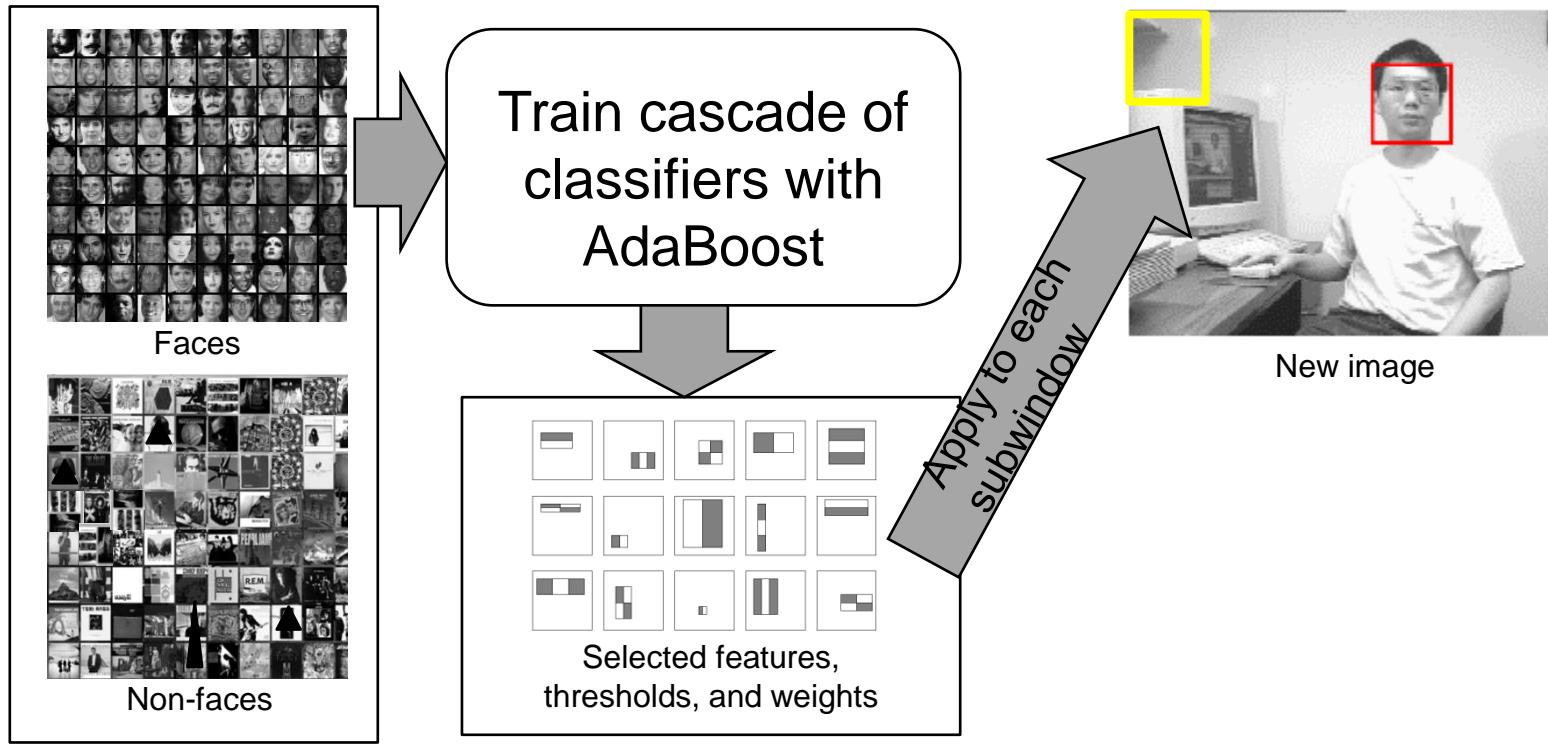
Outputs of a possible rectangle feature on faces and non-faces.

Resulting weak classifier:


$$h_t(x) = \begin{cases} +1 & \text{if } f_t(x) > \theta_t \\ -1 & \text{otherwise} \end{cases}$$

For next round, reweight the examples according to errors, choose another filter/threshold combo.

# Recap: Viola-Jones Face Detector



- Train with 5K positives, 350M negatives
- Real-time detector using 38 layer cascade
- 6061 features in final layer
- [Implementation available in OpenCV:  
<http://sourceforge.net/projects/opencvlibrary/>]

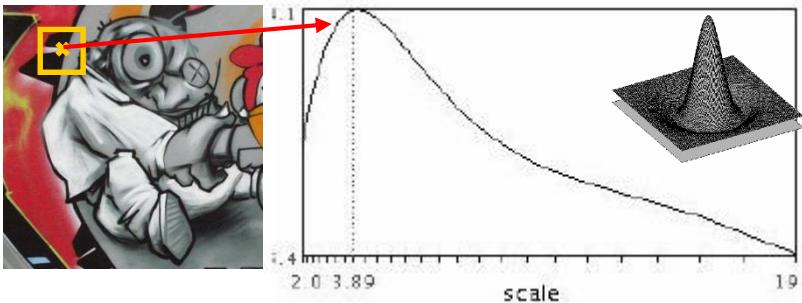
# Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
  - Local Features – Detection and Description
  - Recognition with Local Features
- Deep Learning
- 3D Reconstruction

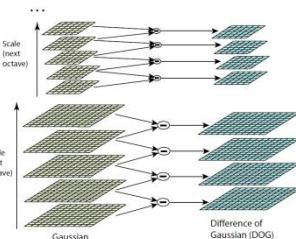
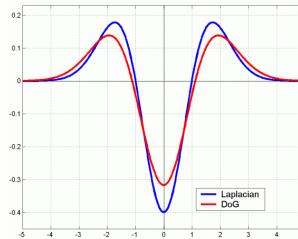
$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

*Harris & Hessian detector*

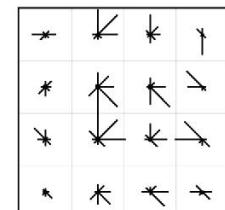
$$Hes(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$



*Laplacian scale selection*

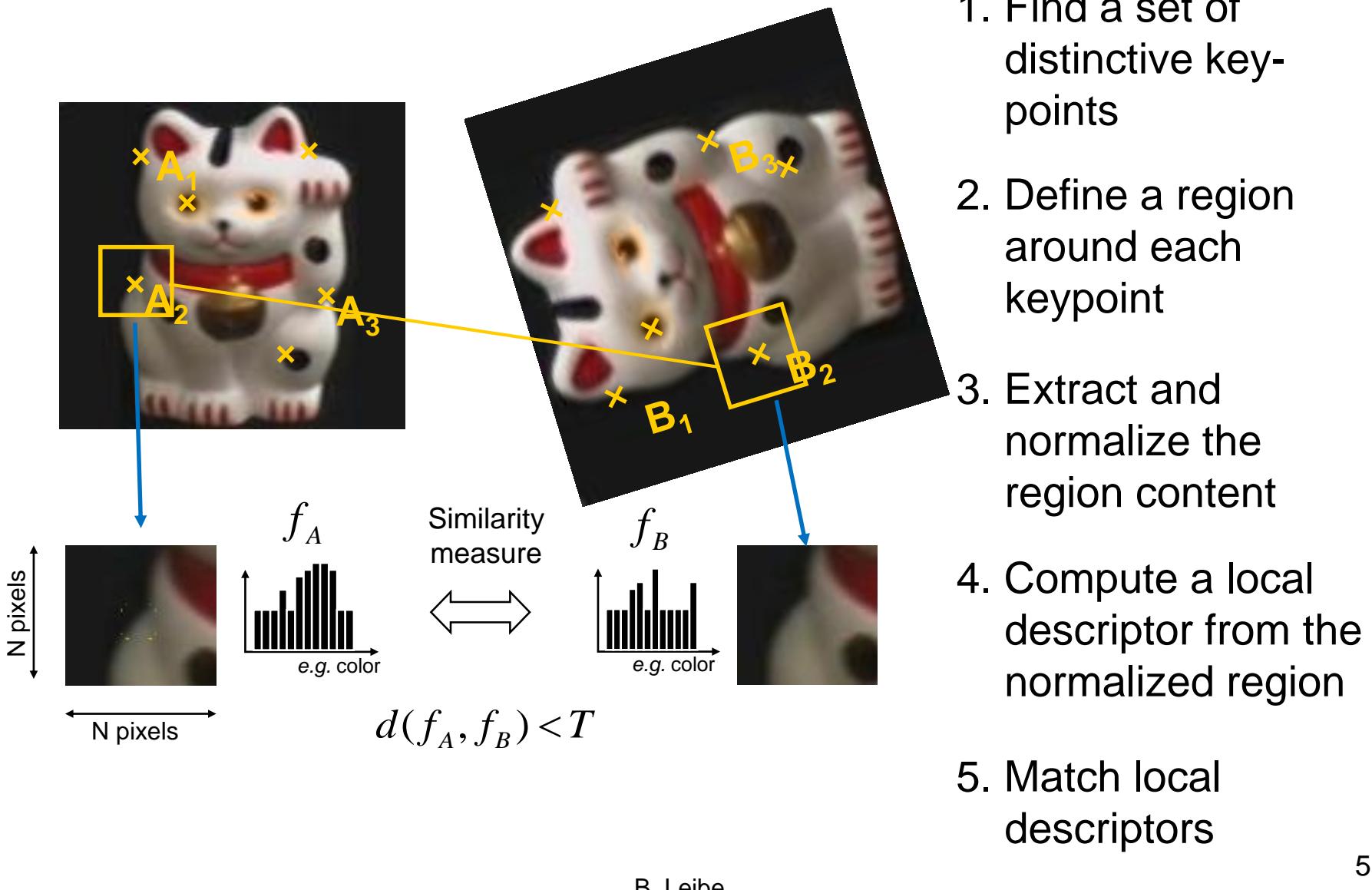


*Difference-of-Gaussian (DoG)*



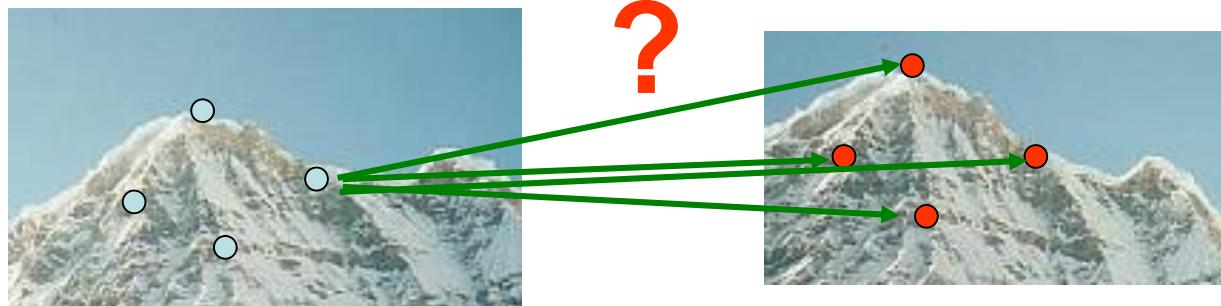
*SIFT descriptor*

# Recap: Local Feature Matching Pipeline



# Recap: Requirements for Local Features

- Problem 1:
  - Detect the same point *independently* in both images
- Problem 2:
  - For each point correctly recognize the corresponding one



We need a repeatable detector!

We need a reliable and distinctive descriptor!

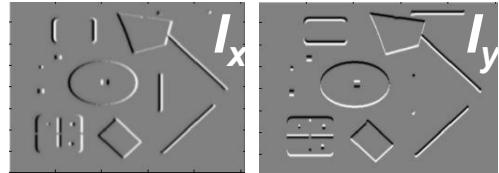
see  
Exercise 3.2!

# Recap: Harris Detector [Harris88]

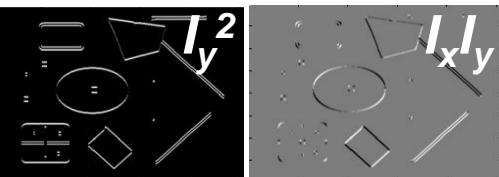
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

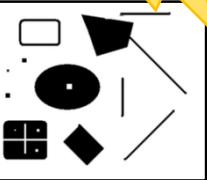
1. Image derivatives



2. Square of derivatives



3. Gaussian filter  $g(\sigma_\nu)$



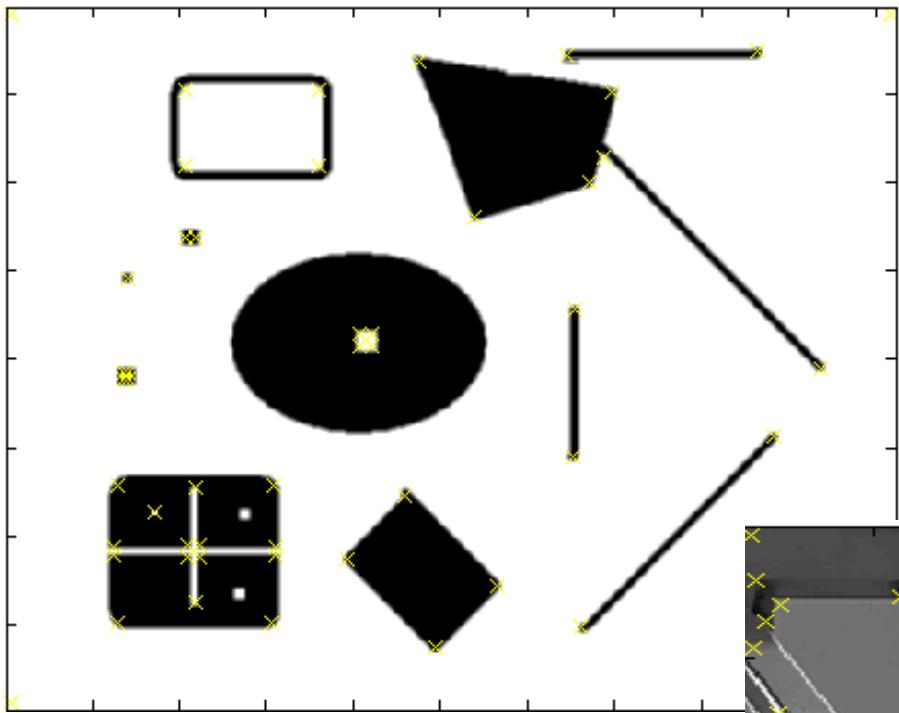
4. Cornerness function – two strong eigenvalues

$$\begin{aligned} R &= \det[M(\sigma_I, \sigma_D)] - \alpha[\text{trace}(M(\sigma_I, \sigma_D))] \\ &= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2 \end{aligned}$$

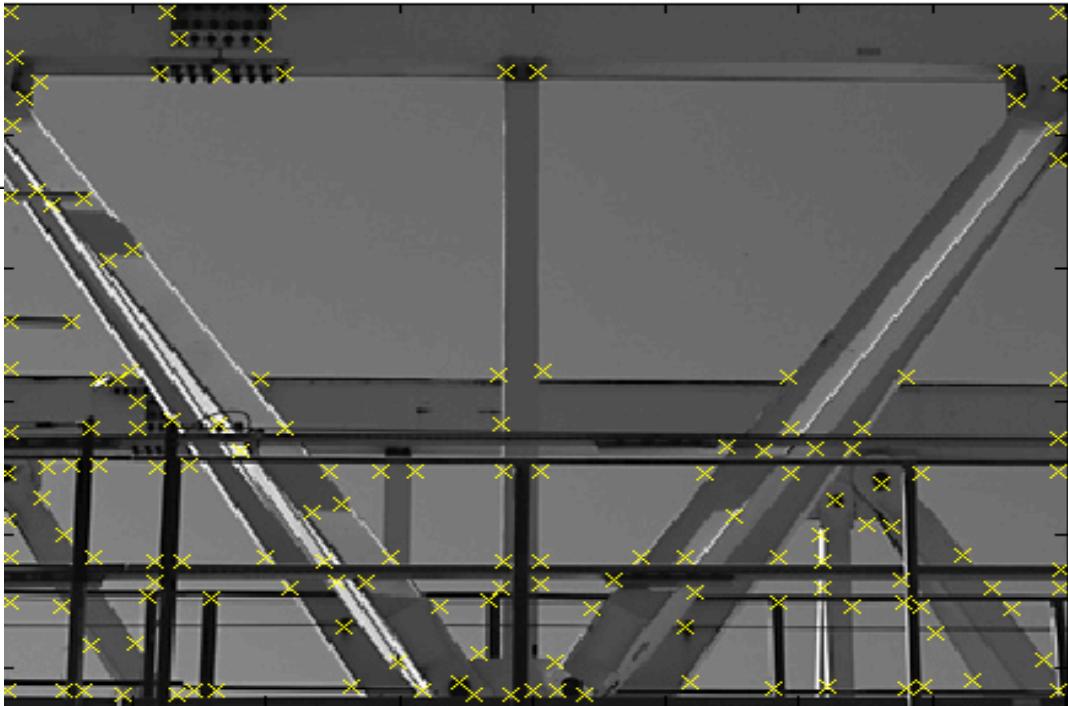
5. Perform non-maximum suppression



# Recap: Harris Detector Responses



*Effect:* A very precise corner detector.



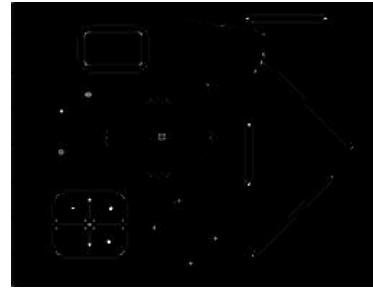
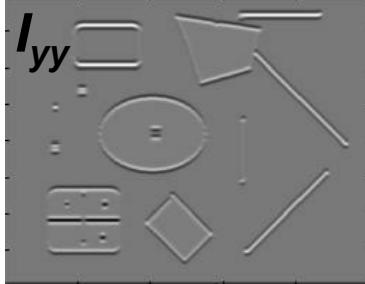
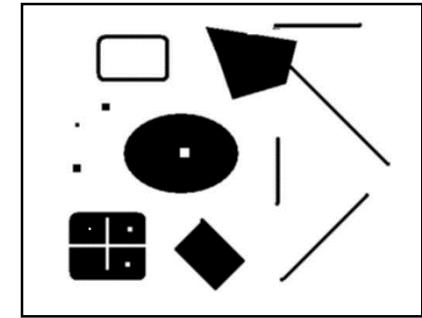
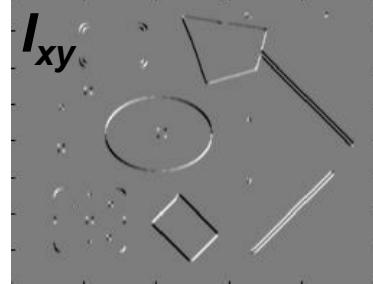
# Recap: Hessian Detector

[Beaudet78]

see  
Exercise 3.2!

- Hessian determinant

$$Hessian(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

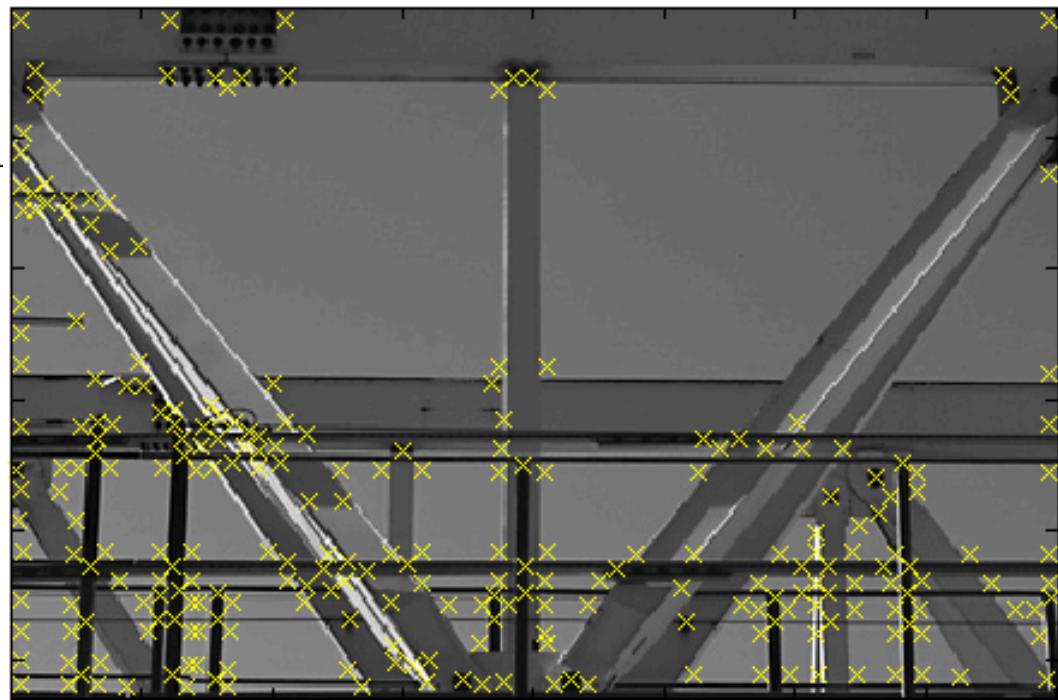
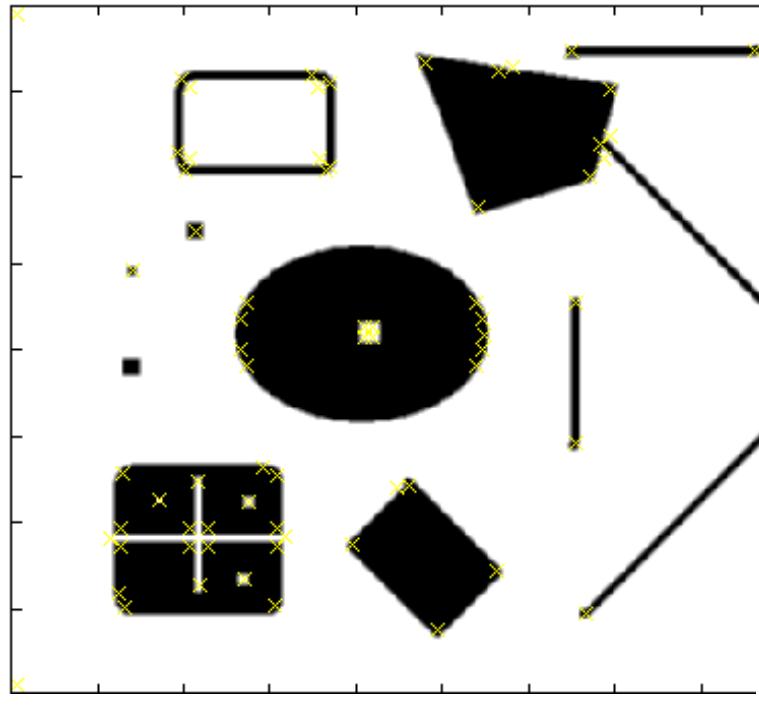


$$\det(Hessian(I)) = I_{xx}I_{yy} - I_{xy}^2$$

In Matlab:

$$I_{xx}.*I_{yy} - (I_{xy})^2$$

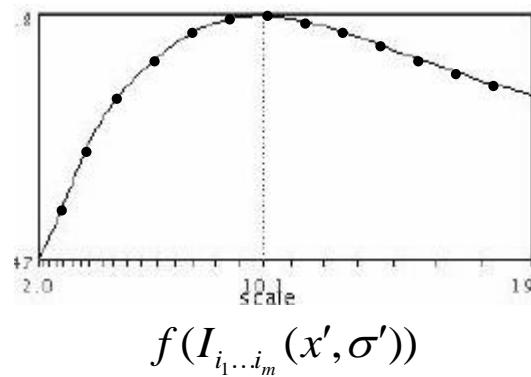
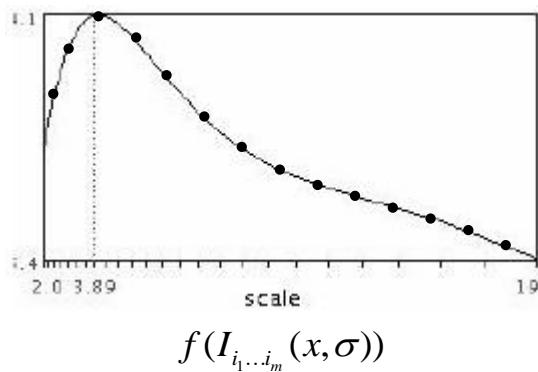
# Hessian Detector – Responses



*Effect:* Responses mainly on corners and strongly textured areas.

# Recap: Automatic Scale Selection

- Function responses for increasing scale (scale signature)

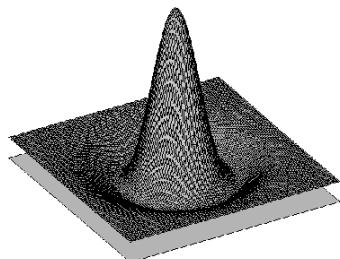


B. Leibe

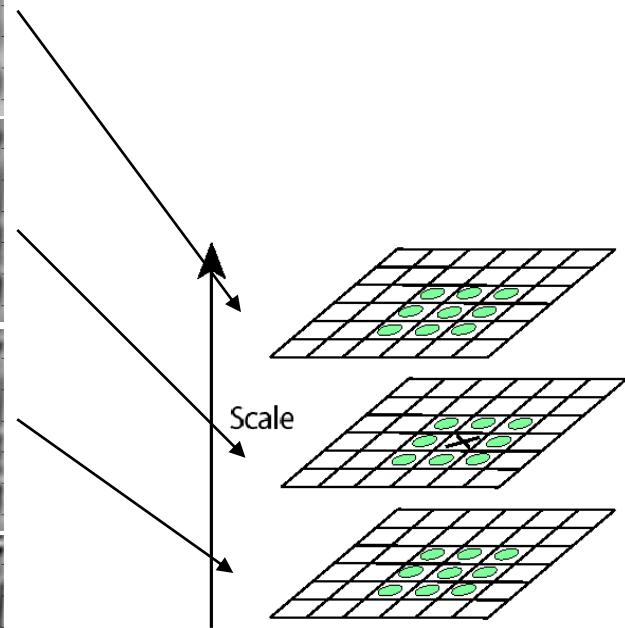
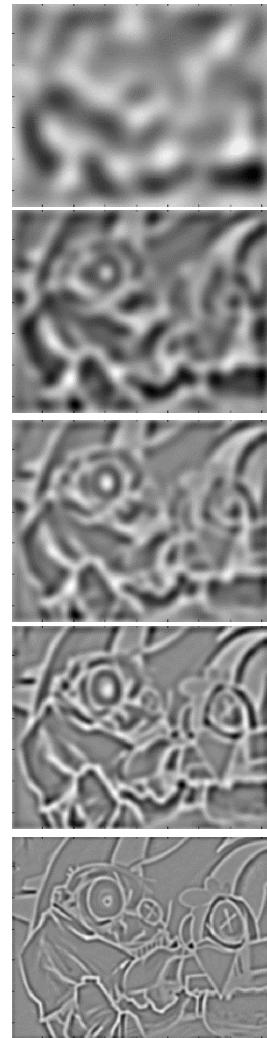
# Recap: Laplacian-of-Gaussian (LoG)

- Interest points:

- Local maxima in scale space of Laplacian-of-Gaussian

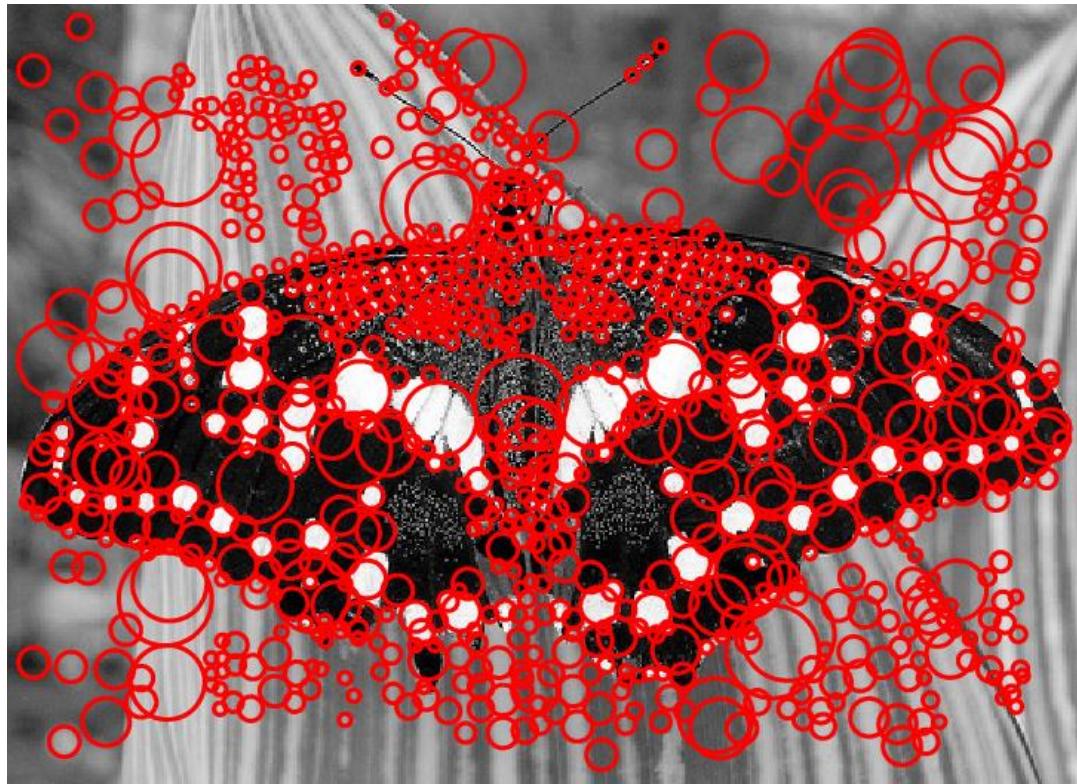


$$L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$$
$$\sigma^2$$
$$\sigma$$



