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Learning Graph Signal Representations with Narrowband Spectral Kernels

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August 22-25, 2022
IEEE MLSP 2022, Xi'an, China

OUTLINE

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