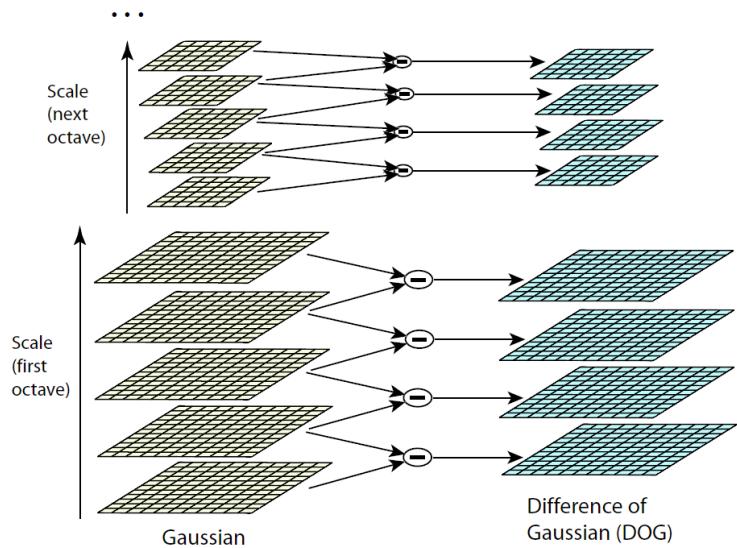
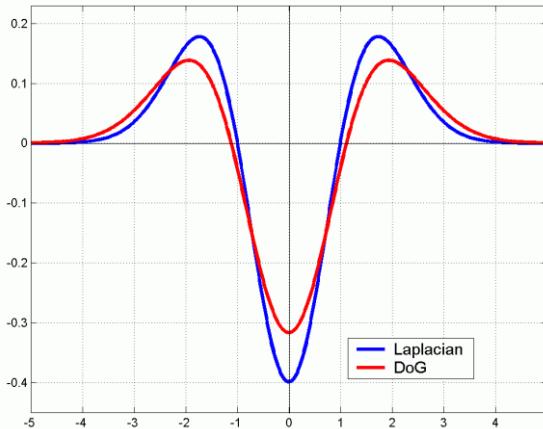
$\Rightarrow$  List of  $(x, y, \sigma)$

# Recap: LoG Detector Responses



# Recap: Key point localization with DoG

- Efficient implementation
  - Approximate LoG with a difference of Gaussians (DoG)
- Approach DoG Detector
  - Detect maxima of difference-of-Gaussian in scale space
  - Reject points with low contrast (threshold)
  - Eliminate edge responses



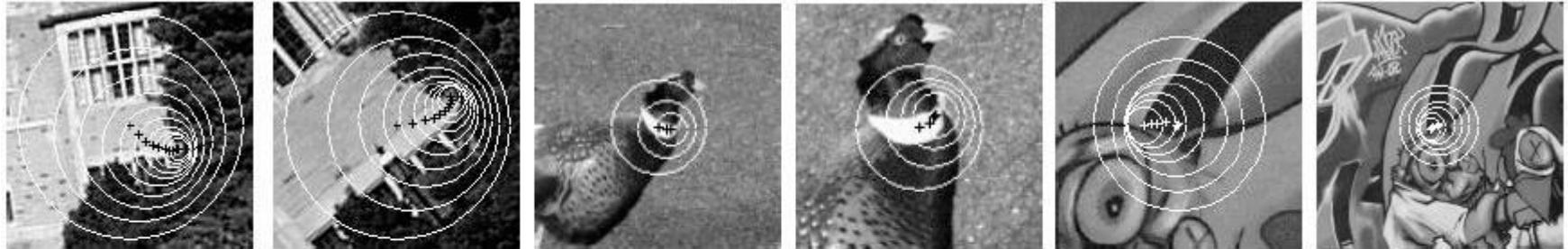
**Candidate keypoints:  
list of  $(x, y, \sigma)$**

Image source: David Lowe

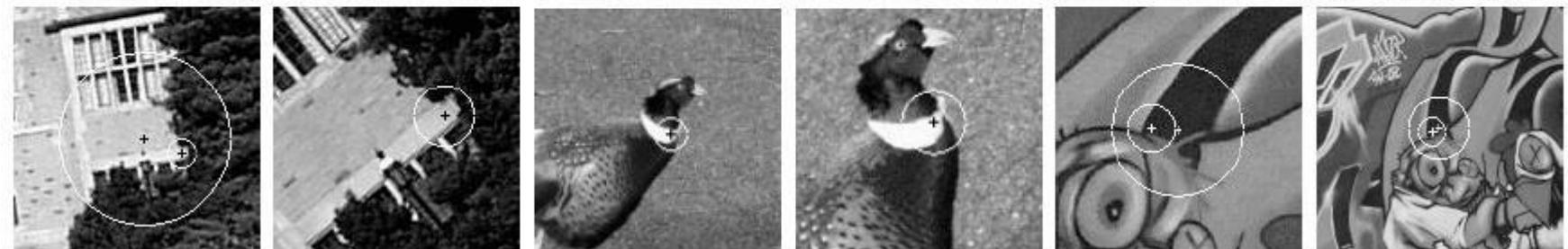
# Recap: Harris-Laplace

1. Initialization: Multiscale Harris corner detection
2. Scale selection based on Laplacian  
(same procedure with Hessian  $\Rightarrow$  Hessian-Laplace)

Harris points

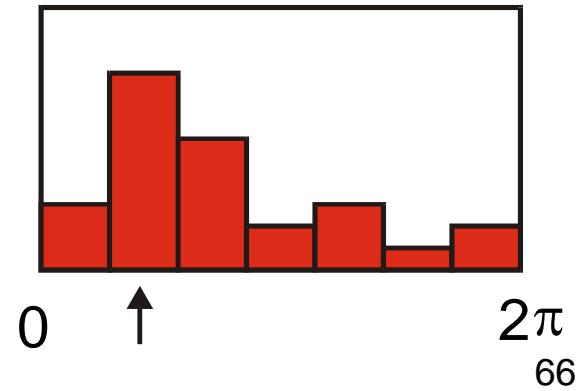
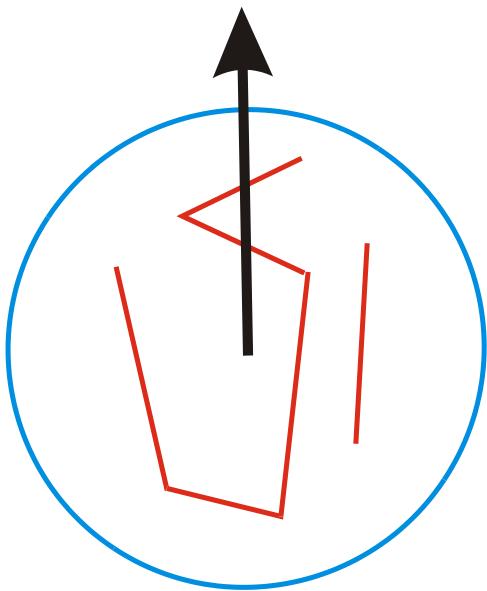


Harris-Laplace points



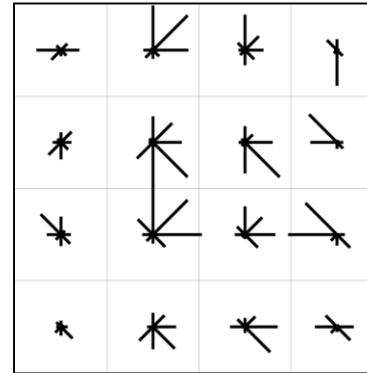
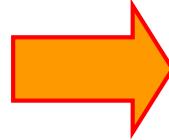
# Recap: Orientation Normalization

- Compute orientation histogram [Lowe, SIFT, 1999]
- Select dominant orientation
- Normalize: rotate to fixed orientation



# Recap: SIFT Feature Descriptor

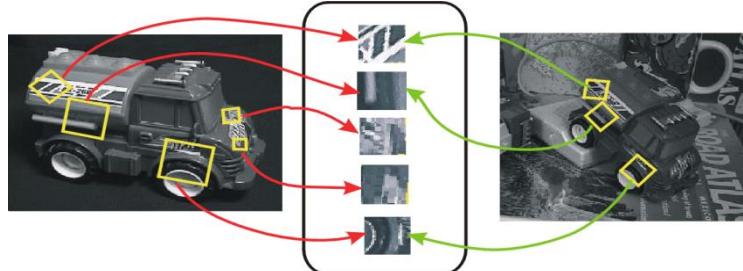
- **Scale Invariant Feature Transform**
- Descriptor computation:
  - Divide patch into  $4 \times 4$  sub-patches: 16 cells
  - Compute histogram of gradient orientations (8 reference angles) for all pixels inside each sub-patch
  - Resulting descriptor:  $4 \times 4 \times 8 = 128$  dimensions



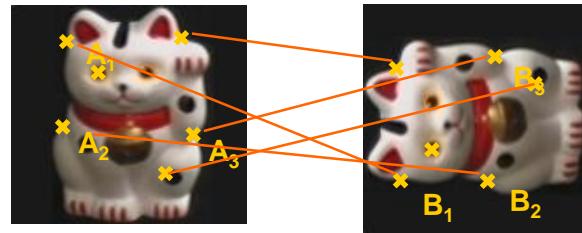
D.G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)" IJCV 60 (2), pp. 91-110, 2004.

# Repetition

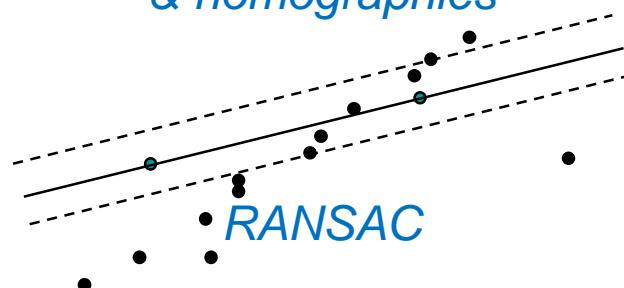
- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
  - Local Features – Detection and Description
  - Recognition with Local Features
- Deep Learning
- 3D Reconstruction



*Recognition pipeline*



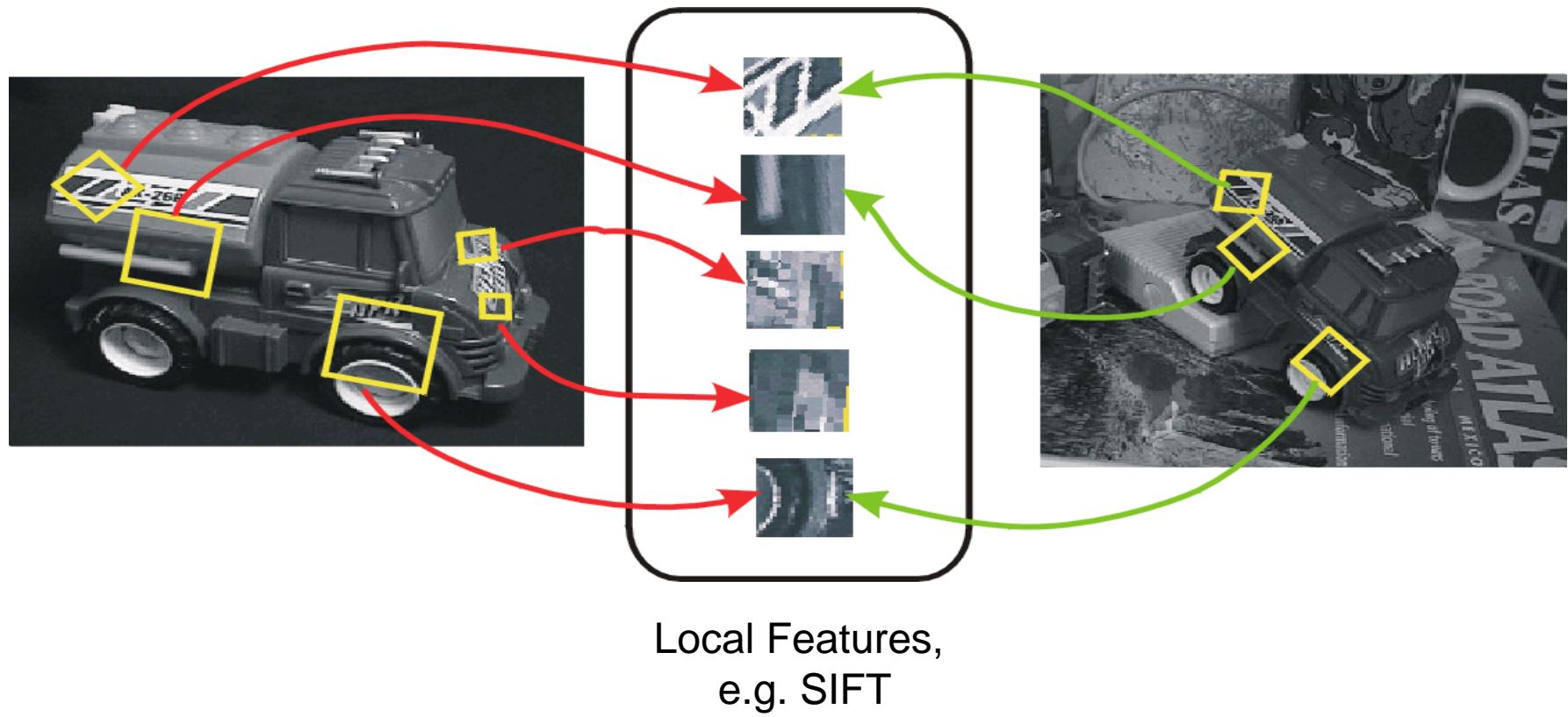
*Fitting affine transformations  
& homographies*



*Gen. Hough Transform*

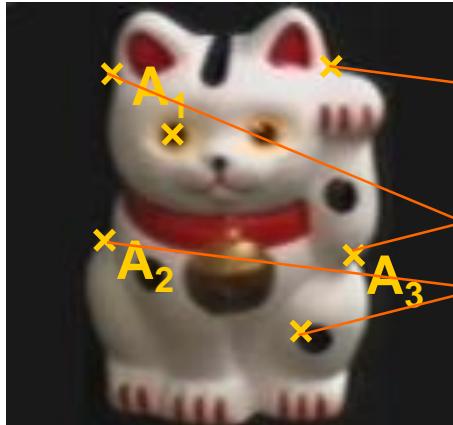
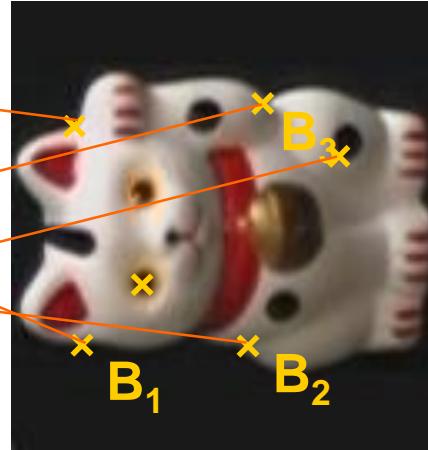
# Recap: Recognition with Local Features

- Image content is transformed into local features that are invariant to translation, rotation, and scale
- Goal: Verify if they belong to a consistent configuration



# Recap: Fitting an Affine Transformation

- Assuming we know the correspondences, how do we get the transformation?

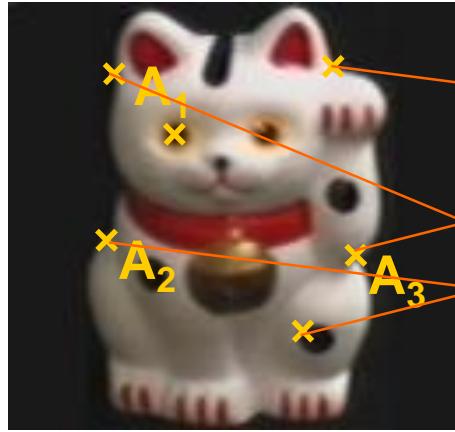
 $(x_i, y_i)$  $(x'_i, y'_i)$ 

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ \dots & & & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix}$$

# Recap: Fitting a Homography

- Estimating the transformation



Homogenous coordinates

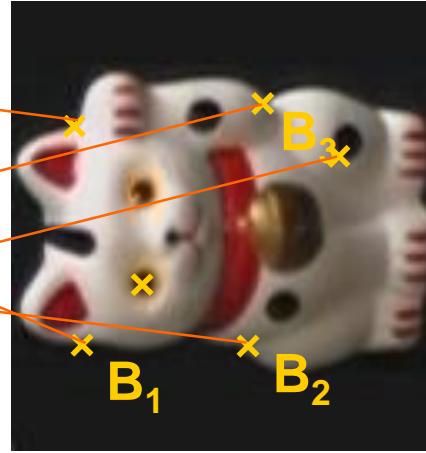


Image coordinates

$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$

$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$

$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$

⋮

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x_{A_1} = \frac{h_{11} x_{B_1} + h_{12} y_{B_1} + h_{13}}{h_{31} x_{B_1} + h_{32} y_{B_1} + 1}$$

B. Leibe

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & z' & \\ & & 1 \end{bmatrix} \cdot \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$y_{A_1} = \frac{h_{21} x_{B_1} + h_{22} y_{B_1} + h_{23}}{h_{31} x_{B_1} + h_{32} y_{B_1} + 1}$$

Matrix notation

$$x' = Hx$$

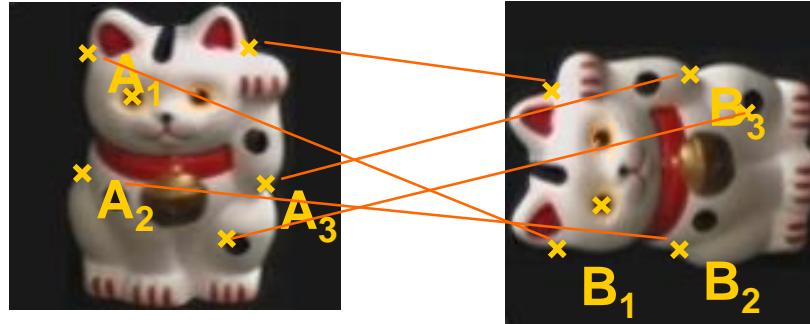
$$x'' = \frac{1}{z'} x'$$

# Recap: Fitting a Homography

- Estimating the transformation

$$h_{11}x_{B_1} + h_{12}y_{B_1} + h_{13} - x_{A_1}h_{31}x_{B_1} - x_{A_1}h_{32}y_{B_1} - x_{A_1} = 0$$

$$h_{21}x_{B_1} + h_{22}y_{B_1} + h_{23} - y_{A_1}h_{31}x_{B_1} - y_{A_1}h_{32}y_{B_1} - y_{A_1} = 0$$



$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$

$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$

$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$

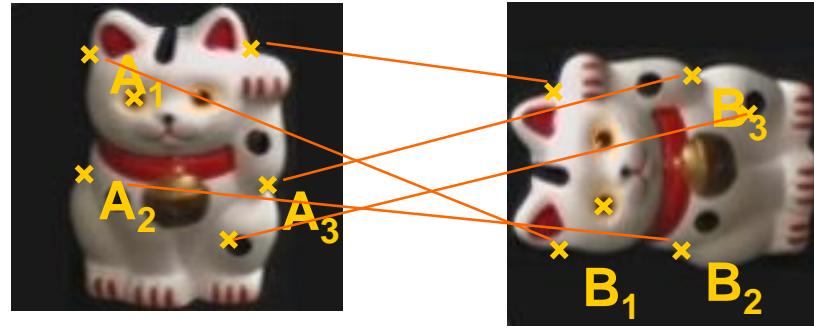
$\vdots$

$$\begin{bmatrix} x_{B_1} & y_{B_1} & 1 & 0 & 0 & 0 & -x_{A_1}x_{B_1} & -x_{A_1}y_{B_1} & -x_{A_1} \\ 0 & 0 & 0 & x_{B_1} & y_{B_1} & 1 & -y_{A_1}x_{B_1} & -y_{A_1}y_{B_1} & -y_{A_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$Ah = 0$$

# Recap: Fitting a Homography

- Estimating the transformation
- Solution:
  - Null-space vector of  $A$
  - Corresponds to smallest eigenvector



SVD

$$Ah = 0$$

$$\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T = \mathbf{U} \begin{bmatrix} d_{11} & \cdots & d_{19} \\ \vdots & \ddots & \vdots \\ d_{91} & \cdots & d_{99} \end{bmatrix} \begin{bmatrix} v_{11} & \cdots & v_{19} \\ \vdots & \ddots & \vdots \\ v_{91} & \cdots & v_{99} \end{bmatrix}^T$$

$$\mathbf{h} = \frac{[v_{19}, \dots, v_{99}]}{v_{99}}$$

Minimizes least square error

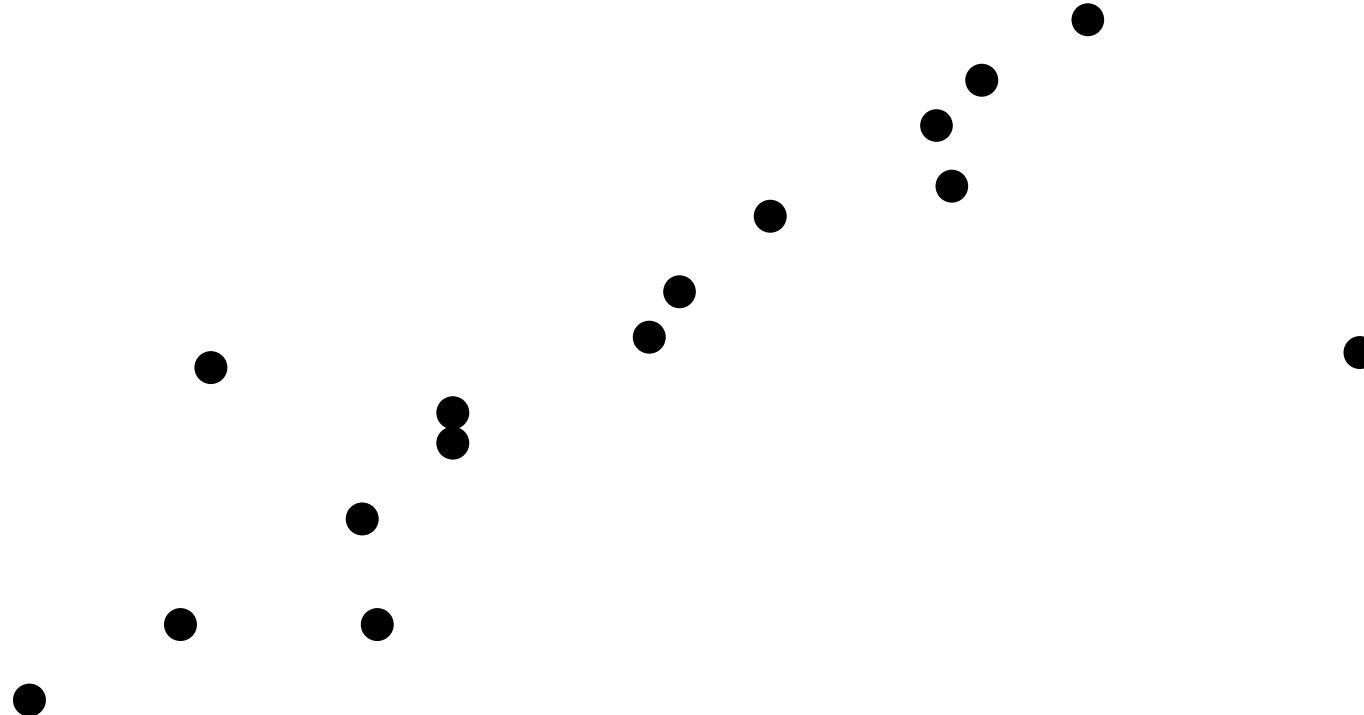
# Recap: RANSAC

## RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
  2. Compute transformation from seed group
  3. Find *inliers* to this transformation
  4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers
- Keep the transformation with the largest number of inliers

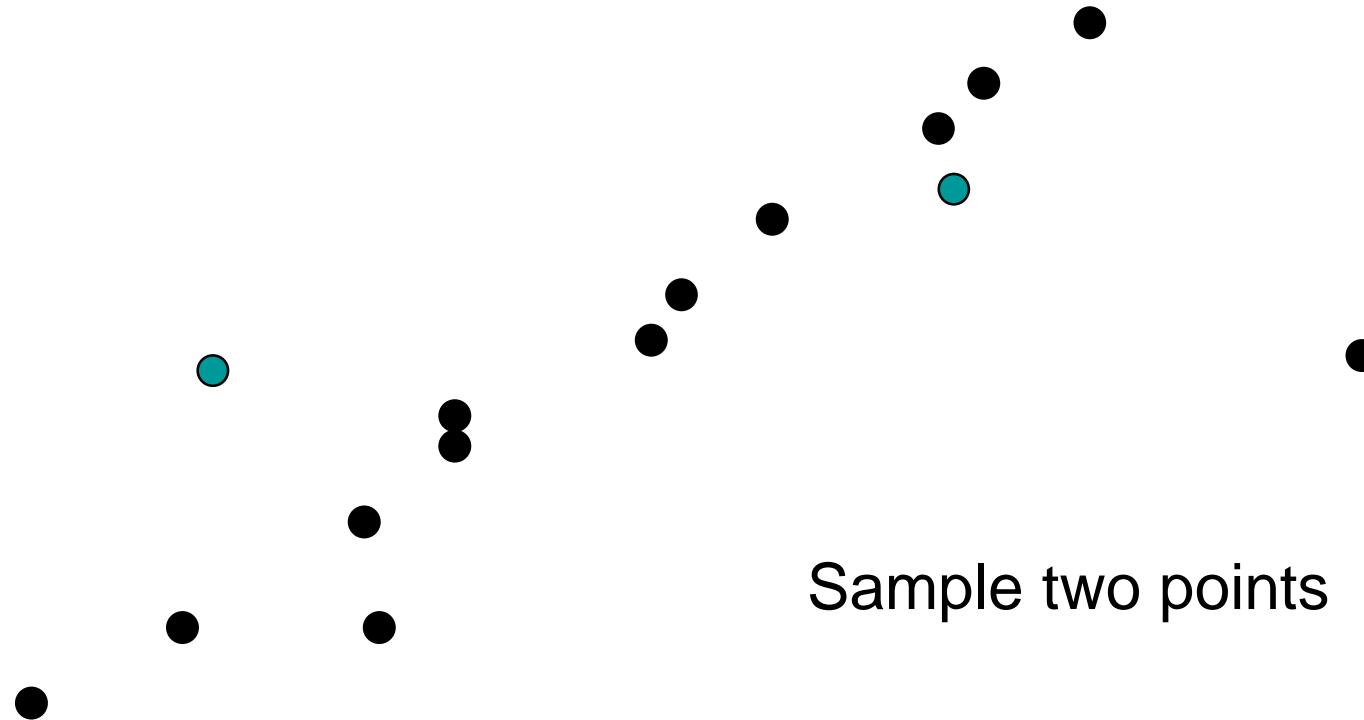
# Recap: RANSAC Line Fitting Example

- Task: Estimate the best line



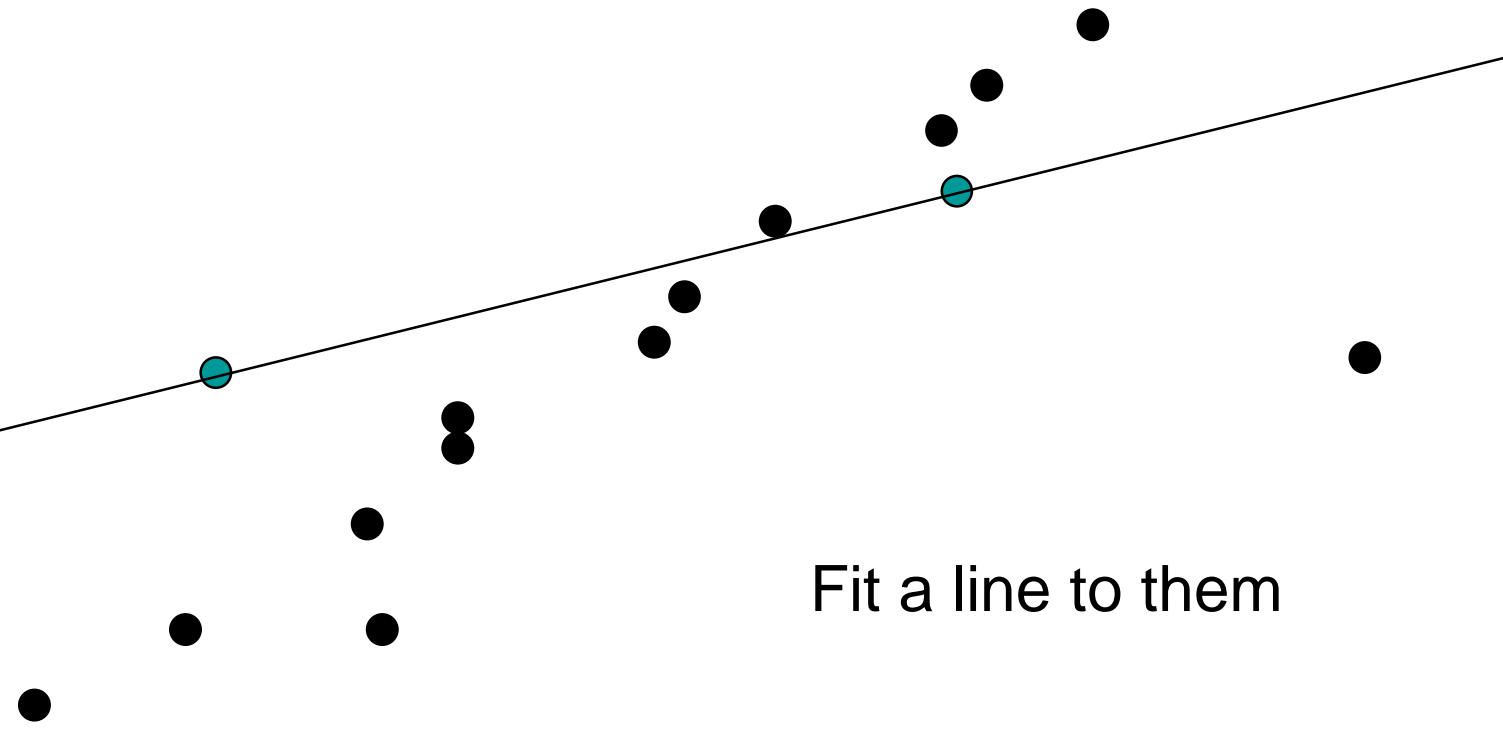
# Recap: RANSAC Line Fitting Example

- Task: Estimate the best line



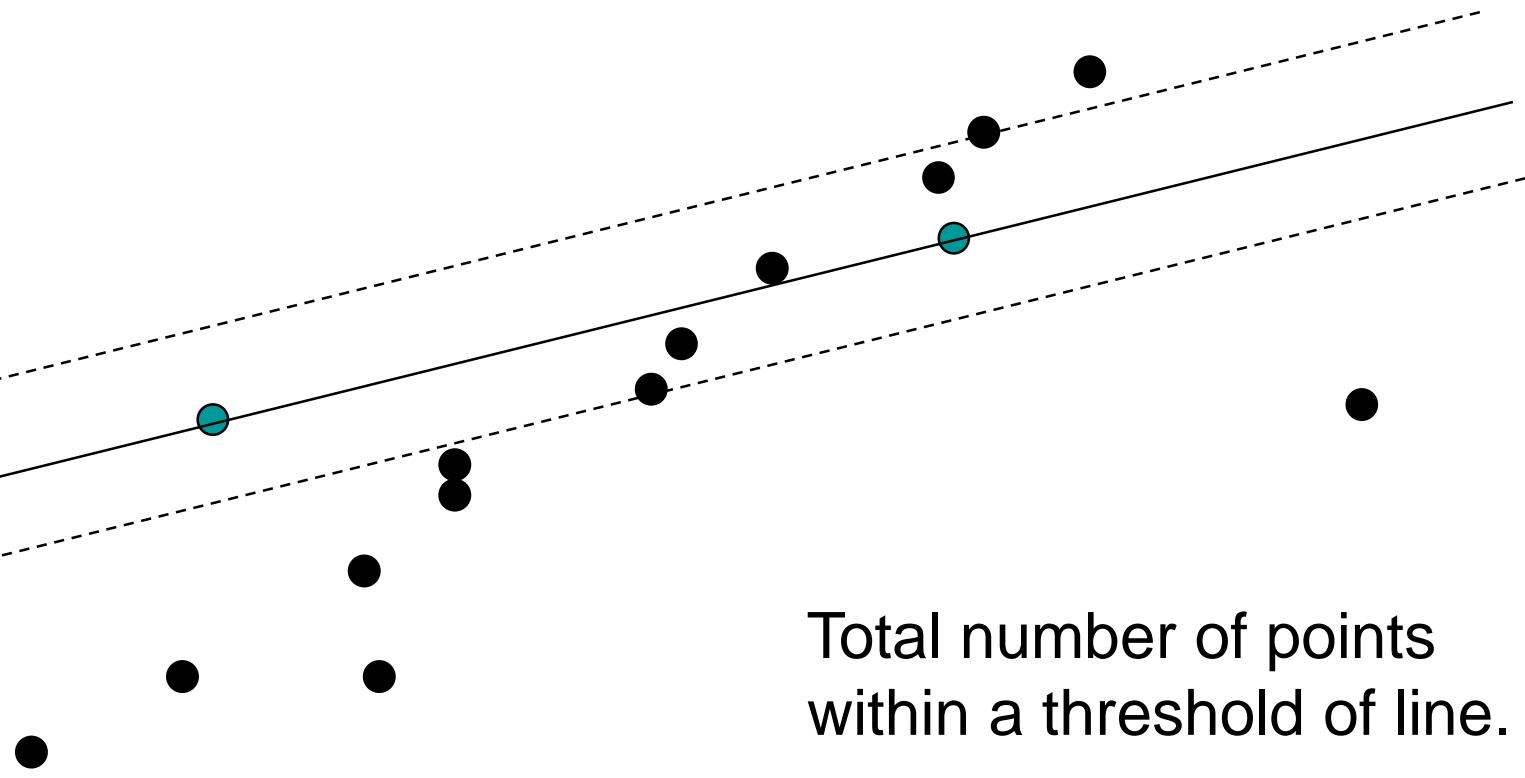
# Recap: RANSAC Line Fitting Example

- Task: Estimate the best line



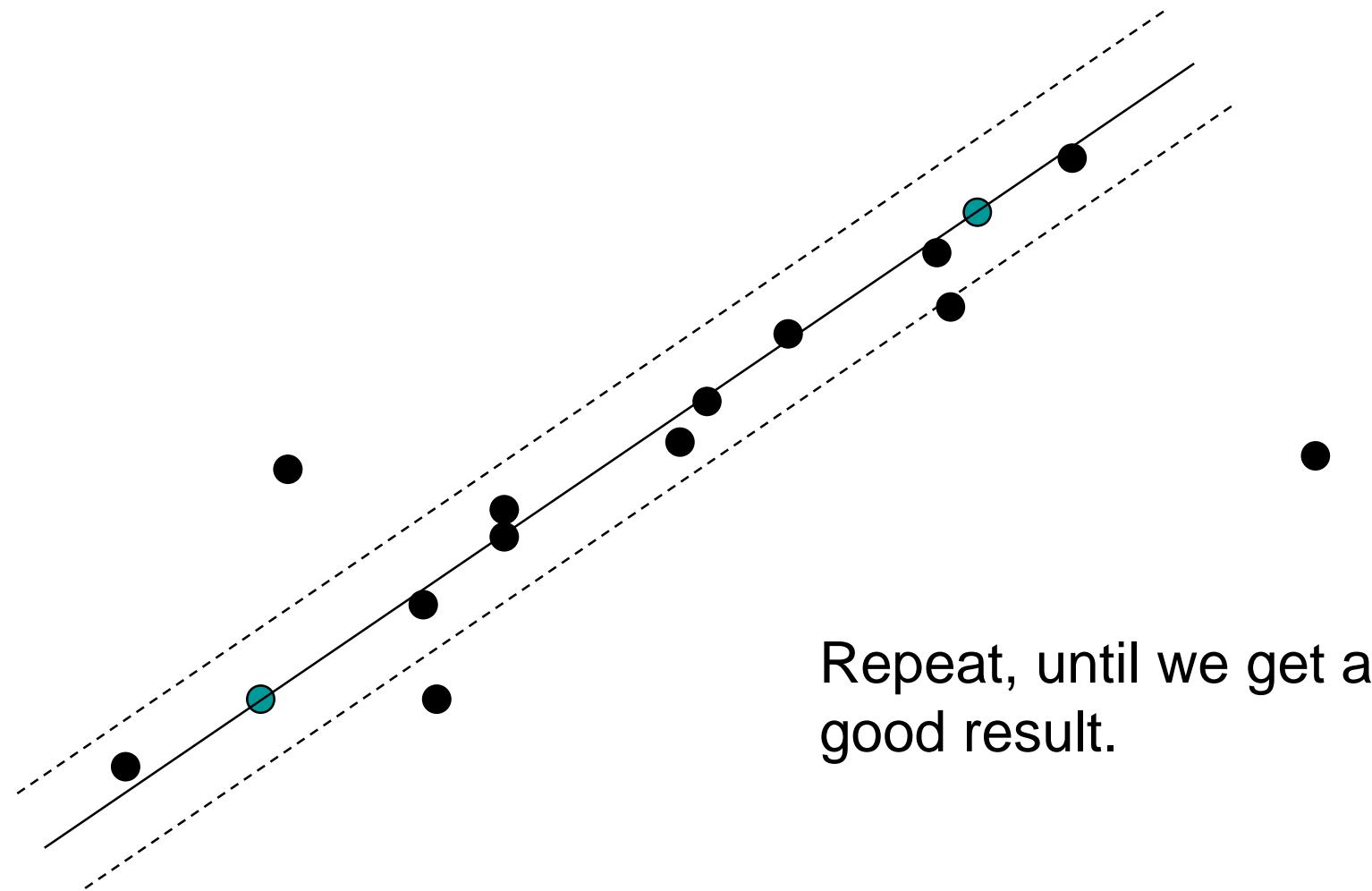
# Recap: RANSAC Line Fitting Example

- Task: Estimate the best line



# Recap: RANSAC Line Fitting Example

- Task: Estimate the best line



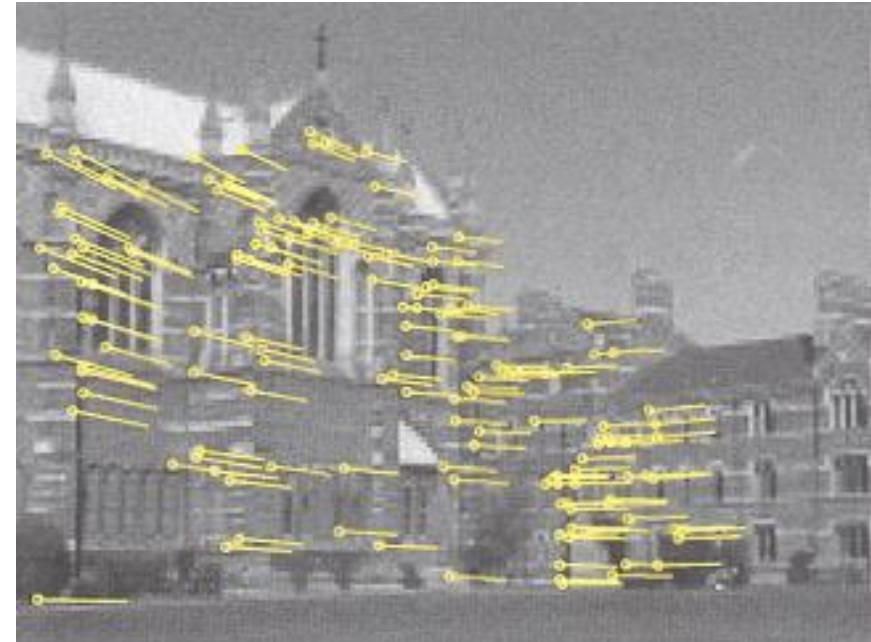
# Recap: Feature Matching Example

- Find best stereo match within a square search window (here 300 pixels<sup>2</sup>)
- Global transformation model: epipolar geometry

before RANSAC



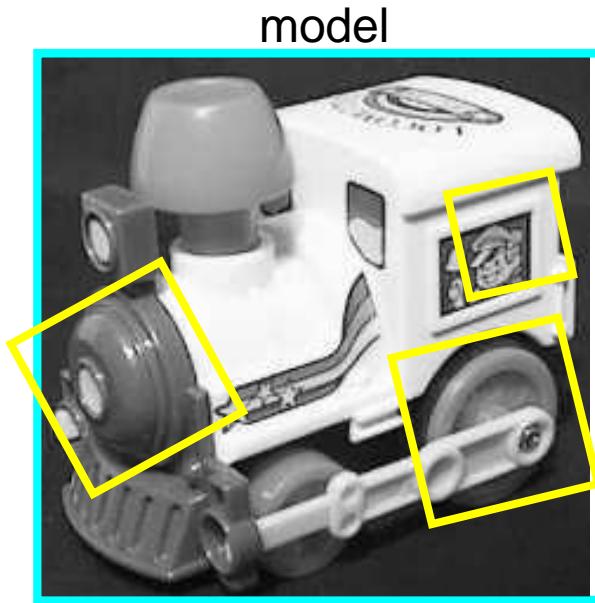
after RANSAC



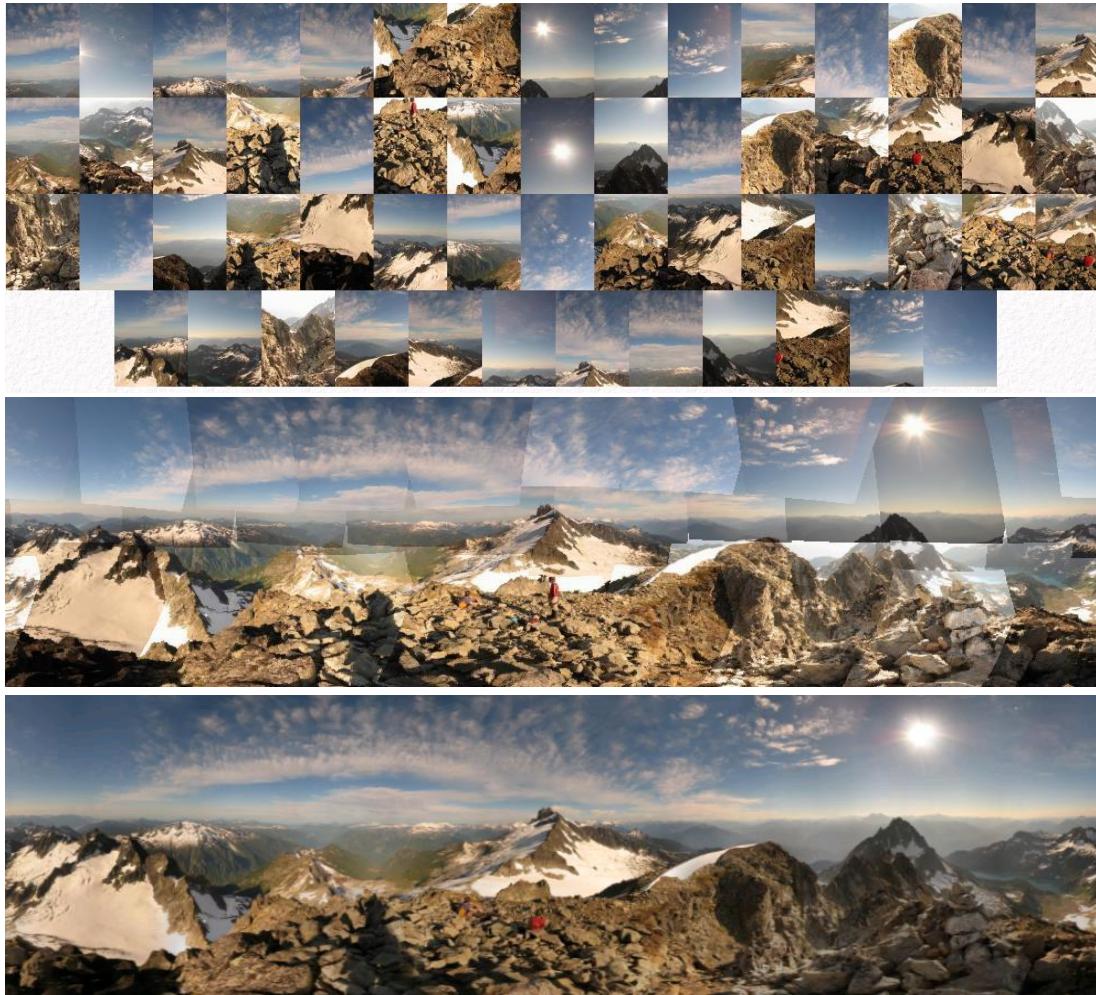
Images from Hartley & Zisserman

# Recap: Generalized Hough Transform

- Suppose our features are scale- and rotation-invariant
  - Then a single feature match provides an alignment hypothesis (translation, scale, orientation).
  - Of course, a hypothesis from a single match is unreliable.
  - Solution: let each match vote for its hypothesis in a Hough space with very coarse bins.



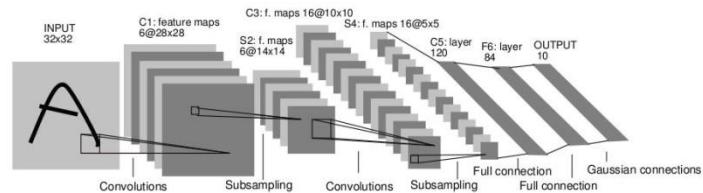
# Application: Panorama Stitching



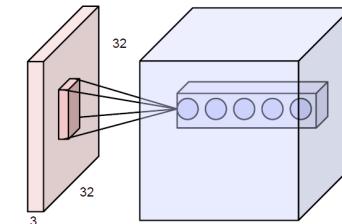
<http://www.cs.ubc.ca/~mbrown/autostitch/autostitch.html>

# Repetition

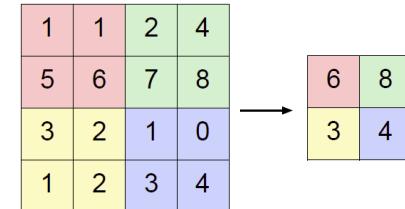
- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
  - Convolutional Neural Networks (CNNs)
  - Deep Learning Background
  - CNNs for Object Detection
  - CNNs for Semantic Segmentation
  - CNNs for Matching & RNNs
- 3D Reconstruction



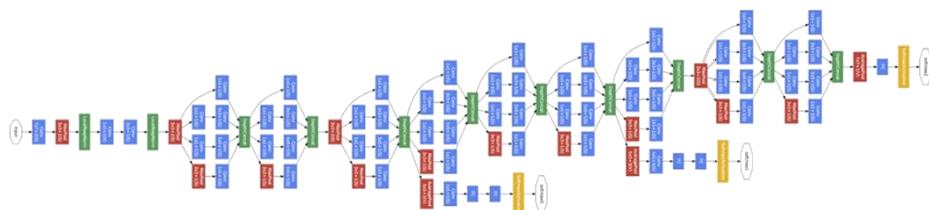
*Convolutional Neural Networks*



*Convolution layers*



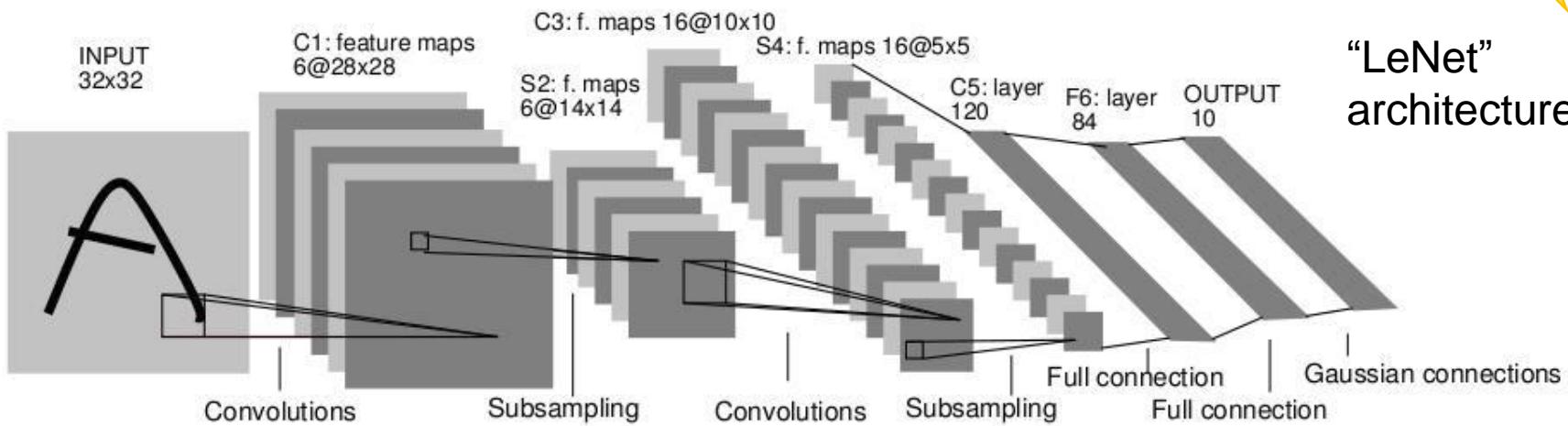
*Pooling layers*



*AlexNet, VGGNet, GoogLeNet, ResNet<sub>83</sub>*

# Recap: Convolutional Neural Networks

“LeNet”  
architecture

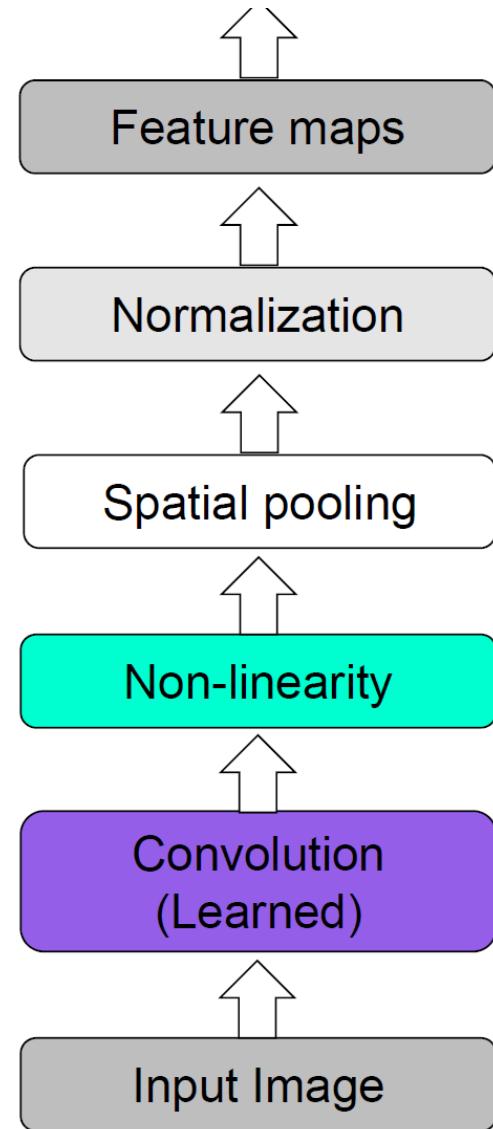


- Neural network with specialized connectivity structure
  - Stack multiple stages of feature extractors
  - Higher stages compute more global, more invariant features
  - Classification layer at the end

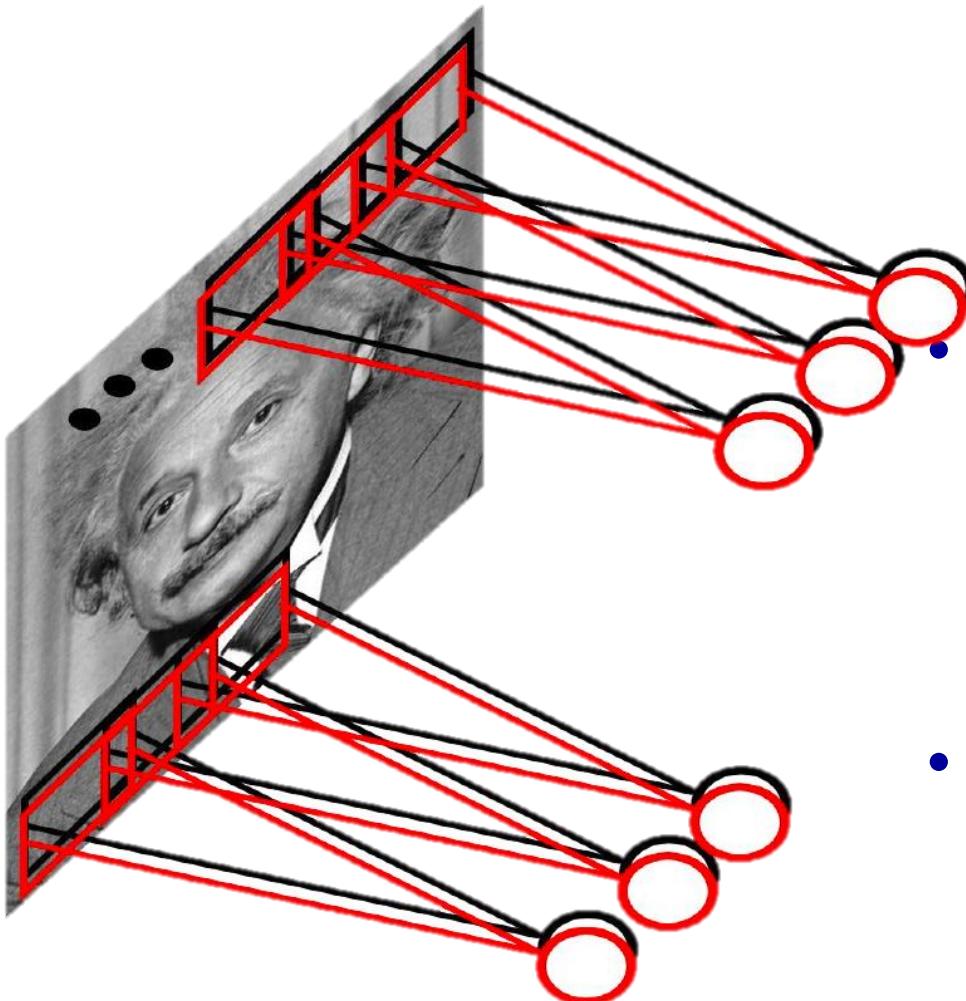
Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner, [Gradient-based learning applied to document recognition](#), Proceedings of the IEEE 86(11): 2278–2324, 1998.

# Recap: CNN Structure

- Feed-forward feature extraction
  1. Convolve input with learned filters
  2. Non-linearity
  3. Spatial pooling
  4. (Normalization)
- Supervised training of convolutional filters by back-propagating classification error

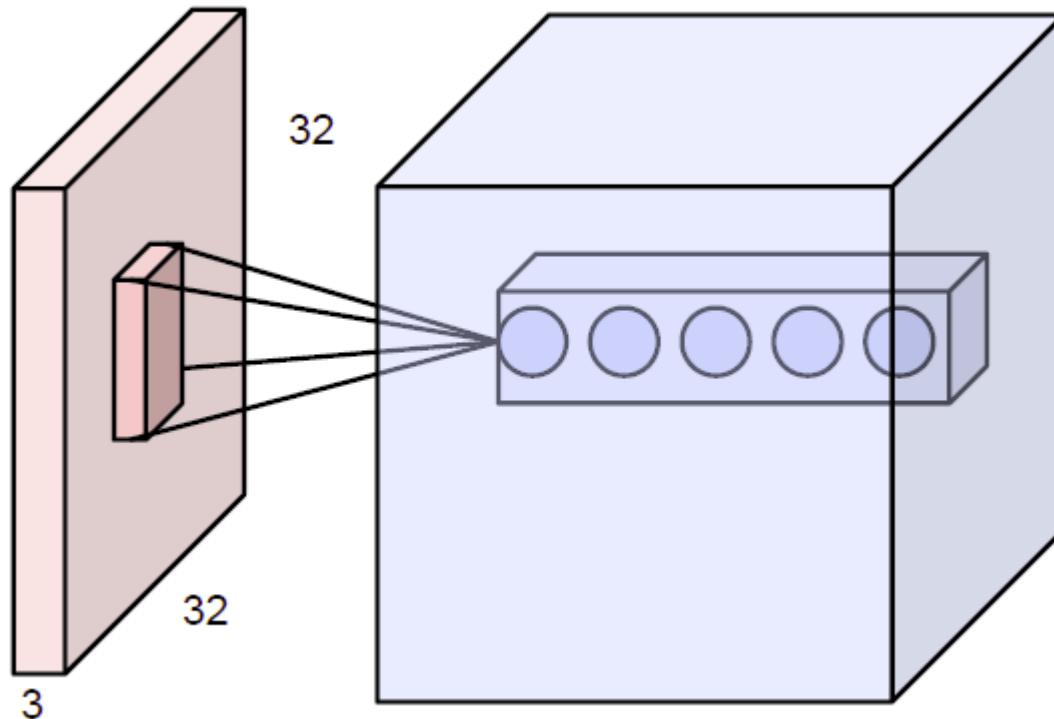


# Recap: Intuition of CNNs

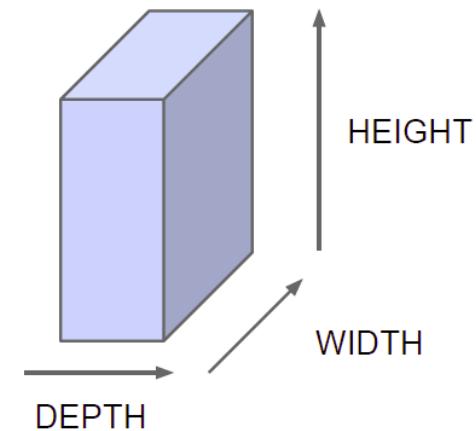


- **Convolutional network**
  - Share the same parameters across different locations
  - Convolutions with learned kernels
- Learn *multiple* filters
  - E.g.  $1000 \times 1000$  image  
100 filters  
 $10 \times 10$  filter size  
⇒ only 10k parameters
- **Result: Response map**
  - size:  $1000 \times 1000 \times 100$
  - Only memory, not params!

# Recap: Convolution Layers



Naming convention:



- All Neural Net activations arranged in 3 dimensions
  - Multiple neurons all looking at the same input region, stacked in depth
  - Form a single  $[1 \times 1 \times \text{depth}]$  depth column in output volume.

# Recap: Activation Maps

Activations:

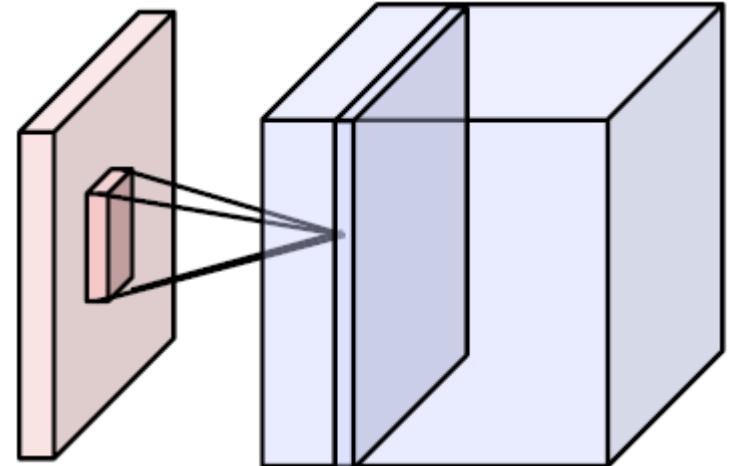


$5 \times 5$  filters

Activation

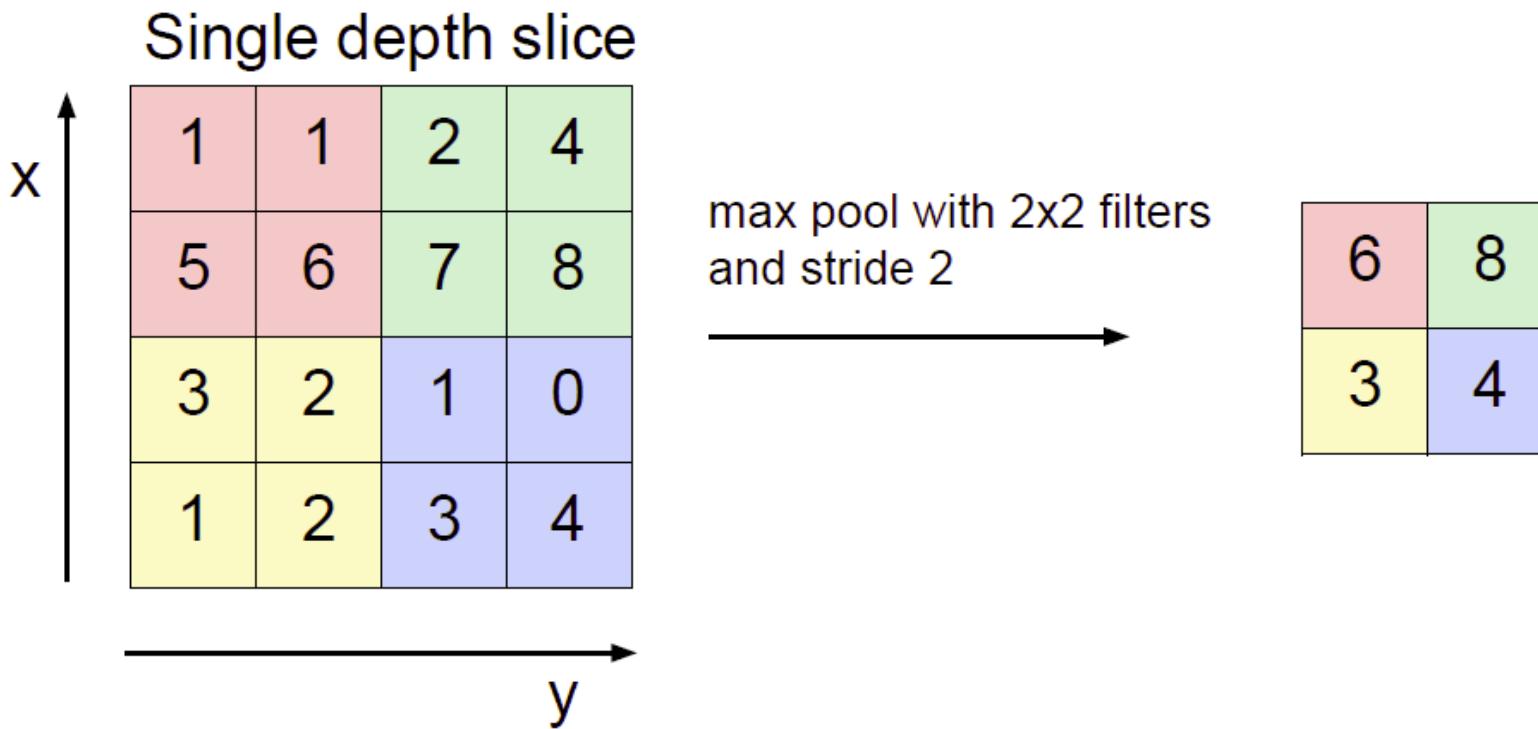


Activation maps



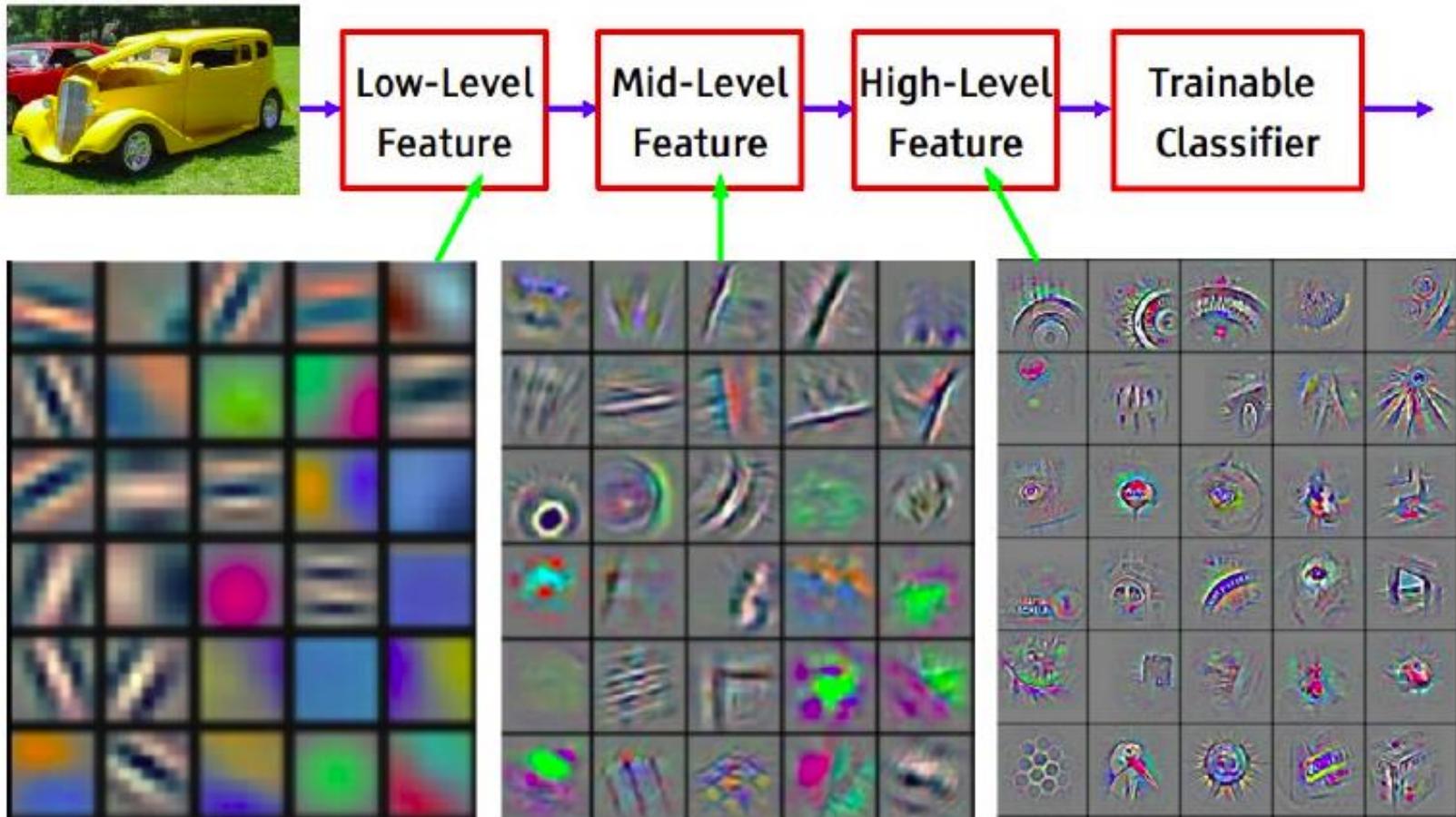
Each activation map is a depth slice through the output volume.

# Recap: Pooling Layers



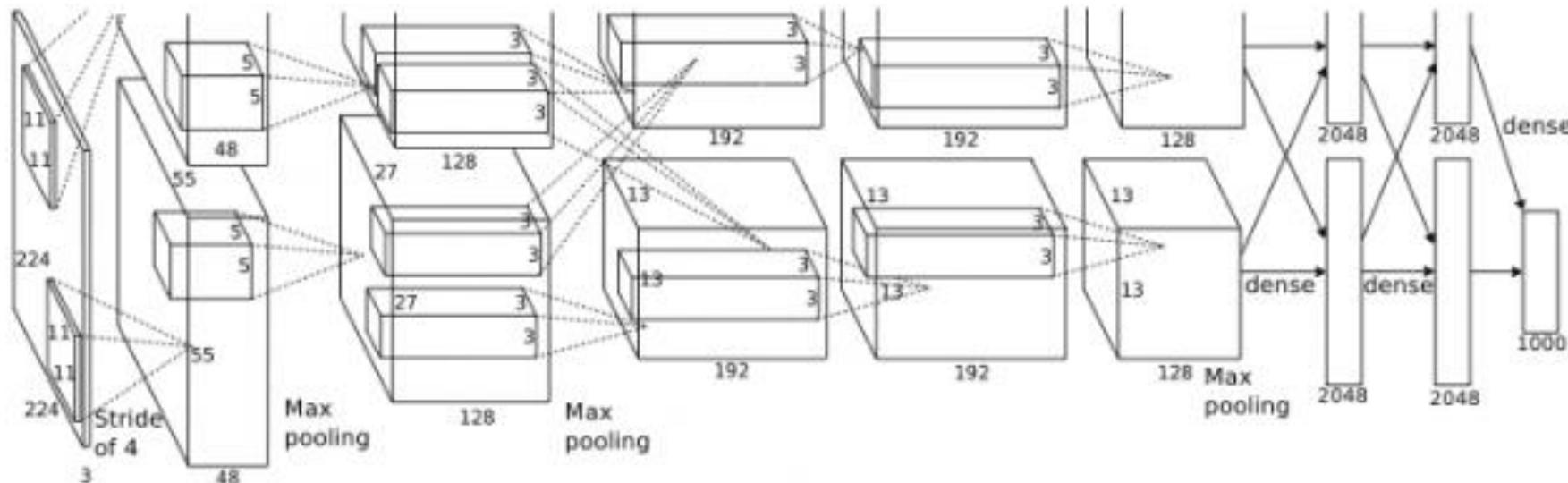
- Effect:
  - Make the representation smaller without losing too much information
  - Achieve robustness to translations

# Recap: Effect of Multiple Convolution Layers



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

# Recap: AlexNet (2012)



- Similar framework as LeNet, but
  - Bigger model (7 hidden layers, 650k units, 60M parameters)
  - More data ( $10^6$  images instead of  $10^3$ )
  - GPU implementation
  - Better regularization and up-to-date tricks for training (Dropout)

A. Krizhevsky, I. Sutskever, and G. Hinton, [ImageNet Classification with Deep Convolutional Neural Networks](#), NIPS 2012.

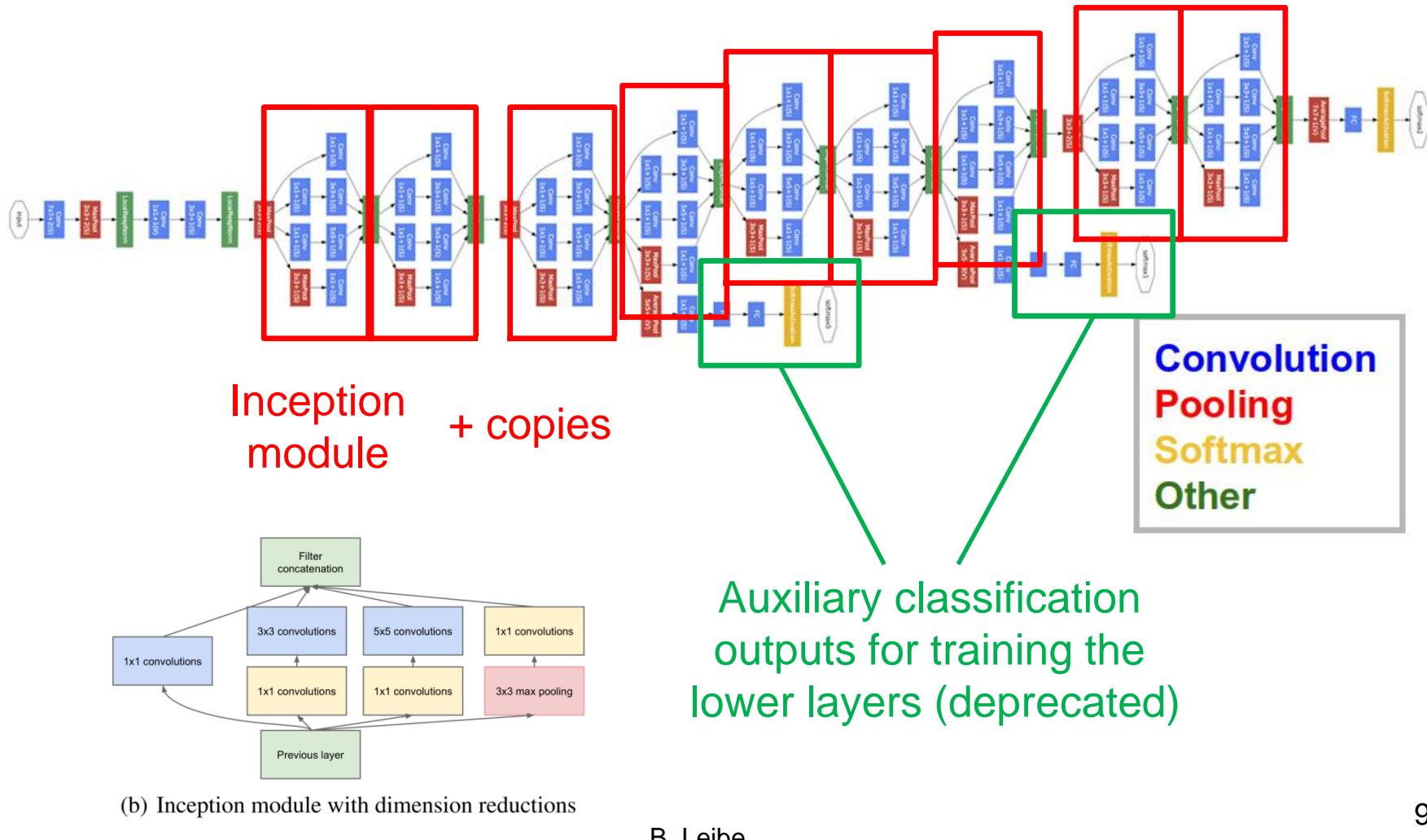
# Recap: VGGNet (2014/15)

- Main ideas
  - Deeper network
  - Stacked convolutional layers with smaller filters (+ nonlinearity)
  - Detailed evaluation of all components
- Results
  - Improved ILSVRC top-5 error rate to 6.7%.

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input ( $224 \times 224$ RGB image)					
conv3-64	conv3-64 LRN	conv3-64 <b>conv3-64</b>	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 <b>conv3-128</b>	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 <b>conv1-256</b>	conv3-256 conv3-256 <b>conv3-256</b>	conv3-256 conv3-256 <b>conv3-256</b> <b>conv3-256</b>
maxpool					
conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 <b>conv1-512</b>	conv3-512 conv3-512 <b>conv3-512</b>	conv3-512 conv3-512 <b>conv3-512</b> <b>conv3-512</b>
maxpool					
conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 <b>conv1-512</b>	conv3-512 conv3-512 <b>conv3-512</b>	conv3-512 conv3-512 <b>conv3-512</b> <b>conv3-512</b>
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Mainly used

# Recap: GoogLeNet (2014)



# Recap: Residual Networks

AlexNet, 8 layers  
(ILSVRC 2012)

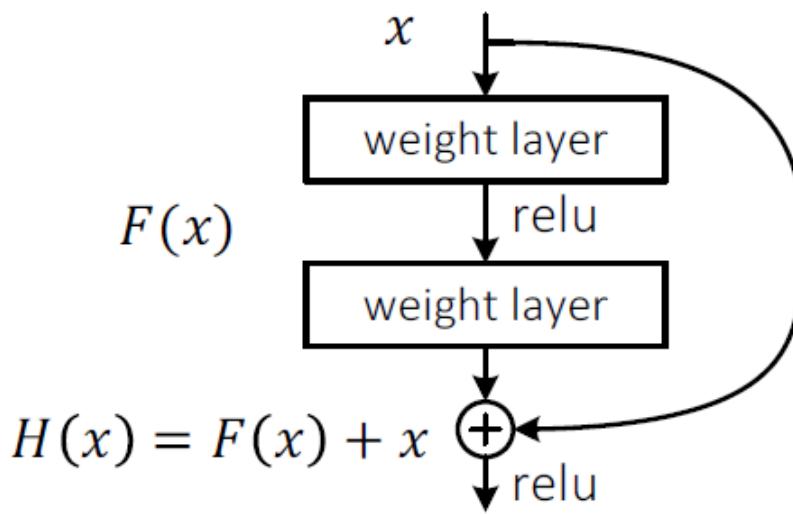


VGG, 19 layers  
(ILSVRC 2014)



ResNet, 152 layers  
(ILSVRC 2015)

- Core component
  - Skip connections bypassing each layer
  - Better propagation of gradients to the deeper layers
  - This makes it possible to train (much) deeper networks.



# Recap: Transfer Learning with CNNs



1. Train on ImageNet
2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

I.e., replace the Softmax layer at the end

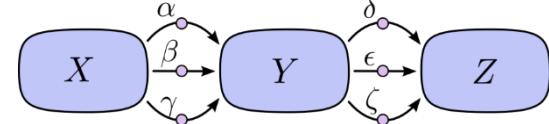
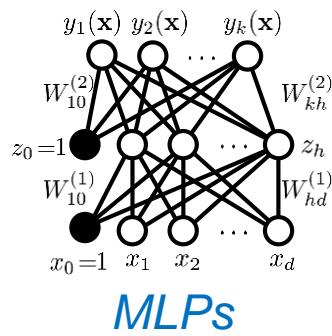


3. If you have a medium sized dataset, “finetune” instead: use the old weights as initialization, train the full network or only some of the higher layers.

Retrain bigger part of the network

# Repetition

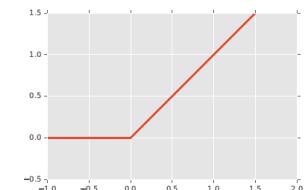
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*Backpropagation Algorithm*

$$w_{kj}^{(\tau+1)} = w_{kj}^{(\tau)} - \eta \left. \frac{\partial E(\mathbf{w})}{\partial w_{kj}} \right|_{\mathbf{w}^{(\tau)}}$$

*Gradient Descent*

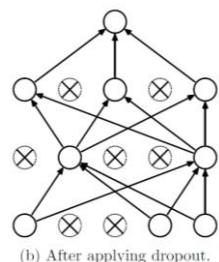


*ReLU*

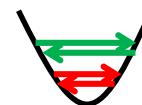
$$\text{Var}(W_i) = \frac{1}{n_{in}}$$

$$\text{Var}(W) = \frac{2}{n_{in}}$$

*Glorot & He Initialization*



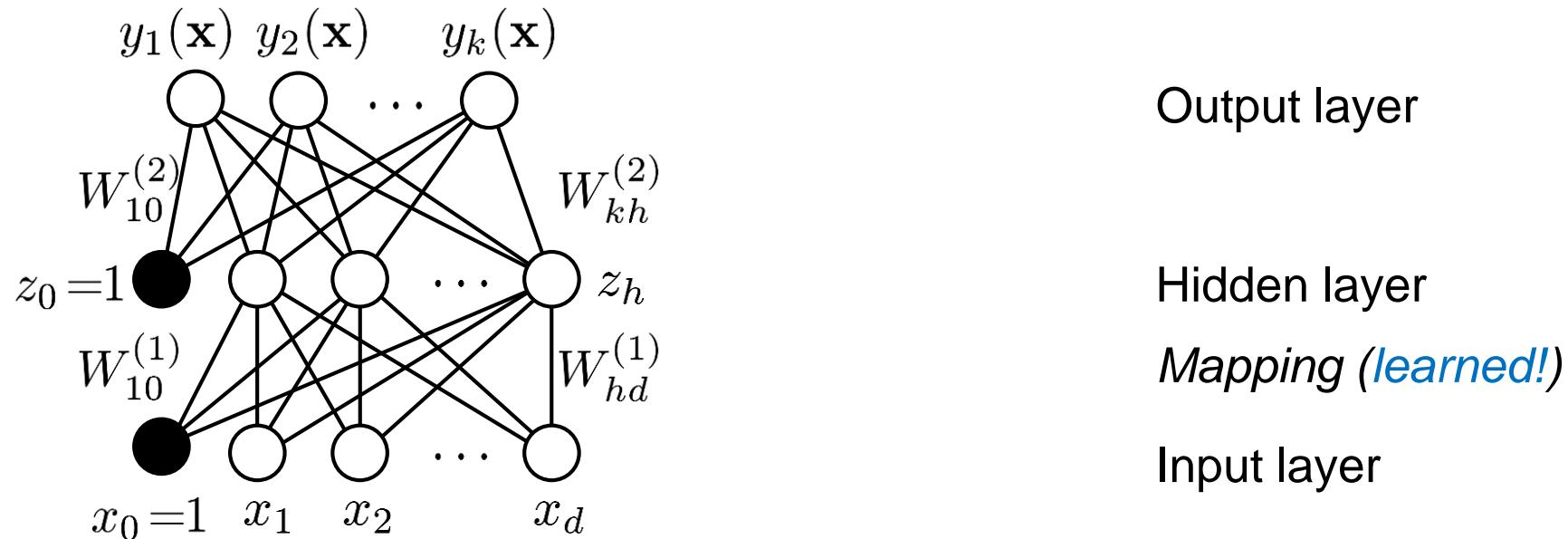
*Dropout*



*Learning Rate*

# Recap: Multi-Layer Perceptrons

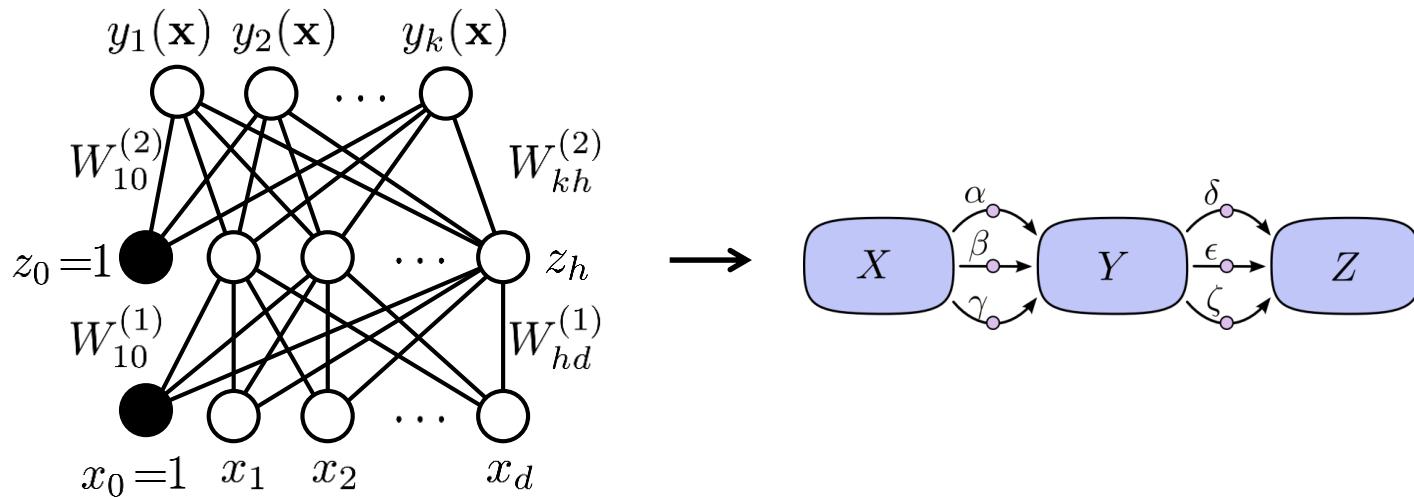
- Deep network = Also learning the feature transformation



- Output

$$y_k(\mathbf{x}) = g^{(2)} \left( \sum_{i=0}^h W_{ki}^{(2)} g^{(1)} \left( \sum_{j=0}^d W_{ij}^{(1)} x_j \right) \right)$$

# Recap: Backpropagation Algorithm



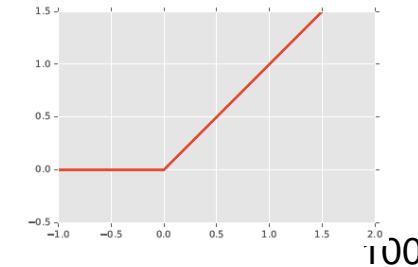
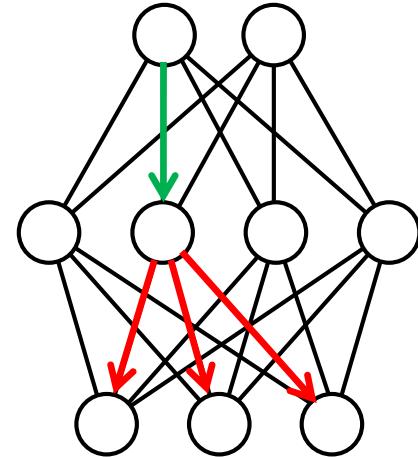
- General formulation (used in deep learning packages)
    - Convert the network into a computational graph.
    - Perform reverse-mode-differentiation this graph
    - Each new layer/module just needs to specify how it affects the
      - forward pass
      - backward pass
$$\mathbf{y} = \text{module.fprop}(\mathbf{x})$$
$$\frac{\partial E}{\partial \mathbf{x}} = \text{module.bprop}\left(\frac{\partial E}{\partial \mathbf{y}}\right)$$
- ⇒ Very general framework, *any differentiable layer* can be used.

# Recap: Supervised Learning

- Two main steps
  1. Computing the gradients for each weight (backprop)
  2. Adjusting the weights in the direction of the gradient
- Gradient Descent: Basic update equation
$$w_{kj}^{(\tau+1)} = w_{kj}^{(\tau)} - \eta \left. \frac{\partial E(\mathbf{w})}{\partial w_{kj}} \right|_{\mathbf{w}^{(\tau)}}$$
- Important considerations
  - On what data do we want to apply this?  $\Rightarrow$  Minibatches
  - How should we choose the step size  $\eta$  (the learning rate)?
  - More advanced optimizers (Momentum, RMSProp, Adam, ...)

# Recap: Practical Considerations

- **Vanishing gradients problem**
  - In multilayer nets, gradients need to be propagated through many layers
  - The **magnitudes of the gradients** are often very different for the different layers, especially if the initial weights are small.  
⇒ Gradients can get very small in the early layers of deep nets.
- When designing deep networks, we need to make sure gradients can be propagated throughout the network
  - By restricting the network depth (shallow networks are easier)
  - By very careful implementation (*numerics matter!*)
  - By choosing suitable nonlinearities (e.g., **ReLU**)
  - By performing proper initialization (**Glorot, He**)



# Recap: Glorot Initialization

[Glorot &amp; Bengio, '10]

- Variance of neuron activations
  - Suppose we have an input  $X$  with  $n$  components and a linear neuron with random weights  $W$  that spits out a number  $Y$ .
  - We want the variance of the input and output of a unit to be the same, therefore  $n \text{Var}(W_i)$  should be 1. This means

$$\text{Var}(W_i) = \frac{1}{n} = \frac{1}{n_{\text{in}}}$$

- Or for the backpropagated gradient

$$\text{Var}(W_i) = \frac{1}{n_{\text{out}}}$$

- As a compromise, Glorot & Bengio propose to use

$$\text{Var}(W) = \frac{2}{n_{\text{in}} + n_{\text{out}}}$$

⇒ Randomly sample the initial weights with this variance.

# Recap: He Initialization

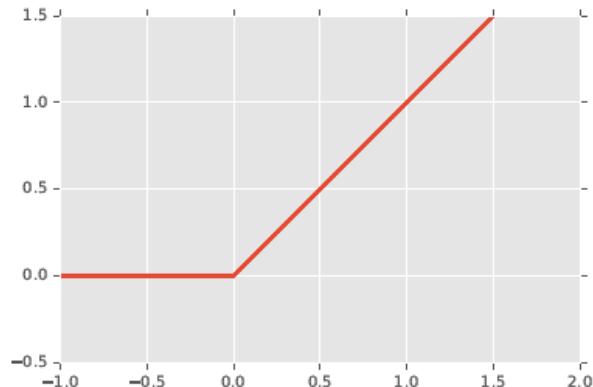
- Extension of Glorot Initialization to ReLU units

- Use Rectified Linear Units (ReLU)

$$g(a) = \max \{0, a\}$$

- Effect: gradient is propagated with a constant factor

$$\frac{\partial g(a)}{\partial a} = \begin{cases} 1, & a > 0 \\ 0, & \text{else} \end{cases}$$



- Same basic idea: Output should have the input variance
  - However, the Glorot derivation was based on tanh units, linearity assumption around zero does not hold for ReLU.
  - He et al. made the derivations, proposed to use instead

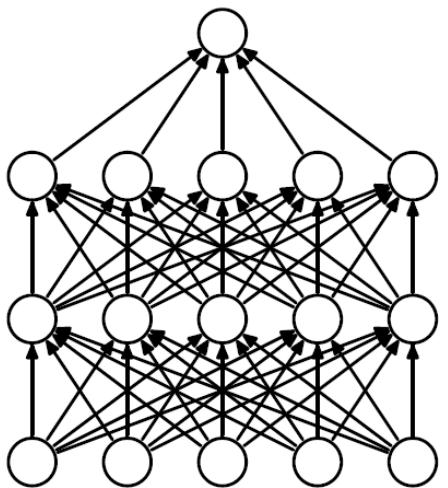
$$\text{Var}(W) = \frac{2}{n_{\text{in}}}$$

# Recap: Batch Normalization

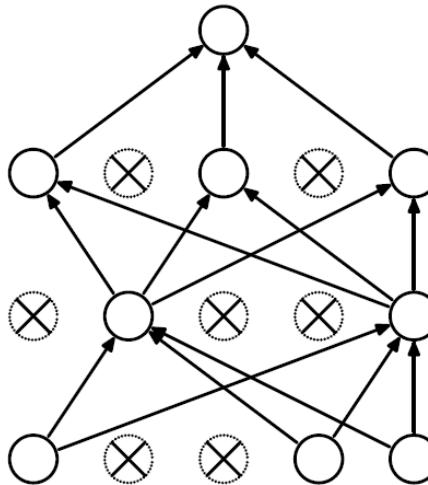
- Motivation
  - Optimization works best if all inputs of a layer are normalized.
- Idea
  - Introduce intermediate layer that centers the activations of the previous layer per minibatch.
  - I.e., perform transformations on all activations and undo those transformations when backpropagating gradients
  - **Complication:** centering + normalization also needs to be done at test time, but minibatches are no longer available at that point.
    - Learn the normalization parameters to compensate for the expected bias of the previous layer (usually a simple moving average)
- Effect
  - Much improved convergence (but parameter values are important!)
  - Widely used in practice

# Recap: Dropout

[Srivastava, Hinton '12]



(a) Standard Neural Net

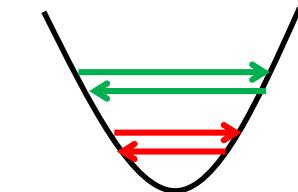
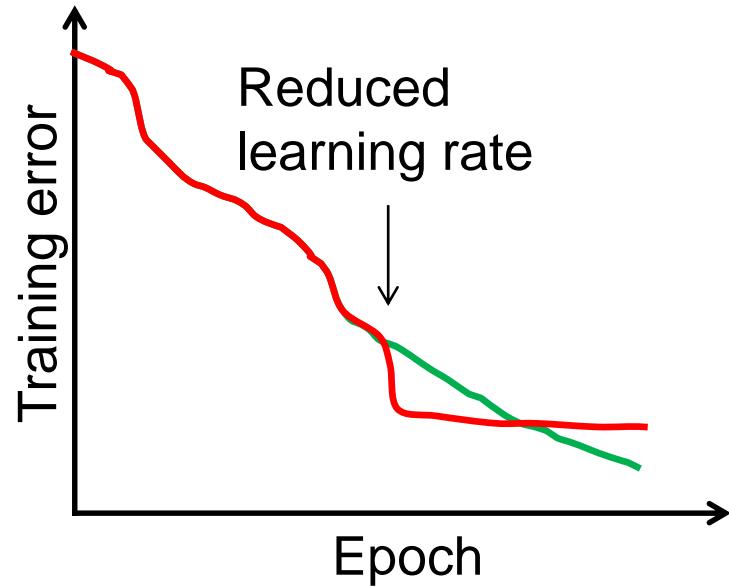


(b) After applying dropout.

- Idea
    - Randomly switch off units during training.
    - Change network architecture for each data point, effectively training many different variants of the network.
    - When applying the trained network, multiply activations with the probability that the unit was set to zero.
- ⇒ Improved performance

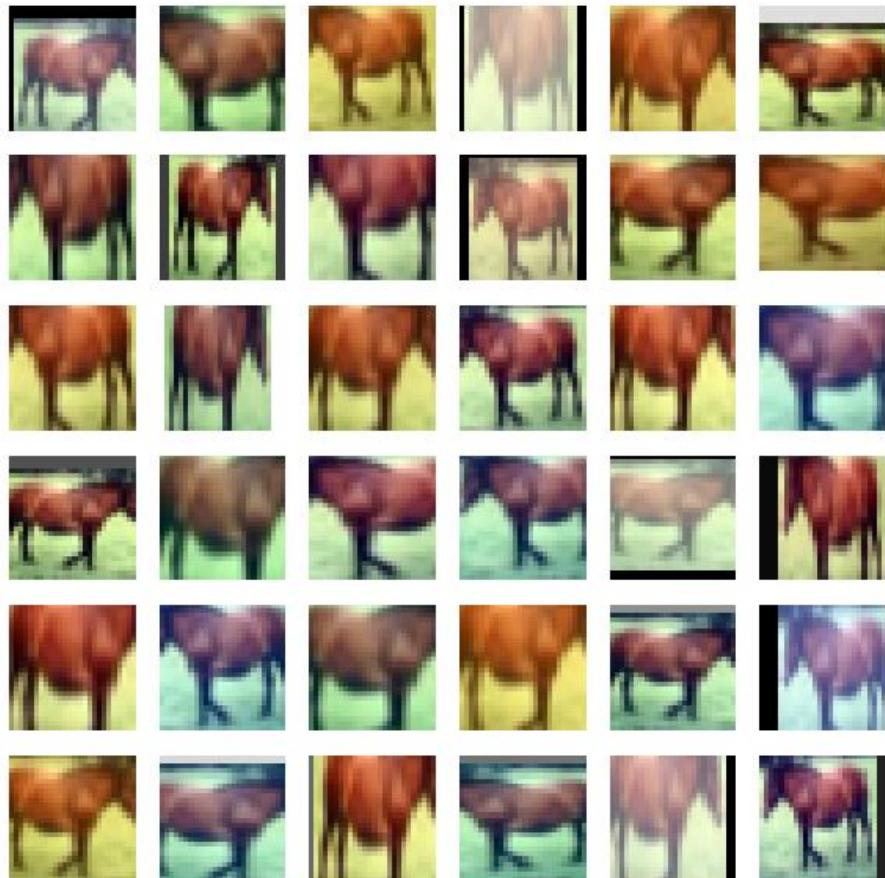
# Recap: Reducing the Learning Rate

- Final improvement step after convergence is reached
  - Reduce learning rate by a factor of 10.
  - Continue training for a few epochs.
  - Do this 1-3 times, then stop training.
- Effect
  - Turning down the learning rate will reduce the random fluctuations in the error due to different gradients on different minibatches.
- *Be careful: Do not turn down the learning rate too soon!*
  - Further progress will be much slower after that.



# Recap: Data Augmentation

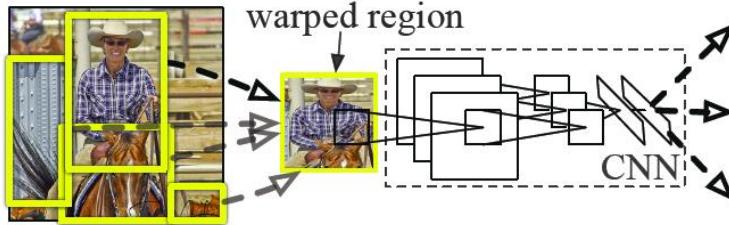
- Effect
  - Much larger training set
  - Robustness against expected variations
- During testing
  - When cropping was used during training, need to again apply crops to get same image size.
  - Beneficial to also apply flipping during test.
  - Applying several ColorPCA variations can bring another ~1% improvement, but at a significantly increased runtime.



Augmented training data  
(from one original image)

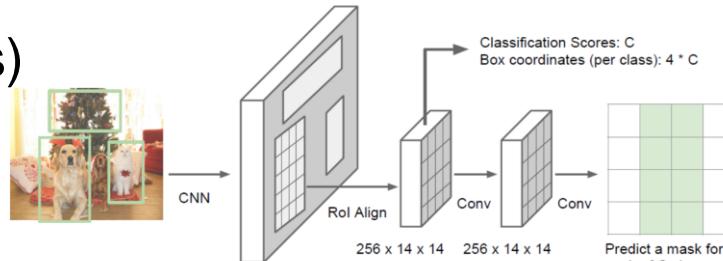
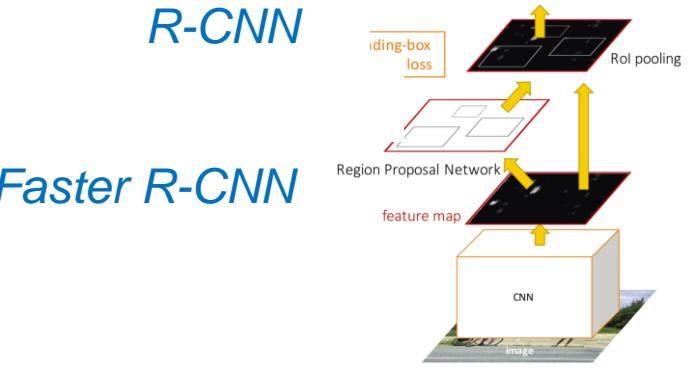
# Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
  - Convolutional Neural Networks (CNNs)
  - Deep Learning Background
  - **CNNs for Object Detection**
  - CNNs for Semantic Segmentation
  - CNNs for Matching & RNNs
- 3D Reconstruction

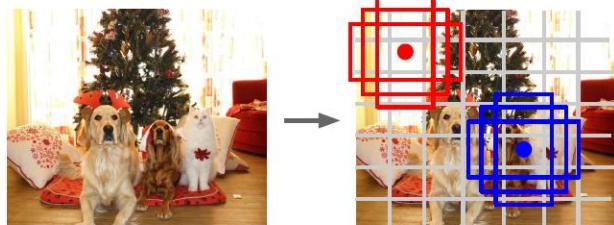


*R-CNN*

*Faster R-CNN*



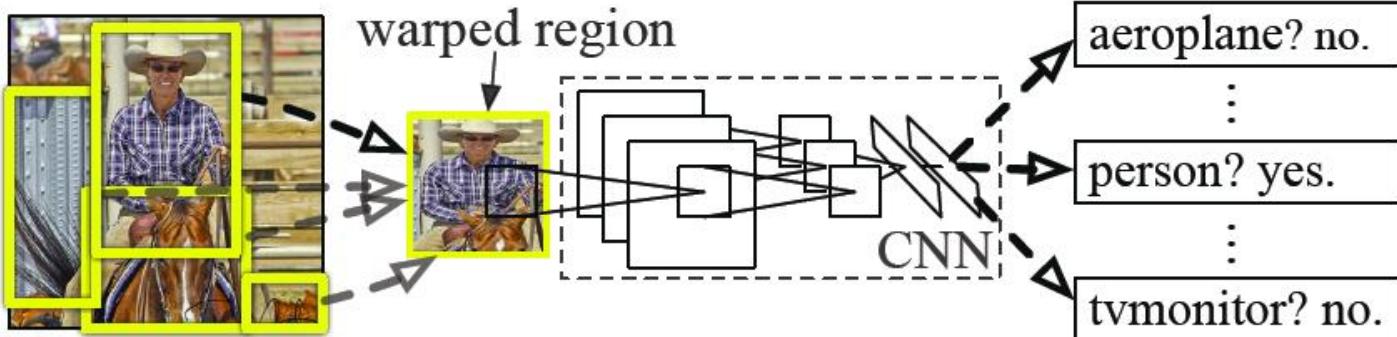
*Mask R-CNN*



*YOLO / SSD*

# Recap: R-CNN for Object Detection

## R-CNN: *Regions with CNN features*

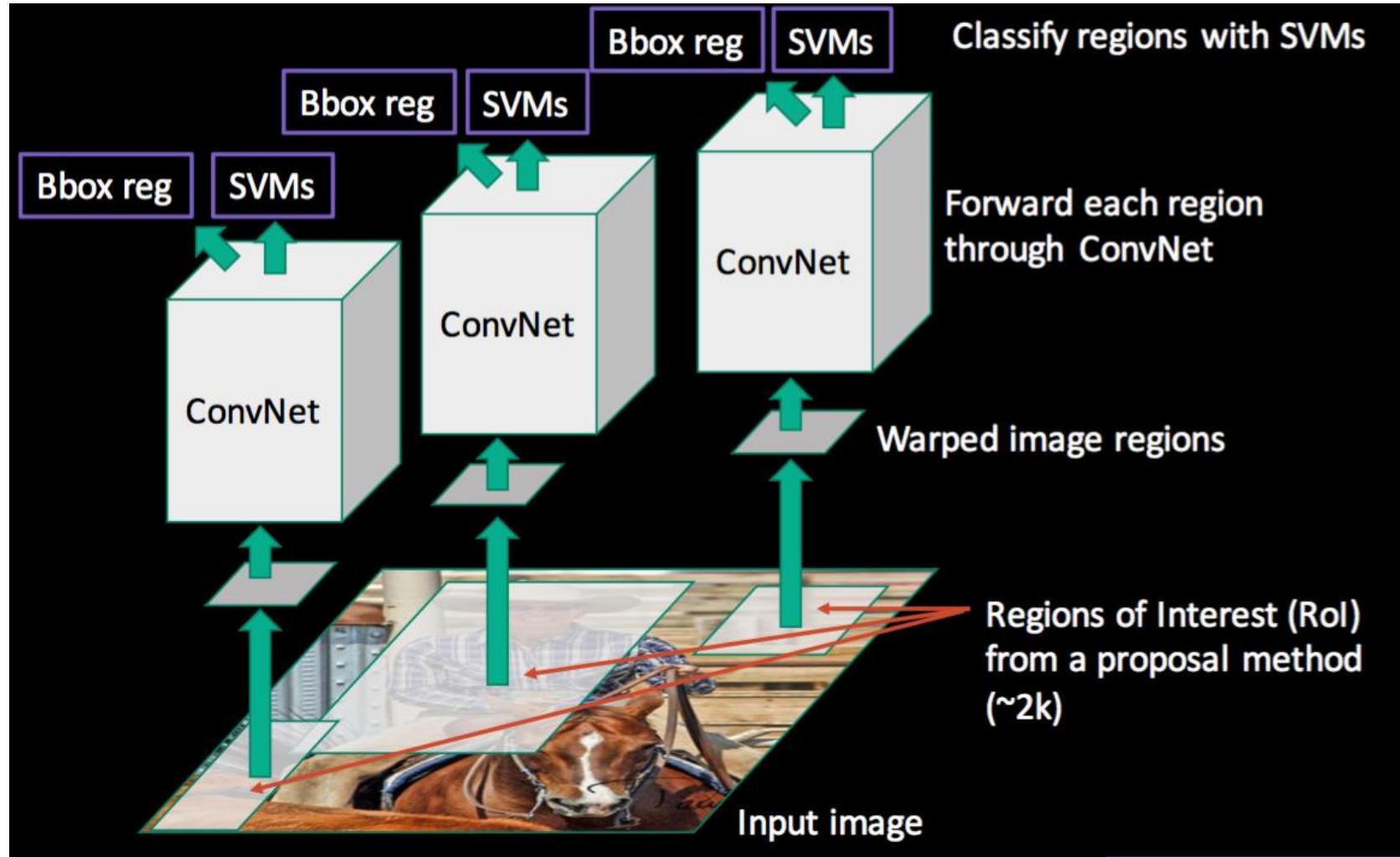


1. Input image
2. Extract region proposals (~2k)
3. Compute CNN features
4. Classify regions

- Key ideas
  - Extract region proposals (Selective Search)
  - Use a pre-trained/fine-tuned classification network as feature extractor (initially AlexNet, later VGGNet) on those regions

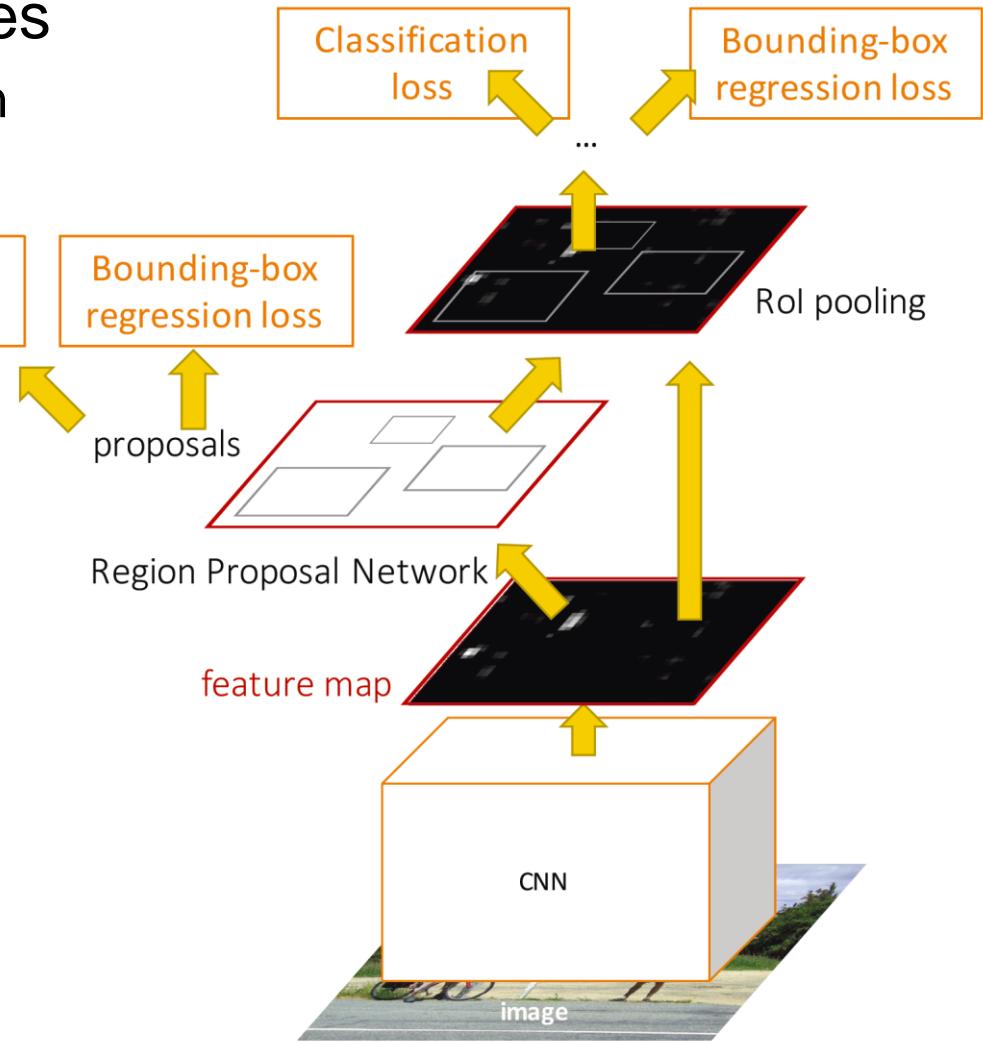
R. Girshick, J. Donahue, T. Darrell, and J. Malik, [Rich Feature Hierarchies for Accurate Object Detection and Semantic Segmentation](#), CVPR 2014

# Recap: R-CNN for Object Detection

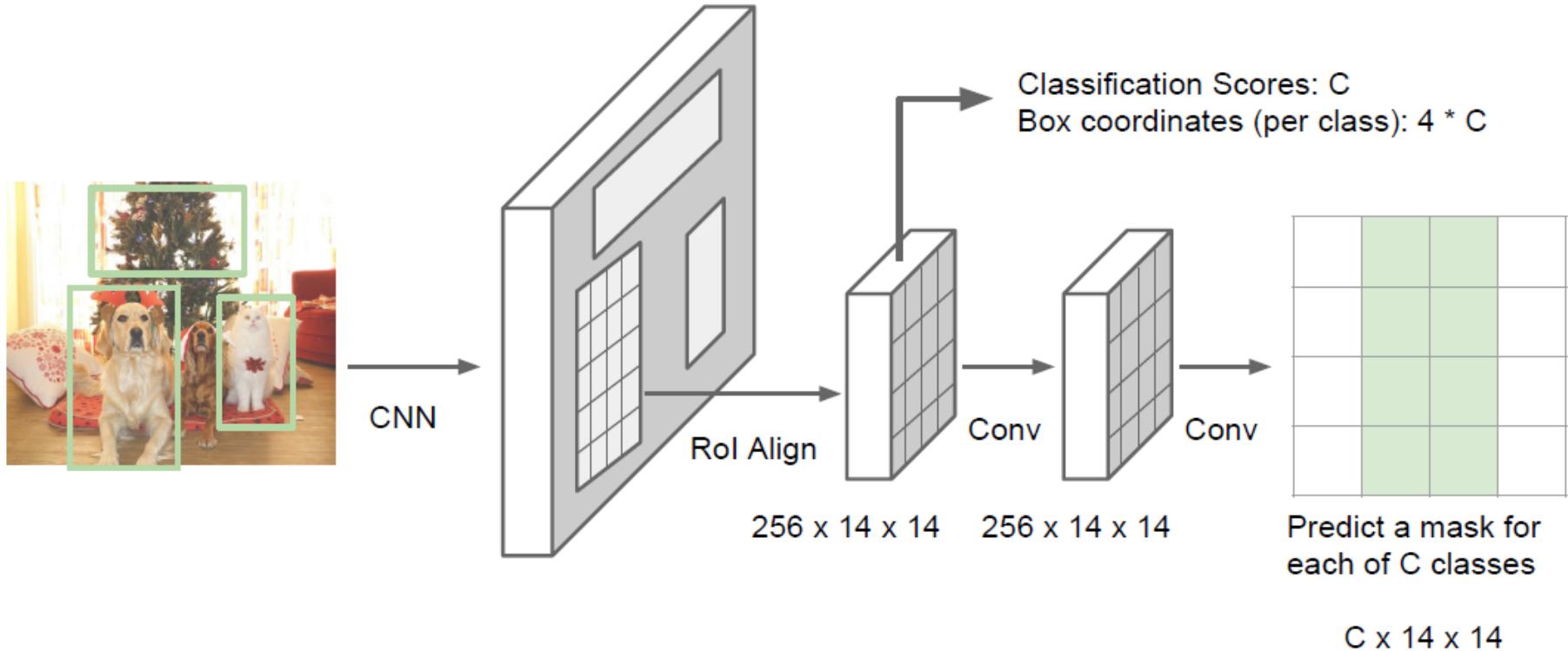


# Recap: Faster R-CNN

- One network, four losses
    - Remove dependence on external region proposal algorithm.
    - Instead, infer region proposals from same CNN.
    - Feature sharing
    - Joint training
- ⇒ Object detection in a single pass becomes possible.



# Recap: Mask R-CNN

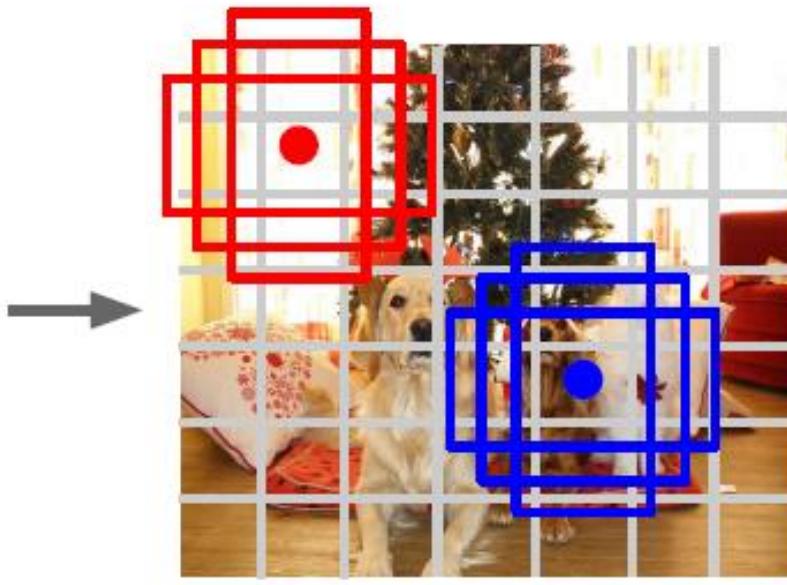


K. He, G. Gkioxari, P. Dollar, R. Girshick, [Mask R-CNN](#), arXiv 1703.06870.

# Recap: YOLO / SSD



Input image  
 $3 \times H \times W$

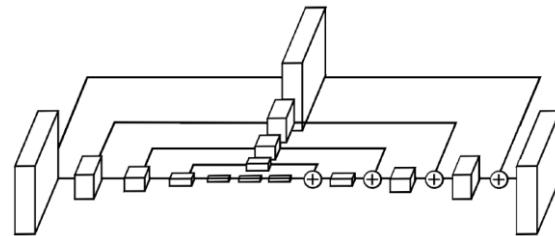


Divide image into grid  
 $7 \times 7$

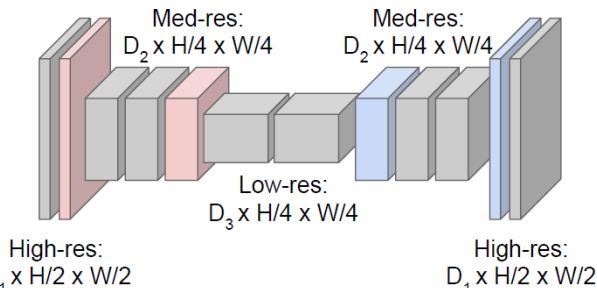
- Idea: Directly go from image to detection scores
- Within each grid cell
  - Start from a set of anchor boxes
  - Regress from each of the  $B$  anchor boxes to a final box
  - Predict scores for each of  $C$  classes (including background)

# Repetition

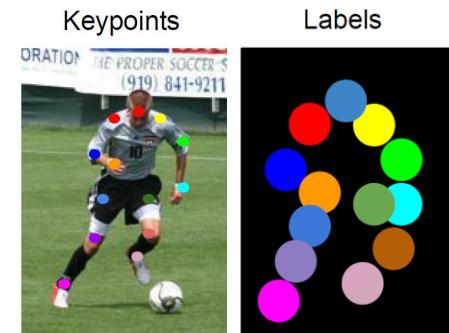
- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
  - Convolutional Neural Networks (CNNs)
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  - **CNNs for Semantic Segmentation**
  - CNNs for Matching & RNNs
- 3D Reconstruction



*Fully Convolutional Networks*



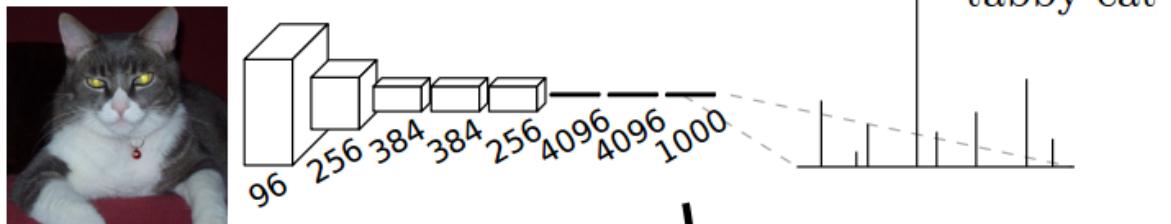
*Encoder-Decoder Architecture*



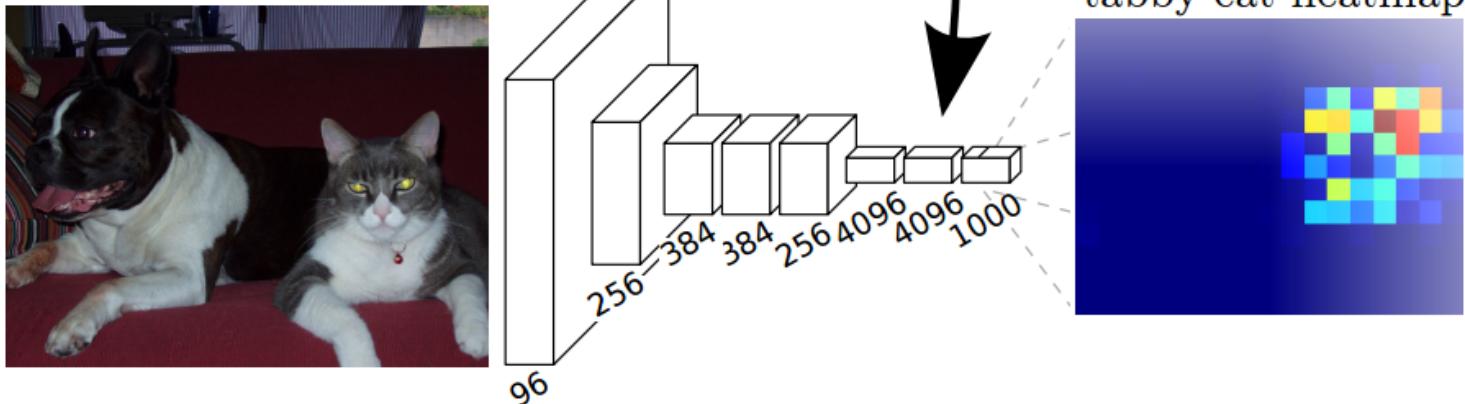
*Human Pose Estimation*

# Recap: Fully Convolutional Networks

- CNN



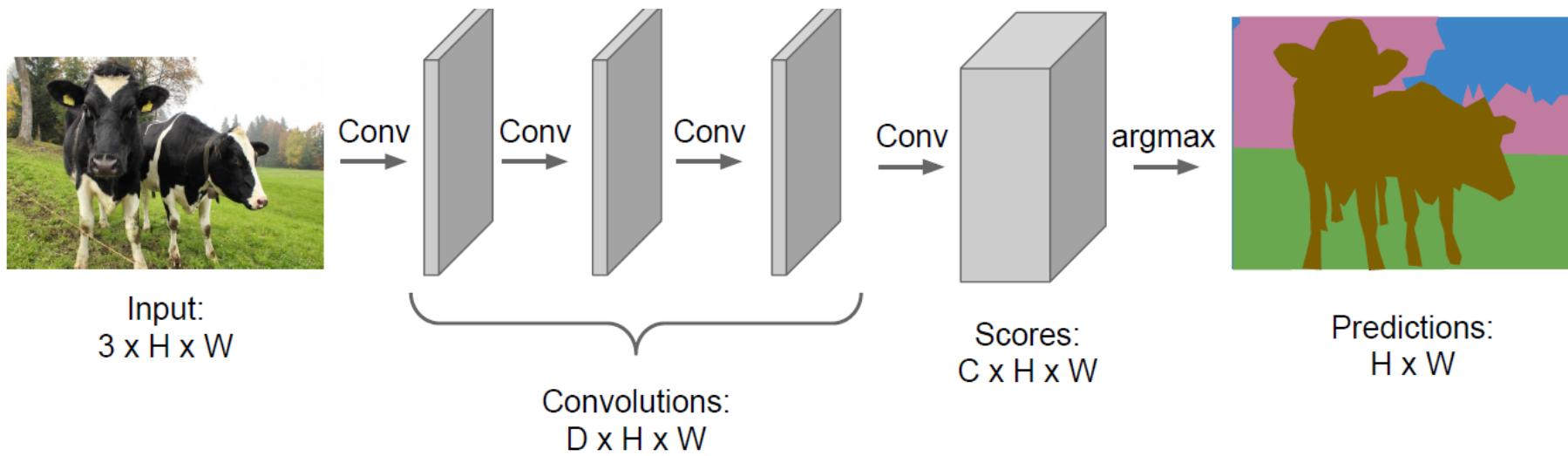
- FCN



- Intuition

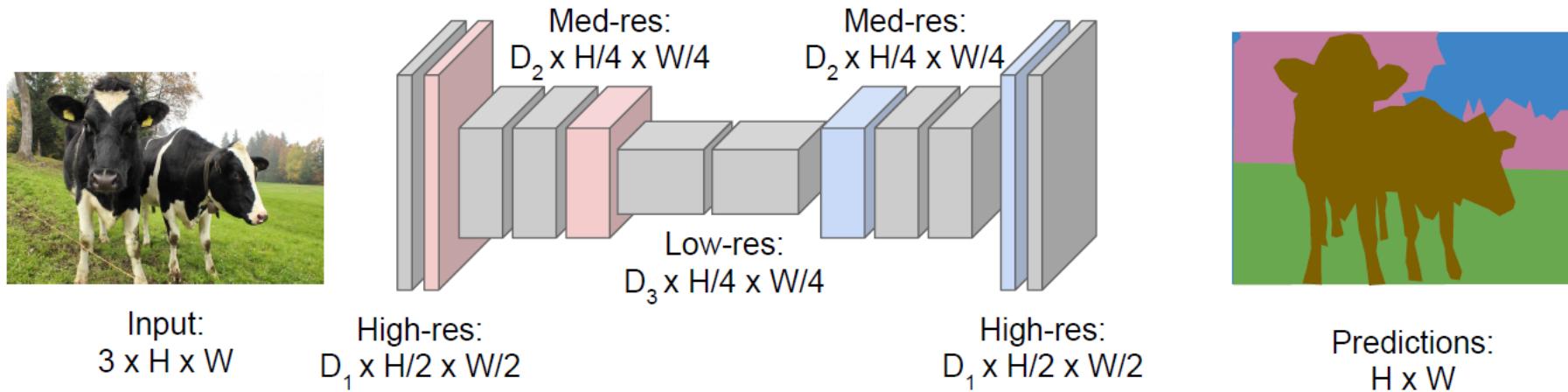
- Think of FCNs as performing a sliding-window classification, producing a heatmap of output scores for each class

# Recap: Fully-Convolutional Networks



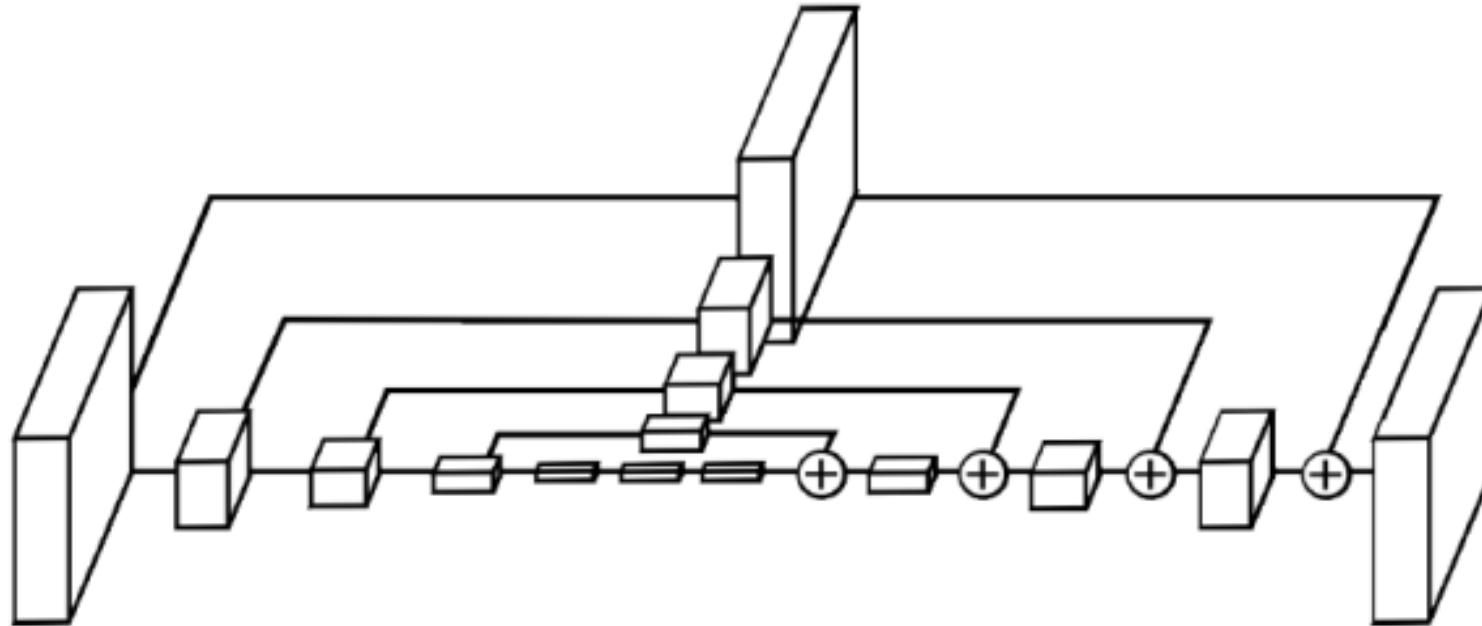
- Design a network as a sequence of convolutional layers
  - To make predictions for all pixels at once
  - **Fully Convolutional Networks (FCNs)**
    - All operations formulated as convolutions
    - Fully-connected layers become  $1 \times 1$  convolutions
    - Advantage: can process arbitrarily sized images

# Recap: Encoder-Decoder Architecture



- Design a network as a sequence of convolutional layers
  - With **downsampling** and **upsampling** inside the network!
  - **Downsampling**
    - Pooling, strided convolution
  - **Upsampling**
    - Unpooling or strided transpose convolution

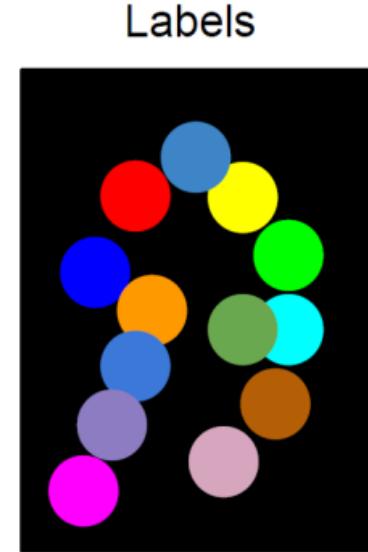
# Recap: Skip Connections



- Encoder-Decoder Architecture with skip connections
  - Problem: downsampling loses high-resolution information
  - Use skip connections to preserve this higher-resolution information

# Recap: FCNs for Human Pose Estimation

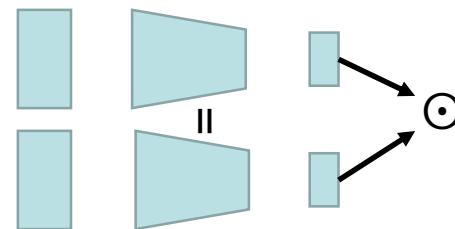
- Input data



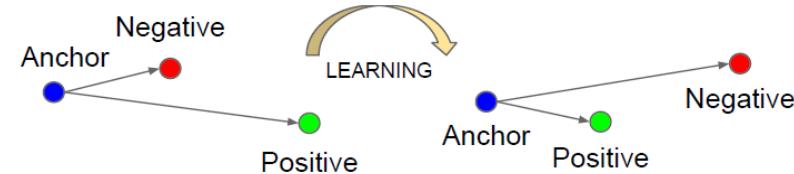
- Formulate pose estimation as a segmentation problem
  - Annotate images with keypoints for skeleton joints
  - Define a target disk around each keypoint with radius  $r$
  - Set the ground-truth label to 1 within each such disk
  - Infer heatmaps for the joints as in semantic segmentation

# Repetition

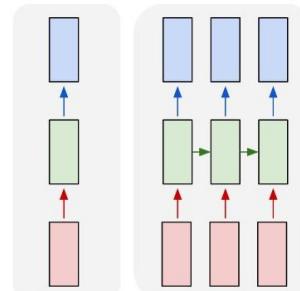
- Image Processing Basics
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  - Convolutional Neural Networks (CNNs)
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  - CNNs for Semantic Segmentation
  - **CNNs for Matching & RNNs**
- 3D Reconstruction



*Siamese Networks*



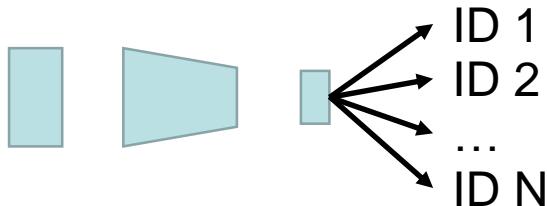
*Triplet Loss*



*Recurrent Neural Networks*

# Recap: Types of Models used for Matching Tasks

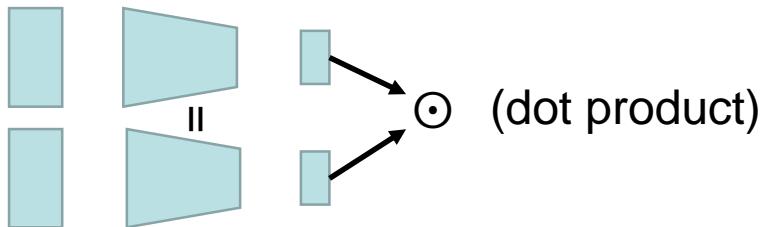
- Identification models (I)



Training

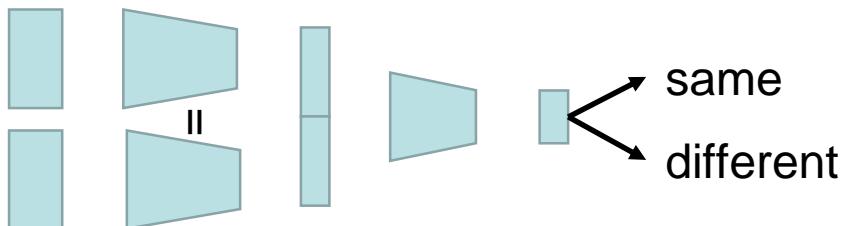
Multi-class  
classification loss

- Embedding models (E)



Large-margin loss,  
Triplet loss

- Verification models (V)



Two-class  
classification loss

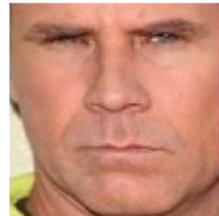
# Triplet Loss Networks

- Learning a discriminative embedding
  - Present the network with triplets of examples

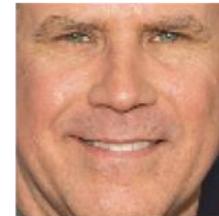
Negative



Anchor

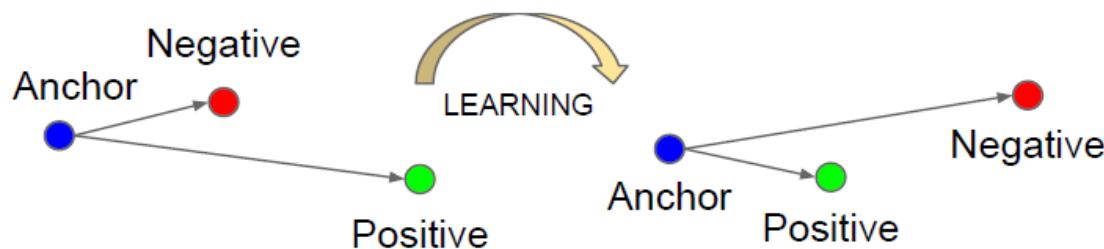


Positive



- Apply triplet loss to learn an embedding  $f(\cdot)$  that groups the positive example closer to the anchor than the negative one.

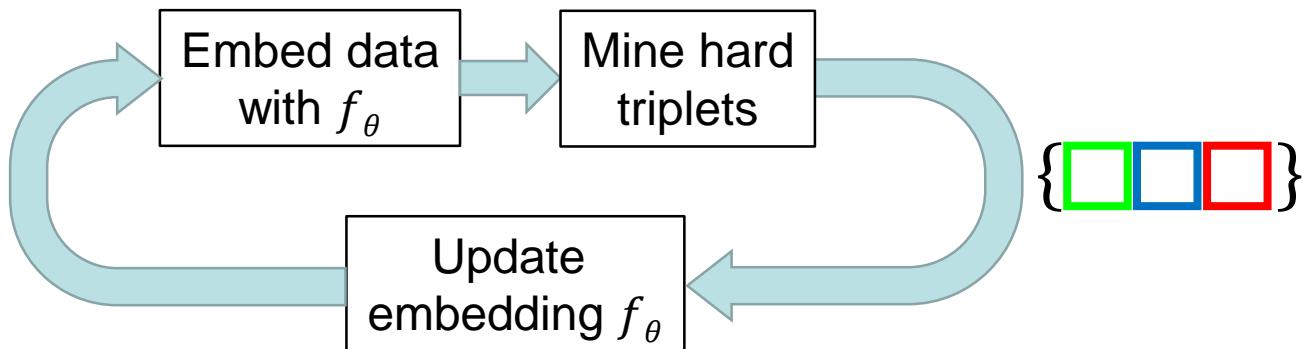
$$\|f(x_i^a) - f(x_i^p)\|_2^2 < \|f(x_i^a) - f(x_i^n)\|_2^2$$



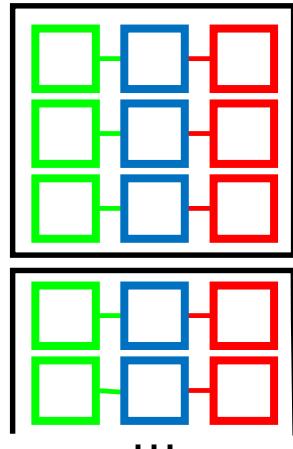
⇒ Used with great success in Google's FaceNet face identification

# Offline Hard Triplet Mining

- Considerable effort needed



- Using the triplets for learning
  - Minibatch learning

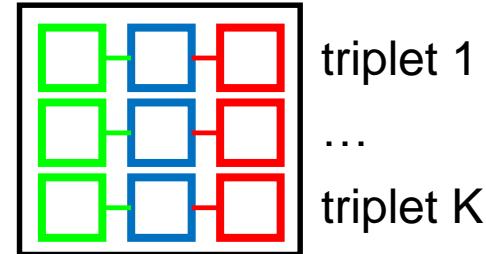


This is a very  
wasteful design!

# Better: Online Hard Triplet Mining

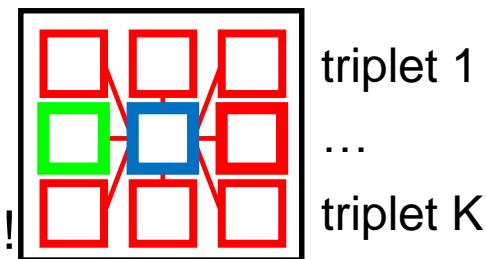
- Core idea

- Core idea
  - The minibatch contains many more potential triplets than the ones that were mined!
  - Why not make use of those also?



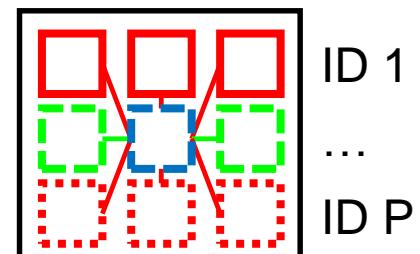
- Possible improvement

- Possible improvement
  - Each member of another triplet becomes an additional negative candidate
  - But: need both hard negatives *and* hard positives!

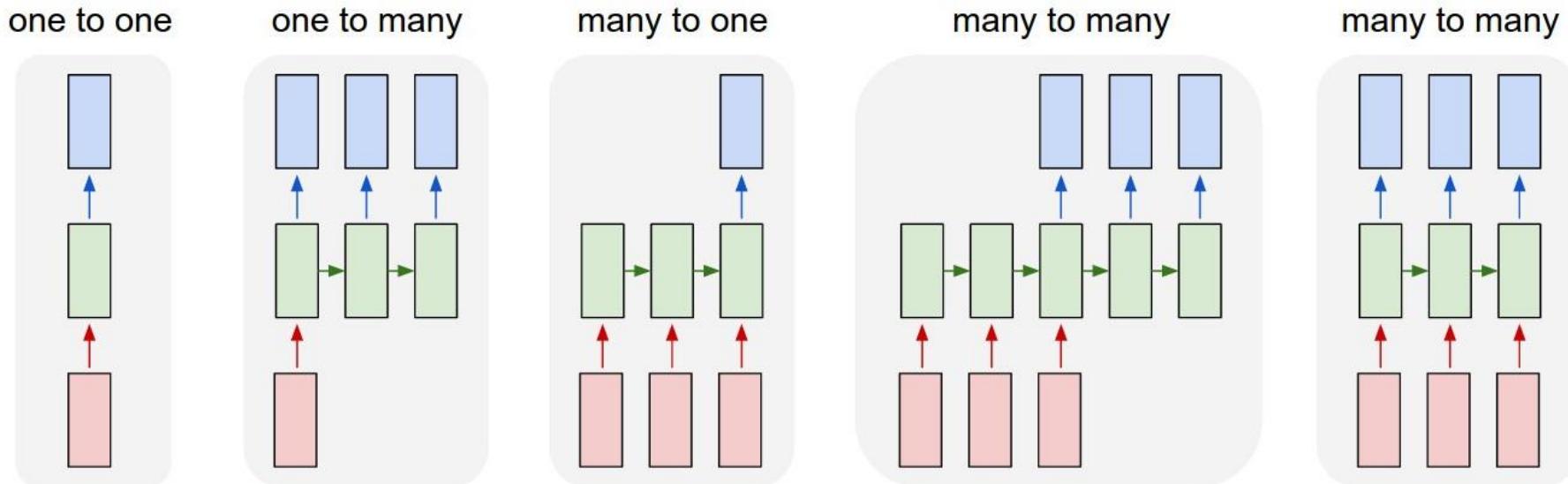


- Better design

- Better design
  - Sample K images from P classes (=people) for each minibatch
  - Triplets are only constructed within the minibatch



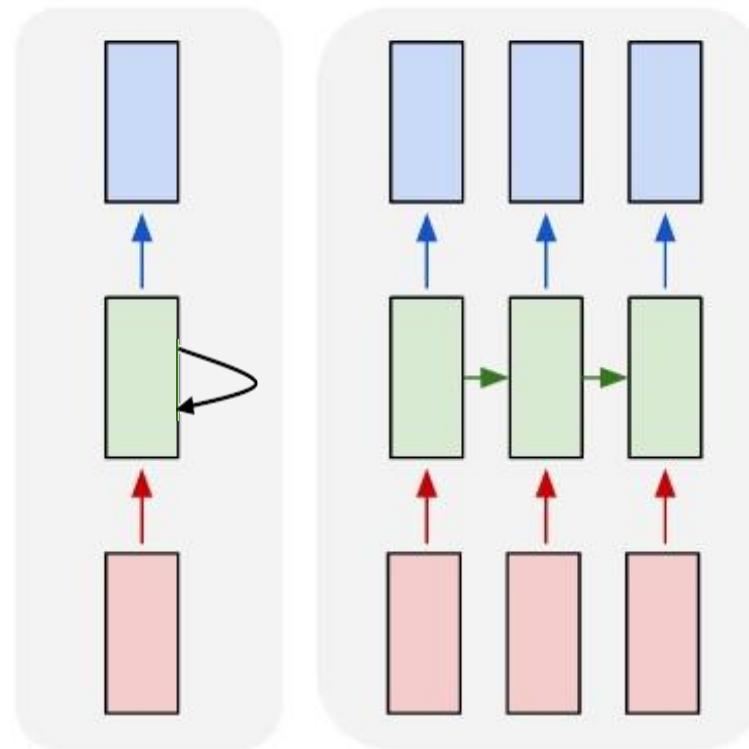
# Recap: Recurrent Neural Networks



- Up to now
  - Simple neural network structure: 1-to-1 mapping of inputs to outputs
- Recurrent Neural Networks
  - Generalize this to arbitrary mappings

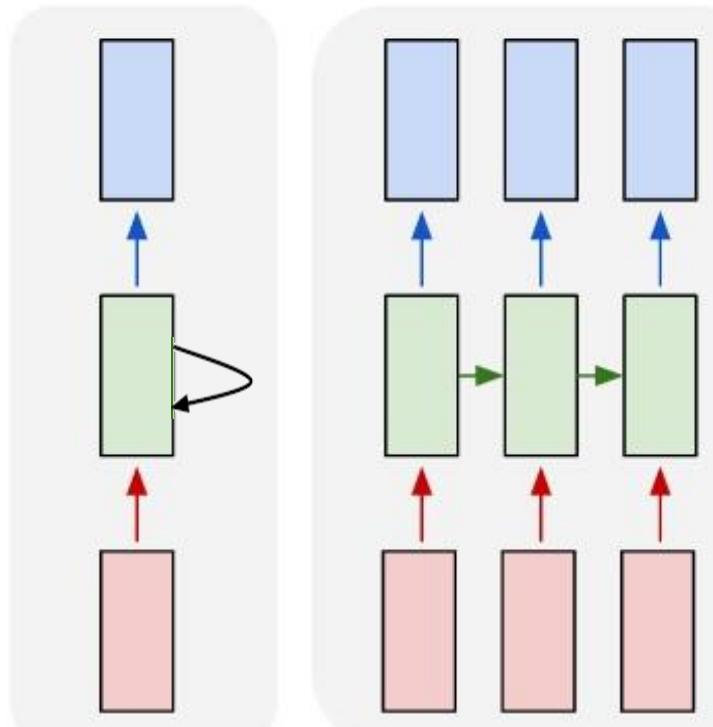
# Recap: RNNs

- RNNs are regular NNs whose hidden units have additional forward connections over time.
  - You can **unroll** them to create a network that extends over time.
  - When you do this, keep in mind that the weights for the hidden units are shared between temporal layers.

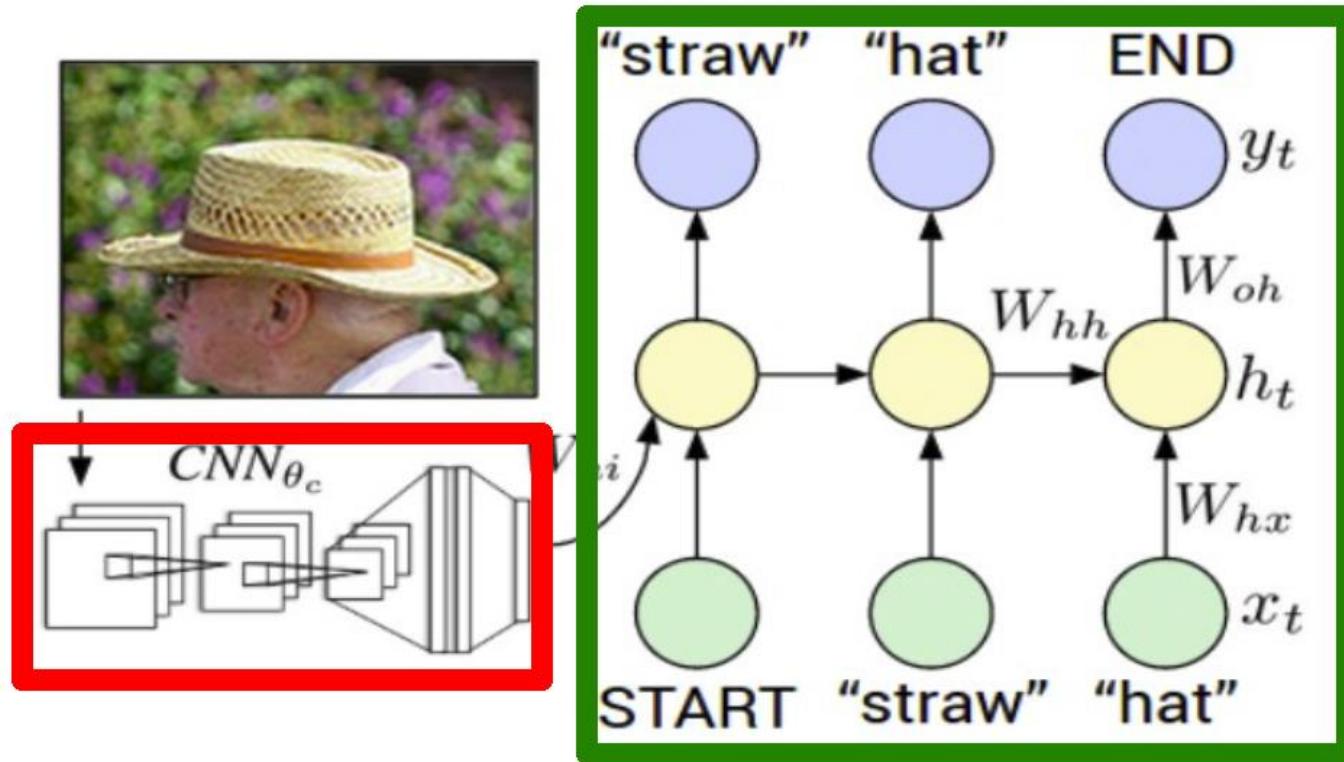


# Recap: RNNs

- RNNs are very powerful, because they combine two properties:
  - Distributed hidden state that allows them to store a lot of information about the past efficiently.
  - Non-linear dynamics that allows them to update their hidden state in complicated ways.
- With enough neurons and time, RNNs can compute anything that can be computed by your computer.
- Training is more challenging (unrolled networks are deep)
  - See *Machine Learning lecture for details...*



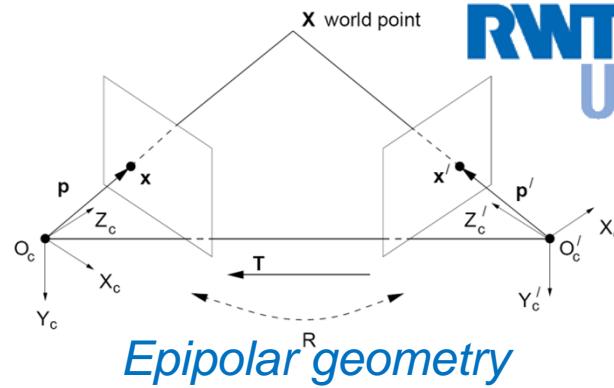
# Recap: Applications – Image Tagging



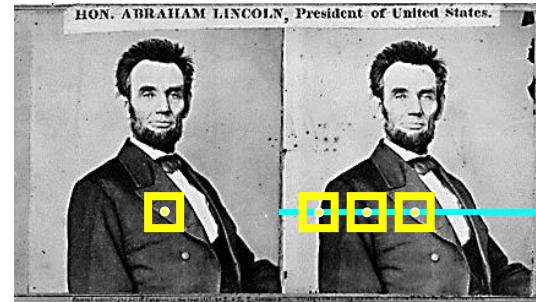
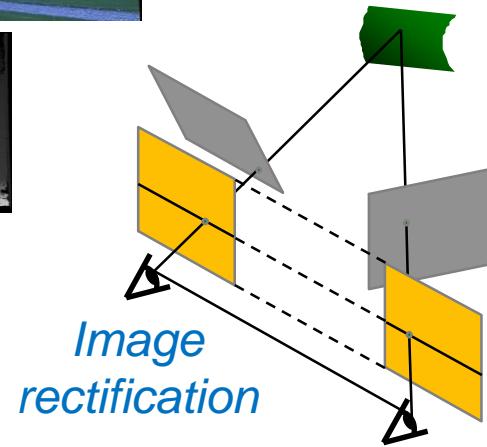
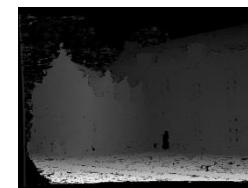
- Simple combination of CNN and RNN
  - Use CNN to define initial state  $\mathbf{h}_0$  of an RNN.
  - Use RNN to produce text description of the image.

# Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
- 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera Calibration & Uncalibrated Reconstruction
  - Structure-from-Motion



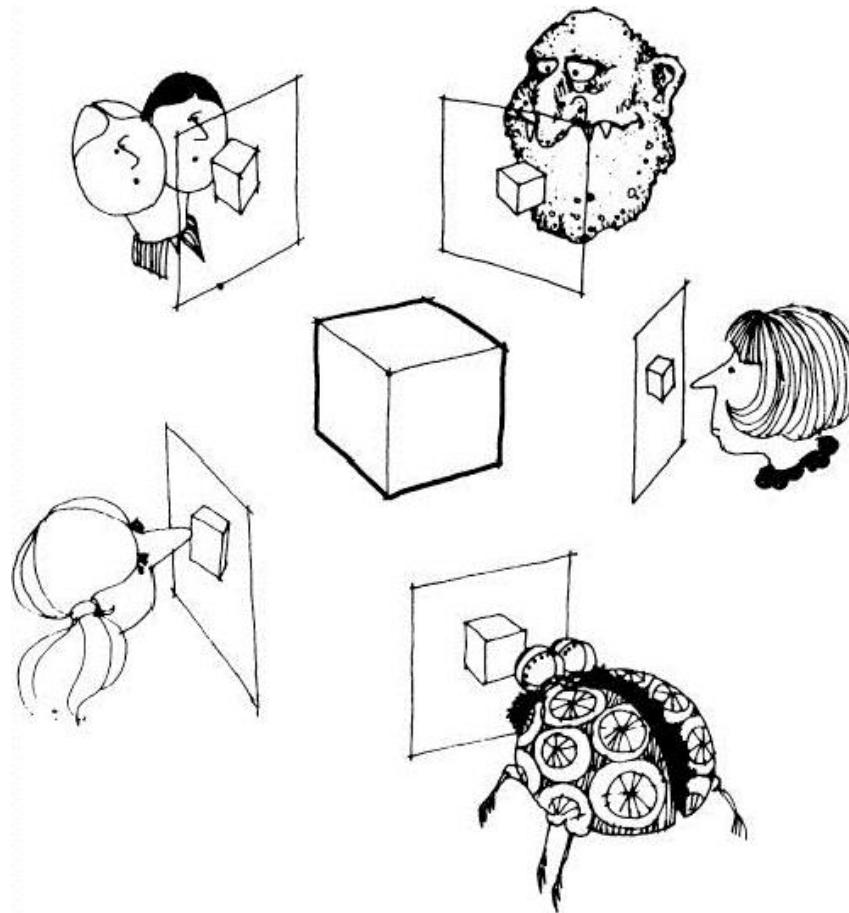
*Disparity*



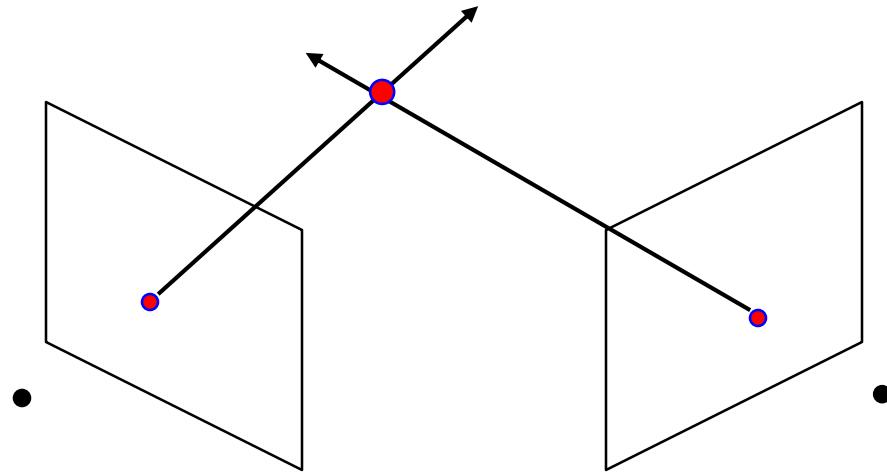
*Dense stereo matching*

# Recap: What Is Stereo Vision?

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape



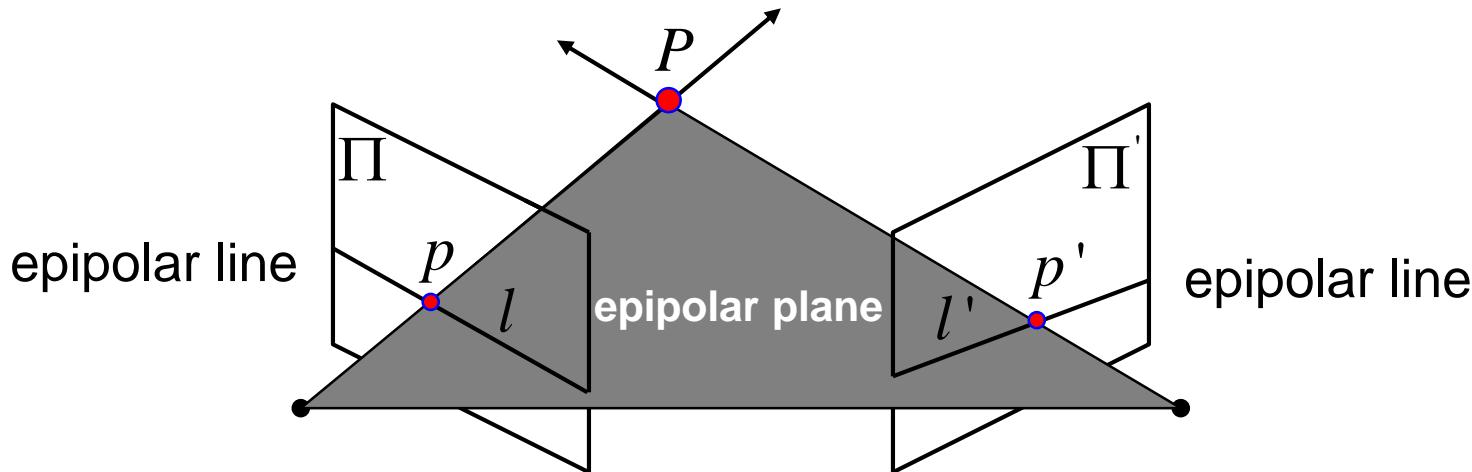
# Recap: Depth with Stereo – Basic Idea



- **Basic Principle: Triangulation**
  - Gives reconstruction as intersection of two rays
  - Requires
    - Camera pose (calibration)
    - Point correspondence

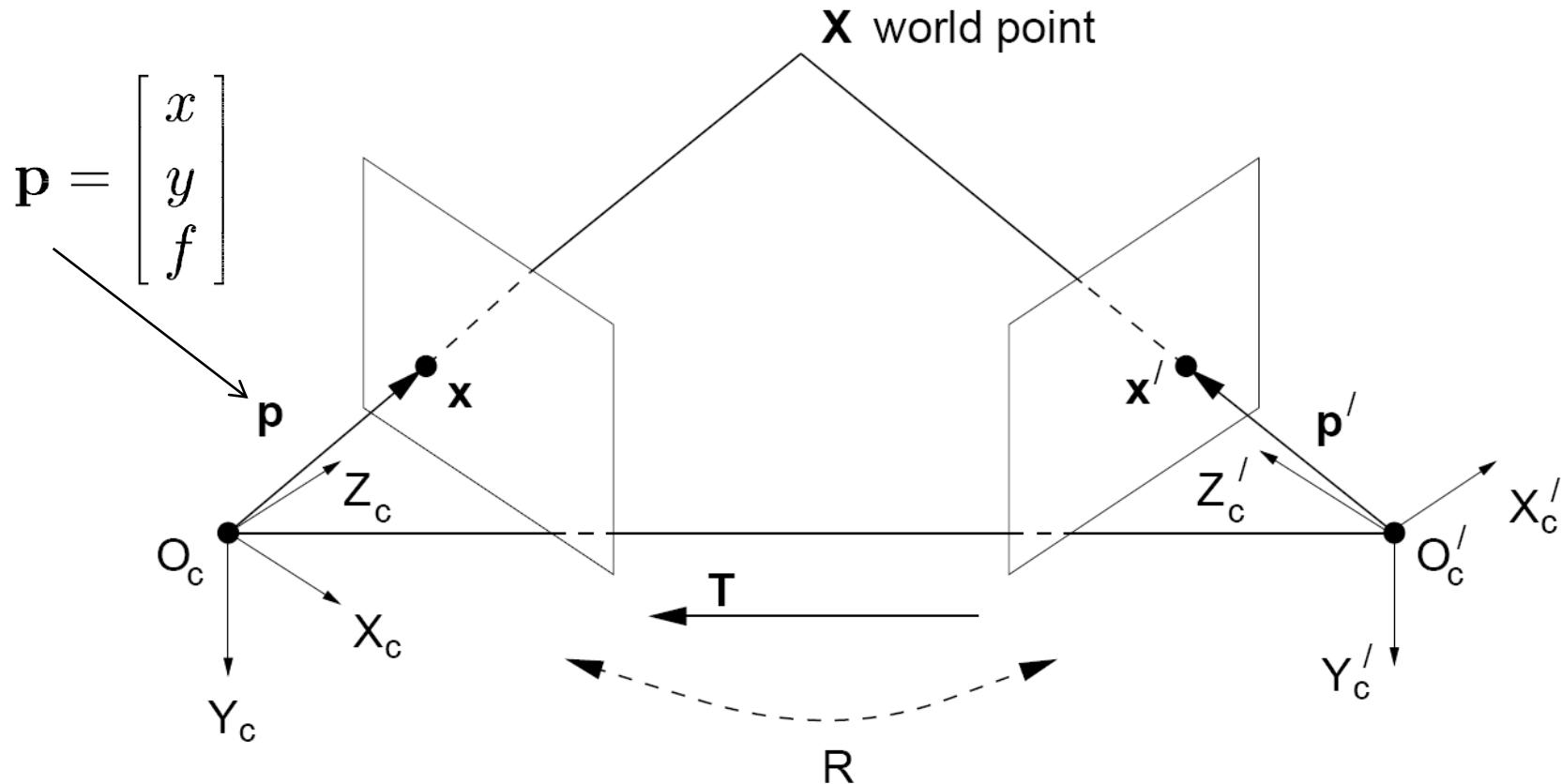
# Recap: Epipolar Geometry

- Geometry of two views allows us to constrain where the corresponding pixel for some image point in the first view must occur in the second view.



- Epipolar constraint:
  - Correspondence for point  $p$  in  $\Pi$  must lie on the epipolar line  $l'$  in  $\Pi'$  (and vice versa).
  - Reduces correspondence problem to 1D search along conjugate epipolar lines.

# Recap: Stereo Geometry With Calibrated Cameras



- Camera-centered coordinate systems are related by known rotation  $R$  and translation  $T$ :

$$\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{T}$$

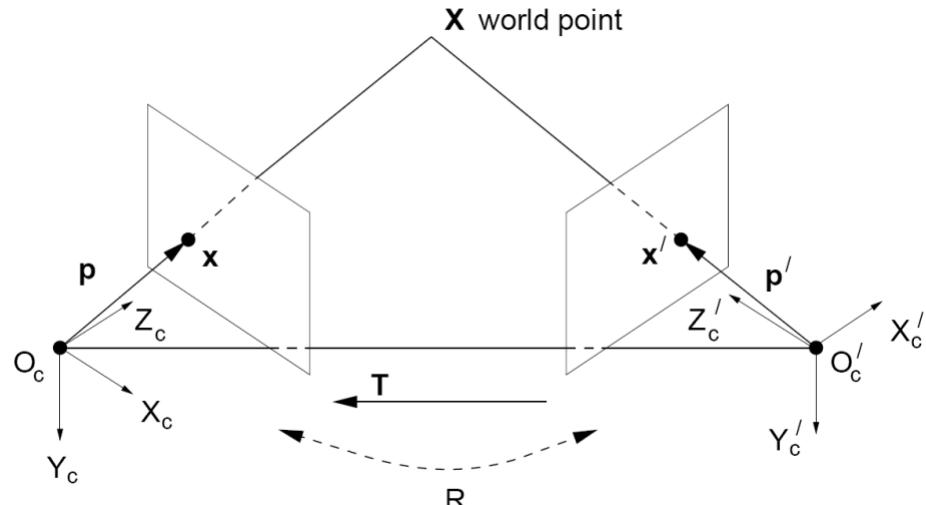
# Recap: Essential Matrix

$$\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X}) = 0$$

$$\mathbf{X}' \cdot (\mathbf{T}_x \mathbf{R}\mathbf{X}) = 0$$

Let  $\mathbf{E} = \mathbf{T}_x \mathbf{R}$

$$\mathbf{X}'^T \mathbf{E} \mathbf{X} = 0$$



- This holds for the rays  $p$  and  $p'$  that are parallel to the camera-centered position vectors  $\mathbf{X}$  and  $\mathbf{X}'$ , so we have:
- $\mathbf{E}$  is called the essential matrix, which relates corresponding image points [Longuet-Higgins 1981]

$$\mathbf{p}'^T \mathbf{E} \mathbf{p} = 0$$

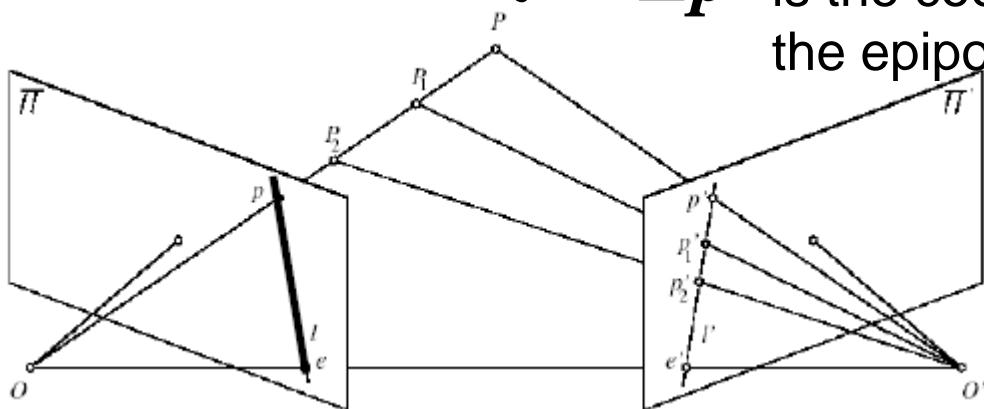
# Recap: Essential Matrix and Epipolar Lines

$$\mathbf{p}'^T \mathbf{E} \mathbf{p} = 0$$

Epipolar constraint: if we observe point  $\mathbf{p}$  in one image, then its position  $\mathbf{p}'$  in second image must satisfy this equation.

$\mathbf{l}' = \mathbf{E} \mathbf{p}$  is the coordinate vector representing the epipolar line for point  $\mathbf{p}$

(i.e., the line is given by:  $\mathbf{l}'^T \mathbf{x} = 0$ )



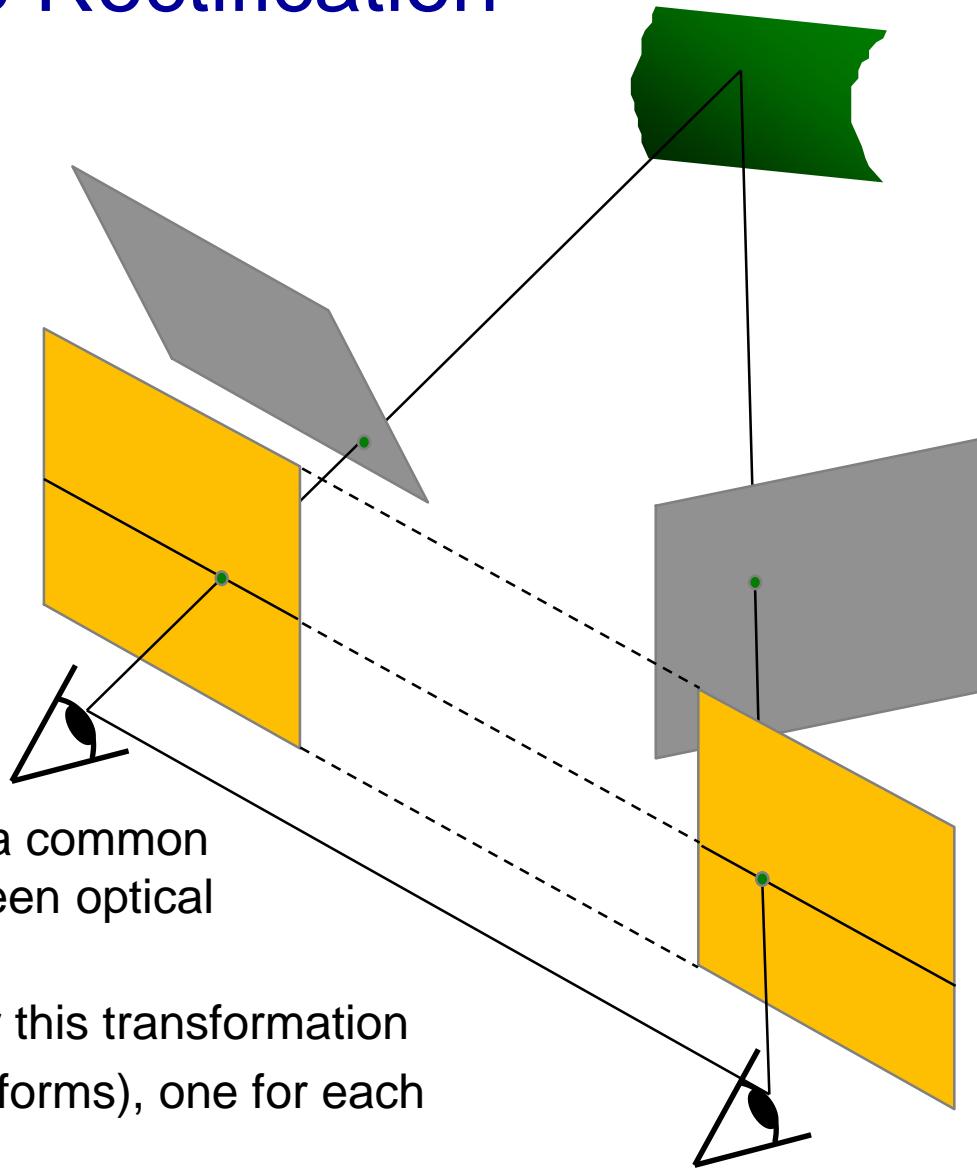
$\mathbf{l} = \mathbf{E}^T \mathbf{p}'$  is the coordinate vector representing the epipolar line for point  $\mathbf{p}'$

# Recap: Stereo Image Rectification

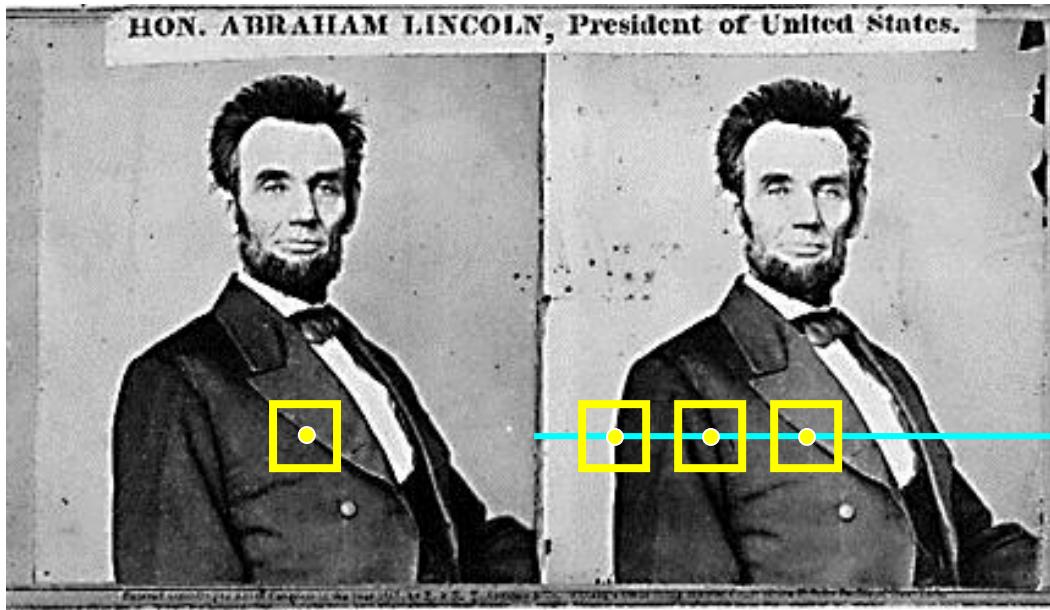
- In practice, it is convenient if image scanlines are the epipolar lines.

- Algorithm

- Reproject image planes onto a common plane parallel to the line between optical centers
- Pixel motion is horizontal after this transformation
- Two homographies (3x3 transforms), one for each input image reprojection



# Recap: Dense Correspondence Search

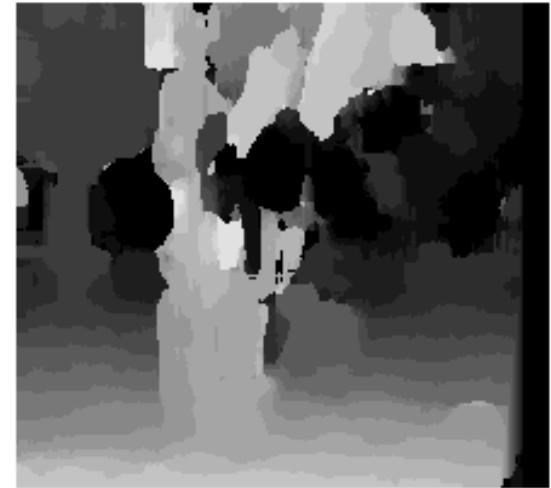


- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match (e.g. SSD, correlation)
  - Triangulate the matches to get depth information
- This is easiest when epipolar lines are scanlines  
⇒ Rectify images first

# Recap: Effect of Window Size



$W = 3$



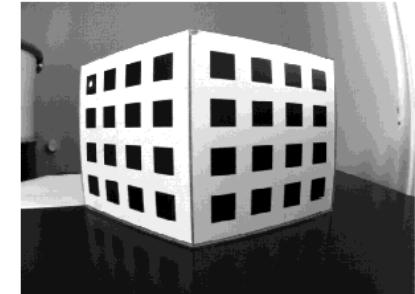
$W = 20$

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

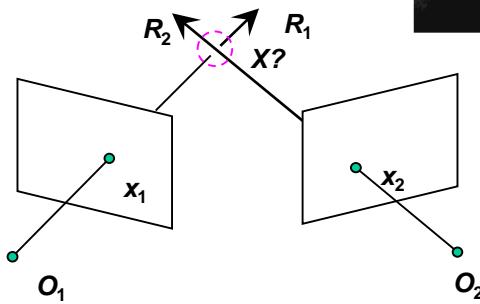
# Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
- 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera Calibration & Uncalibrated Reconstruction
  - Structure-from-Motion

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$



*Camera calibration*



*Triangulation*

*Essential matrix,  
Fundamental matrix*

$$x^T E x' = 0$$

$$x^T F x' = 0$$

$$\begin{bmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 & 1 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 & 1 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 & 1 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 & 1 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 & 1 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 & 1 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

*Eight-point  
algorithm*

# Recap: A General Point

- Equations of the form

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

- How do we solve them? (always!)
  - Apply SVD

$$\xrightarrow{\text{SVD}} \mathbf{A} = \mathbf{U} \mathbf{D} \mathbf{V}^T = \mathbf{U} \begin{bmatrix} d_{11} & & \\ & \ddots & \\ & & d_{NN} \end{bmatrix} \begin{bmatrix} v_{11} & \cdots & v_{1N} \\ \vdots & \ddots & \vdots \\ v_{N1} & \cdots & v_{NN} \end{bmatrix}^T$$

Singular values      Singular vectors

- Singular values of  $\mathbf{A}$  = square roots of the eigenvalues of  $\mathbf{A}^T\mathbf{A}$ .
- The solution of  $\mathbf{A}\mathbf{x}=0$  is the *nullspace* vector of  $\mathbf{A}$ .
- This corresponds to the *smallest singular vector* of  $\mathbf{A}$ .

# Recap: Camera Parameters

- **Intrinsic parameters**

- Principal point coordinates
- Focal length
- Pixel magnification factors
- *Skew (non-rectangular pixels)*
- *Radial distortion*

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & \textcolor{red}{s} & p_x \\ f & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & \textcolor{red}{s}' & x_0 \\ \alpha_y & & y_0 \\ & & 1 \end{bmatrix}$$

- **Extrinsic parameters**

- Rotation R
- Translation t  
(both relative to world coordinate system)

- **Camera projection matrix**

- ⇒ General pinhole camera: 9 DoF
- ⇒ CCD Camera with square pixels: 10 DoF
- ⇒ General camera: 11 DoF

$$P = K[R \mid t]$$

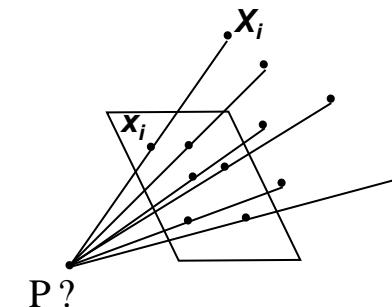
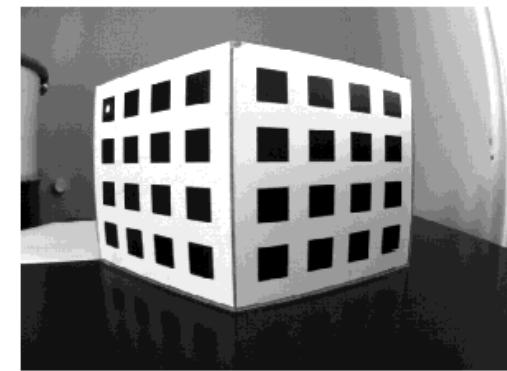
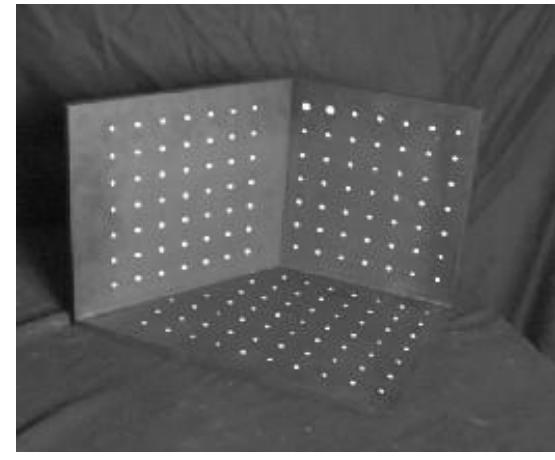
# Recap: Calibrating a Camera

## Goal

- Compute intrinsic and extrinsic parameters using observed camera data.

## Main idea

- Place “calibration object” with known geometry in the scene
- Get correspondences
- Solve for mapping from scene to image: estimate  $P = P_{\text{int}} P_{\text{ext}}$

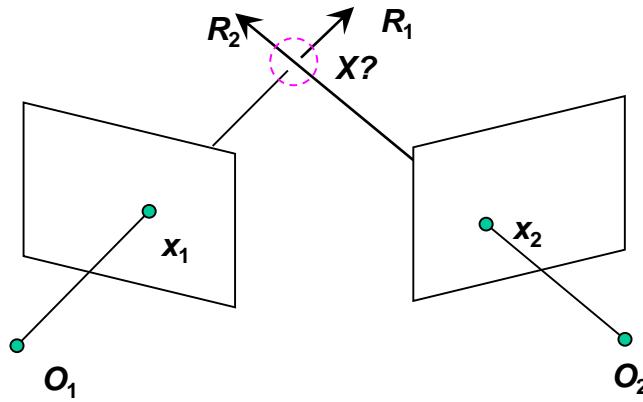


# Recap: Camera Calibration (DLT Algorithm)

$$\begin{bmatrix} 0^T & \mathbf{X}_1^T & -y_1\mathbf{X}_1^T \\ \mathbf{X}_1^T & 0^T & -x_1\mathbf{X}_1^T \\ \dots & \dots & \dots \\ 0^T & \mathbf{X}_n^T & -y_n\mathbf{X}_n^T \\ \mathbf{X}_n^T & 0^T & -x_n\mathbf{X}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{pmatrix} = 0 \quad \mathbf{A}\mathbf{p} = 0$$

- $\mathbf{P}$  has 11 degrees of freedom.
- Two linearly independent equations per independent 2D/3D correspondence.
- Solve with SVD (similar to homography estimation)
  - Solution corresponds to smallest singular vector.
- 5 ½ correspondences needed for a minimal solution.

# Recap: Triangulation – Lin. Alg. Approach



$$\lambda_1 x_1 = P_1 X$$

$$x_1 \times P_1 X = 0$$

$$[x_{1\times}]P_1 X = 0$$

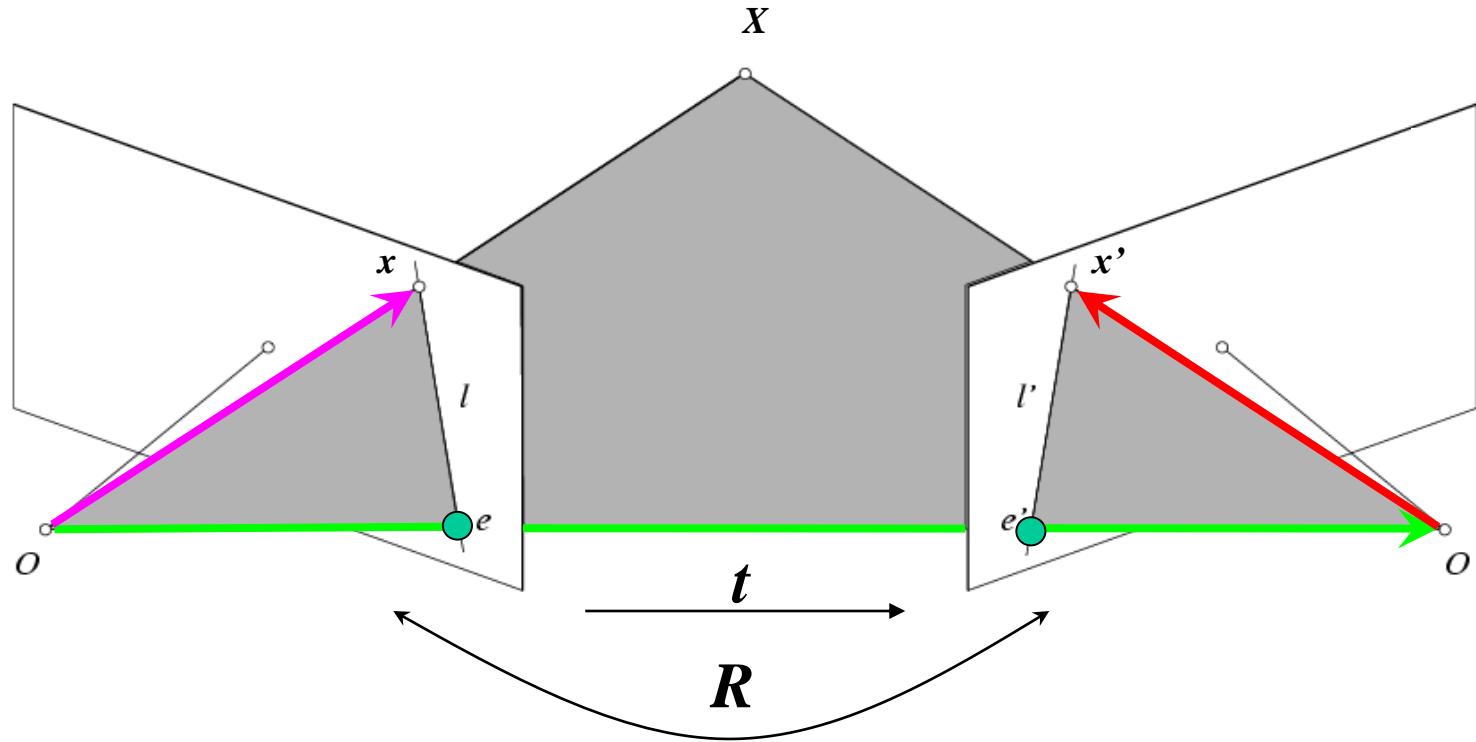
$$\lambda_2 x_2 = P_2 X$$

$$x_2 \times P_2 X = 0$$

$$[x_{2\times}]P_2 X = 0$$

- Two independent equations each in terms of three unknown entries of  $X$ .
- Stack equations and solve with SVD.
- This approach nicely generalizes to multiple cameras.

# Recap: Epipolar Geometry – Calibrated Case



Camera matrix:  $[I|0]$

$$X = (u, v, w, 1)^T$$

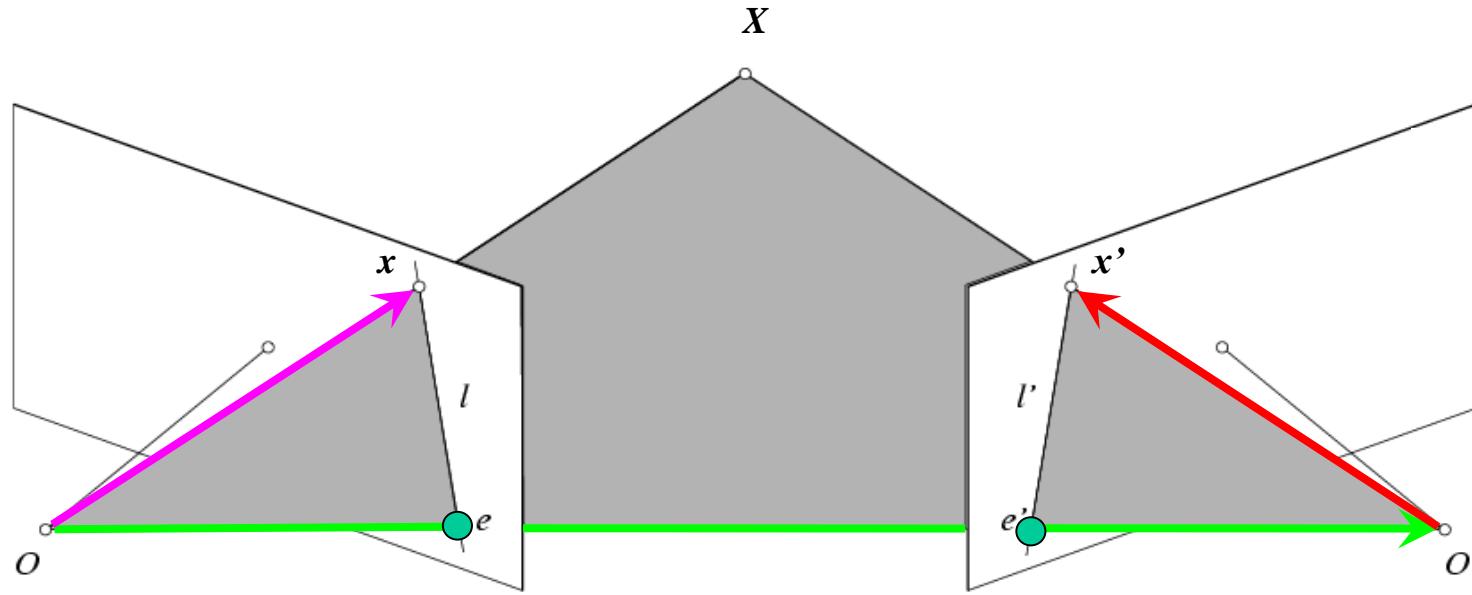
$$x = (u, v, w)^T$$

Camera matrix:  $[R^T | -R^T t]$

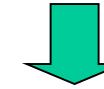
Vector  $x'$  in second coord.  
system has coordinates  $Rx'$   
in the first one.

The vectors  $x$ ,  $t$ , and  $Rx'$  are coplanar

# Recap: Epipolar Geometry – Calibrated Case

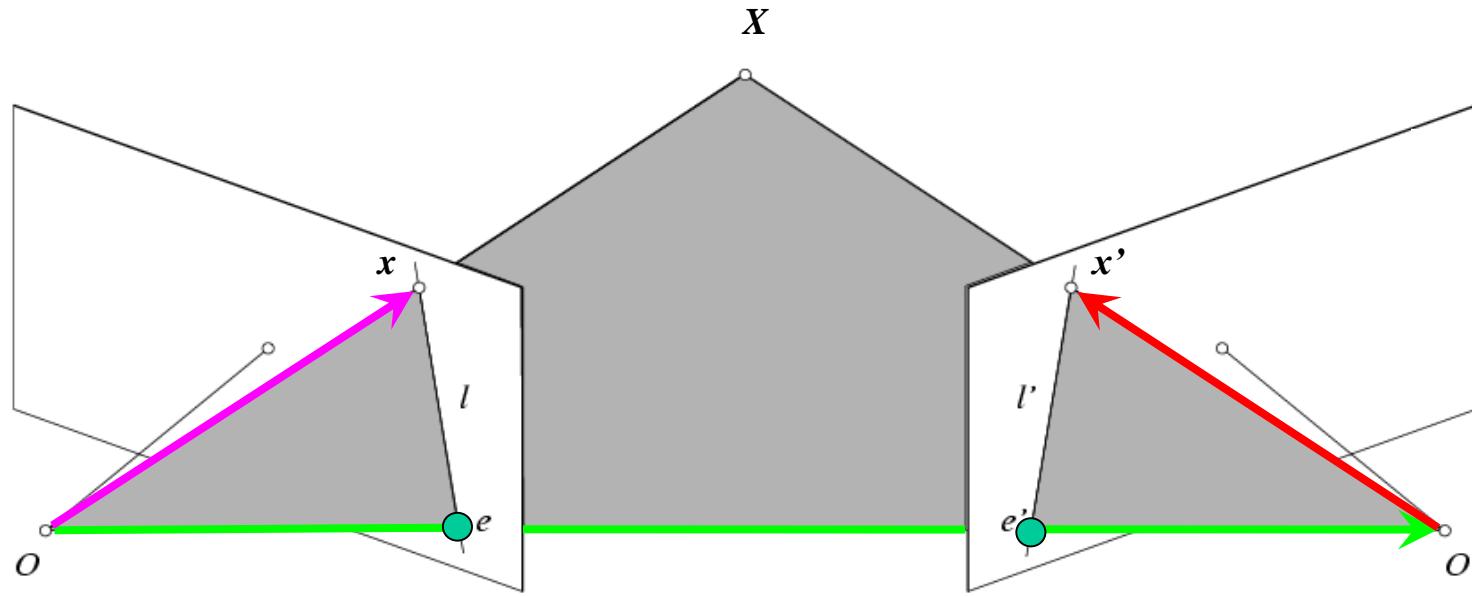


$$x \cdot [t \times (Rx')] = 0 \quad \rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_x]R$$



Essential Matrix  
(Longuet-Higgins, 1981)

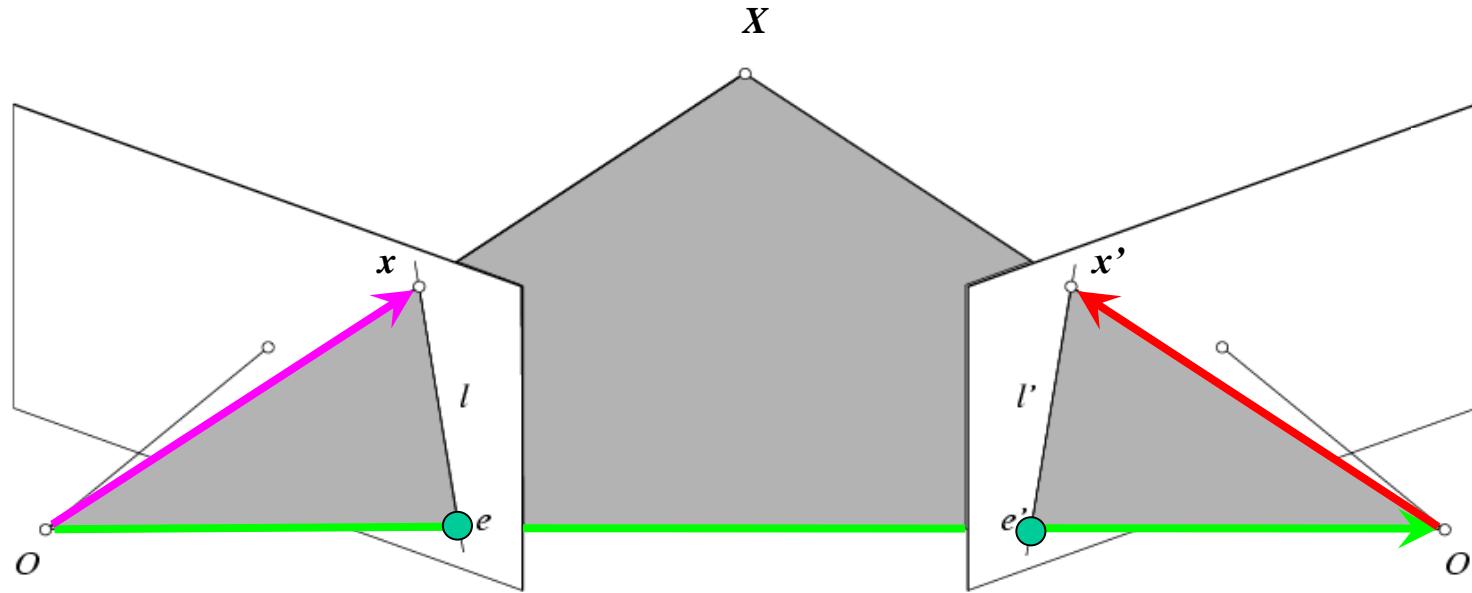
# Recap: Epipolar Geometry – Calibrated Case



$$x \cdot [t \times (Rx')] = 0 \quad \rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_x]R$$

- $E x'$  is the epipolar line associated with  $x'$  ( $l = E x'$ )
- $E^T x$  is the epipolar line associated with  $x$  ( $l' = E^T x$ )
- $E e' = 0$  and  $E^T e = 0$
- $E$  is singular (rank two)
- $E$  has five degrees of freedom (up to scale)

# Recap: Epipolar Geometry – Uncalibrated Case

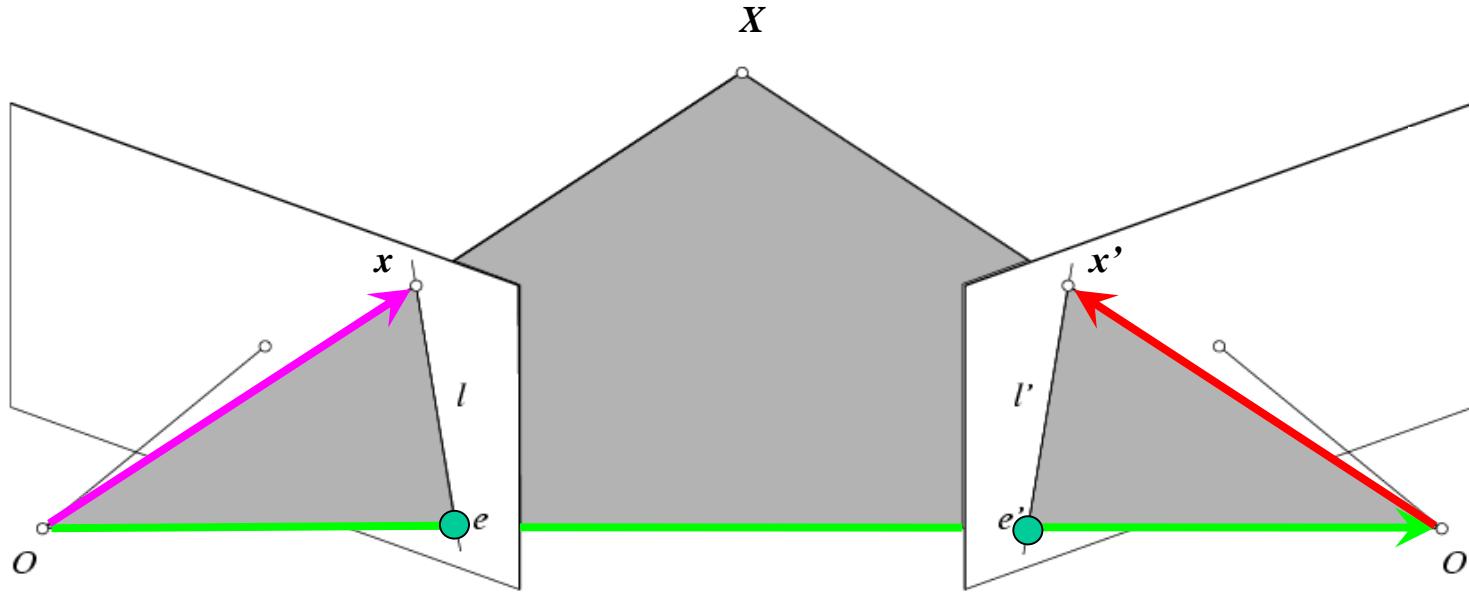


- The calibration matrices  $K$  and  $K'$  of the two cameras are unknown
- We can write the epipolar constraint in terms of *unknown* normalized coordinates:

$$\hat{x}^T E \hat{x}' = 0$$

$$x = K \hat{x}, \quad x' = K' \hat{x}'$$

# Recap: Epipolar Geometry – Uncalibrated Case



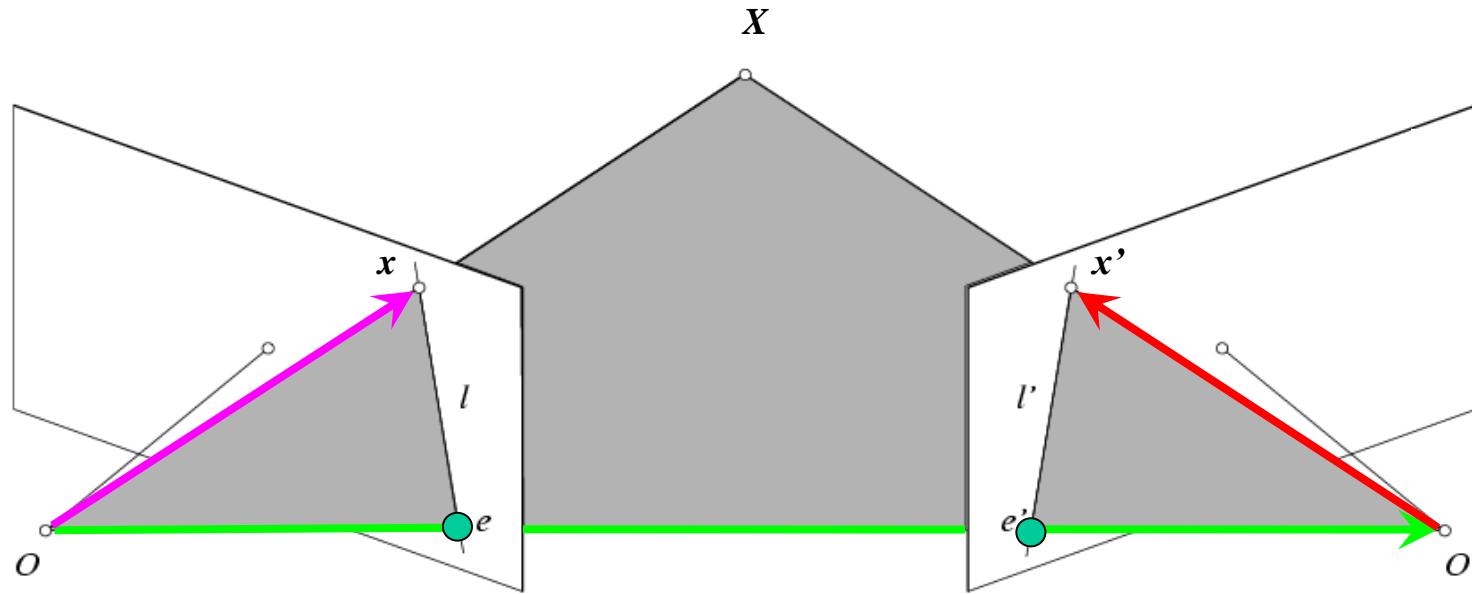
$$\hat{x}^T E \hat{x}' = 0 \quad \rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

$$x = K \hat{x}$$

$$x' = K' \hat{x}'$$

Fundamental Matrix  
(Faugeras and Luong, 1992)

# Recap: Epipolar Geometry – Uncalibrated Case



$$\hat{x}^T E \hat{x}' = 0 \quad \rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $Fx'$  is the epipolar line associated with  $x'$  ( $l = Fx'$ )
- $F^Tx$  is the epipolar line associated with  $x$  ( $l' = F^Tx$ )
- $Fe' = 0$  and  $F^Te = 0$
- $F$  is singular (rank two)
- $F$  has seven degrees of freedom

see  
Exercise 6.1!

# Recap: The Eight-Point Algorithm

$$x = (u, v, 1)^T, \quad x' = (u', v', 1)^T$$

$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \quad \rightarrow \quad (uu', uv', u, vu', vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

$$\begin{bmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 & 1 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 & 1 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 & 1 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 & 1 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 & 1 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 & 1 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0 \quad \rightarrow$$

- 1.) Solve with SVD.  
This minimizes
 
$$\sum_{i=1}^N (x_i^T F x_i')^2$$
- 2.) Enforce rank-2 constraint using SVD

- Problem: poor numerical conditioning

# Recap: Normalized Eight-Point Alg.

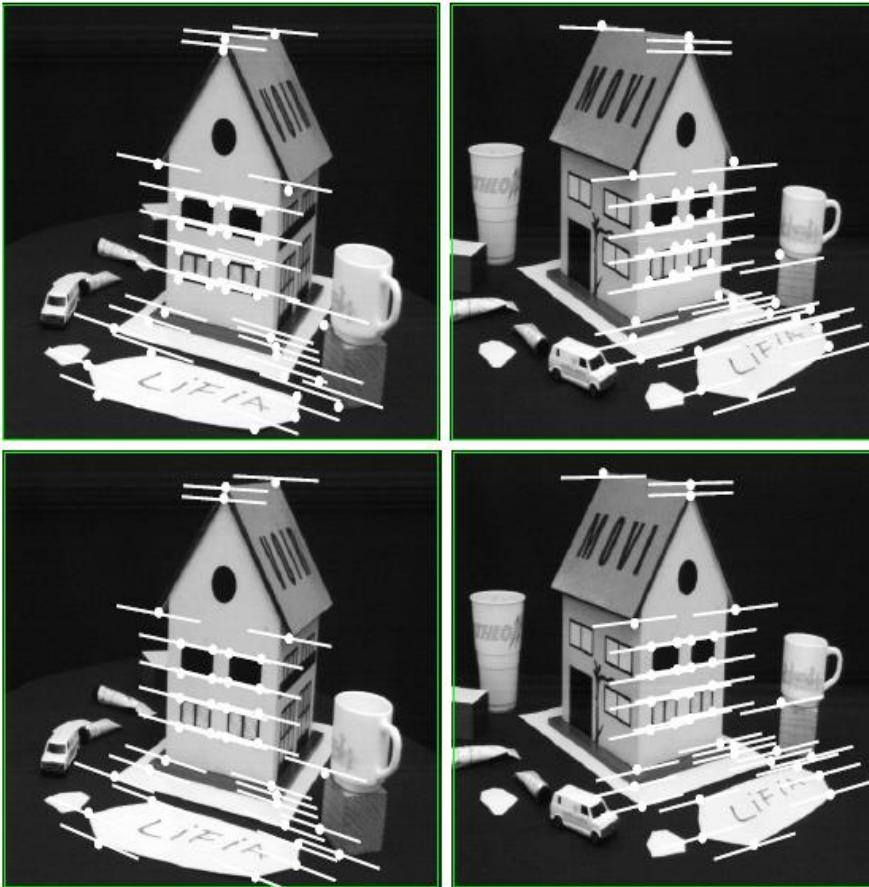
1. Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels.
2. Use the eight-point algorithm to compute  $F$  from the normalized points.
3. Enforce the rank-2 constraint using SVD.

$$\xrightarrow{\text{SVD}} F = \mathbf{U} \mathbf{D} \mathbf{V}^T = \mathbf{U} \begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix} \begin{bmatrix} v_{11} & \cdots & v_{13} \\ \vdots & \ddots & \vdots \\ v_{31} & \cdots & v_{33} \end{bmatrix}^T$$

Set  $d_{33}$  to zero and reconstruct  $F$

4. Transform fundamental matrix back to original units: if  $T$  and  $T'$  are the normalizing transformations in the two images, than the fundamental matrix in original coordinates is  $T^T F T'$ .

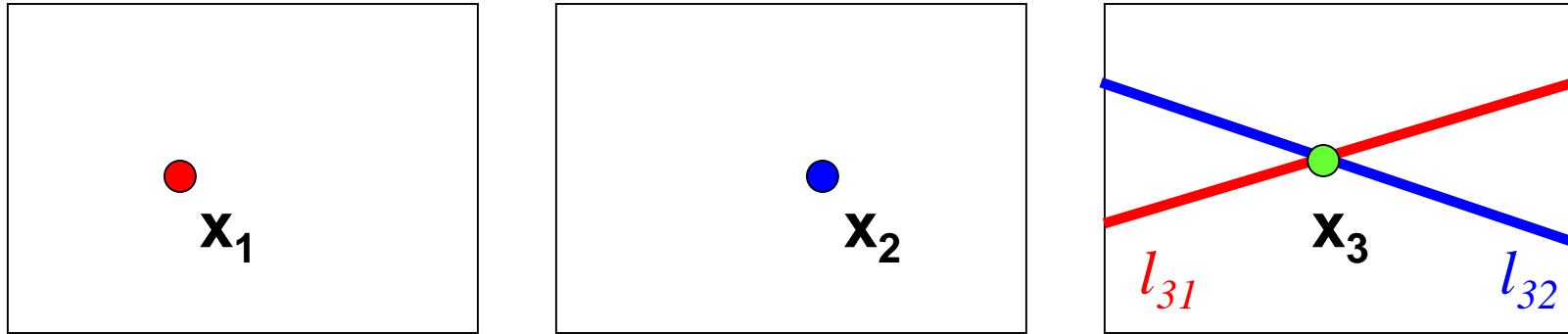
# Recap: Comparison of Estimation Algorithms



	8-point	Normalized 8-point	Nonlinear least squares
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel

# Recap: Epipolar Transfer

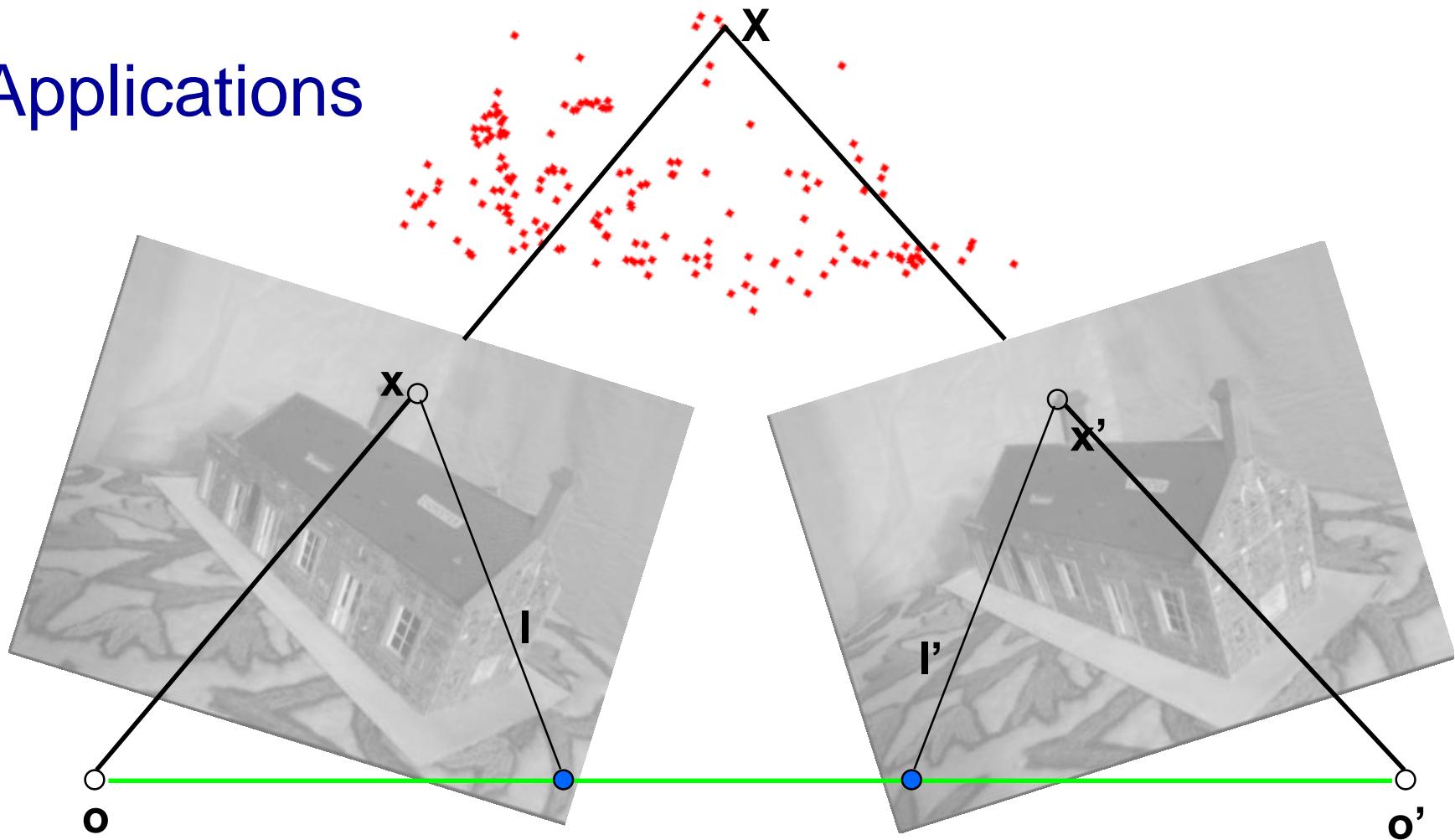
- Assume the epipolar geometry is known
- Given projections of the same point in two images, how can we compute the projection of that point in a third image?



$$l_{31} = F^T_{13} x_1$$

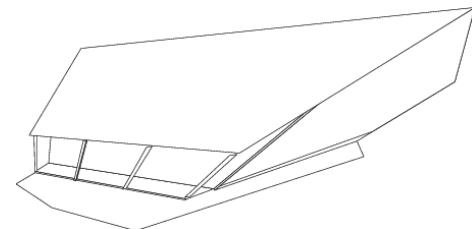
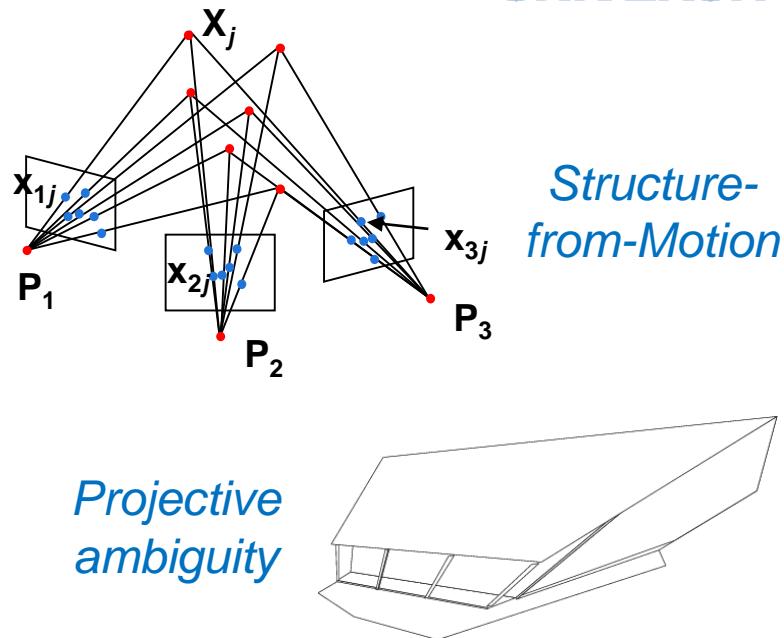
$$l_{32} = F^T_{23} x_2$$

## Applications

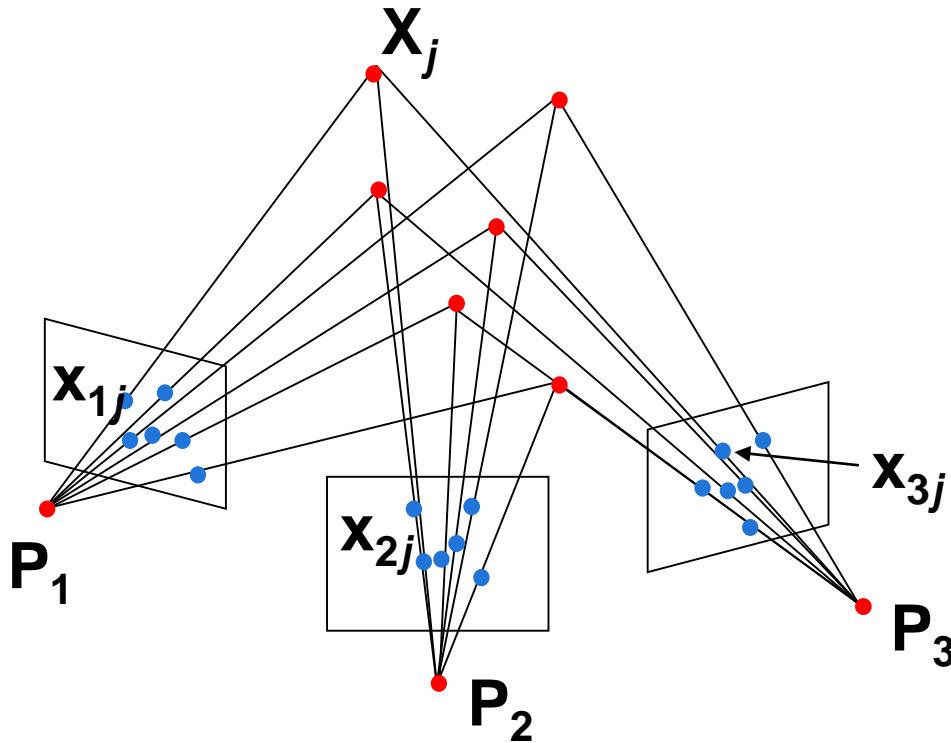


# Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera Calibration & Uncalibrated Reconstruction
  - **Structure-from-Motion**



# Recap: Structure from Motion



- Given:  $m$  images of  $n$  fixed 3D points

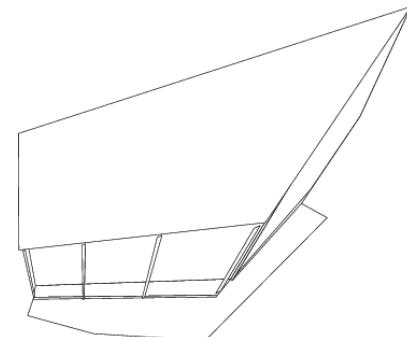
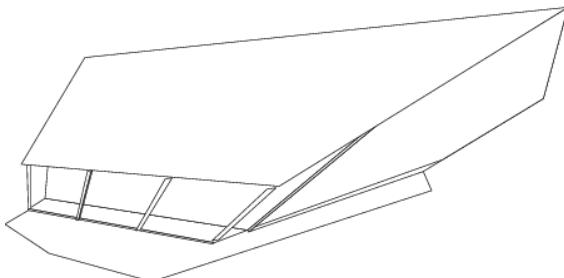
$$\mathbf{x}_{ij} = \mathbf{P}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$$

- Problem: estimate  $m$  projection matrices  $\mathbf{P}_i$  and  $n$  3D points  $\mathbf{X}_j$  from the  $mn$  correspondences  $\mathbf{x}_{ij}$

# Recap: Structure from Motion Ambiguity

- If we scale the entire scene by some factor  $k$  and, at the same time, scale the camera matrices by the factor of  $1/k$ , the projections of the scene points in the image remain exactly the same.
- More generally: if we transform the scene using a transformation  $\mathbf{Q}$  and apply the inverse transformation to the camera matrices, then the images do not change

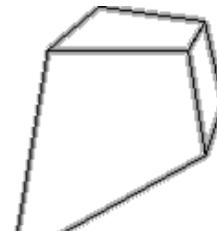
$$\mathbf{x} = \mathbf{P}\mathbf{X} = (\mathbf{P}\mathbf{Q}^{-1})\mathbf{Q}\mathbf{X}$$



# Recap: Hierarchy of 3D Transformations

Projective  
15dof

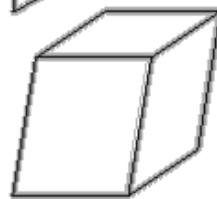
$$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$$



Preserves intersection and tangency

Affine  
12dof

$$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$$



Preserves parallelism, volume ratios

Similarity  
7dof

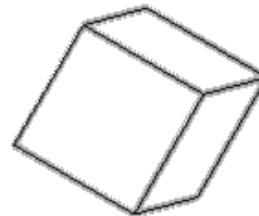
$$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$$



Preserves angles, ratios of length

Euclidean  
6dof

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$$



Preserves angles, lengths

- With no constraints on the camera calibration matrix or on the scene, we get a *projective* reconstruction.
- Need additional information to *upgrade* the reconstruction to affine, similarity, or Euclidean.

# Any More Questions?

*Good luck for the exam!*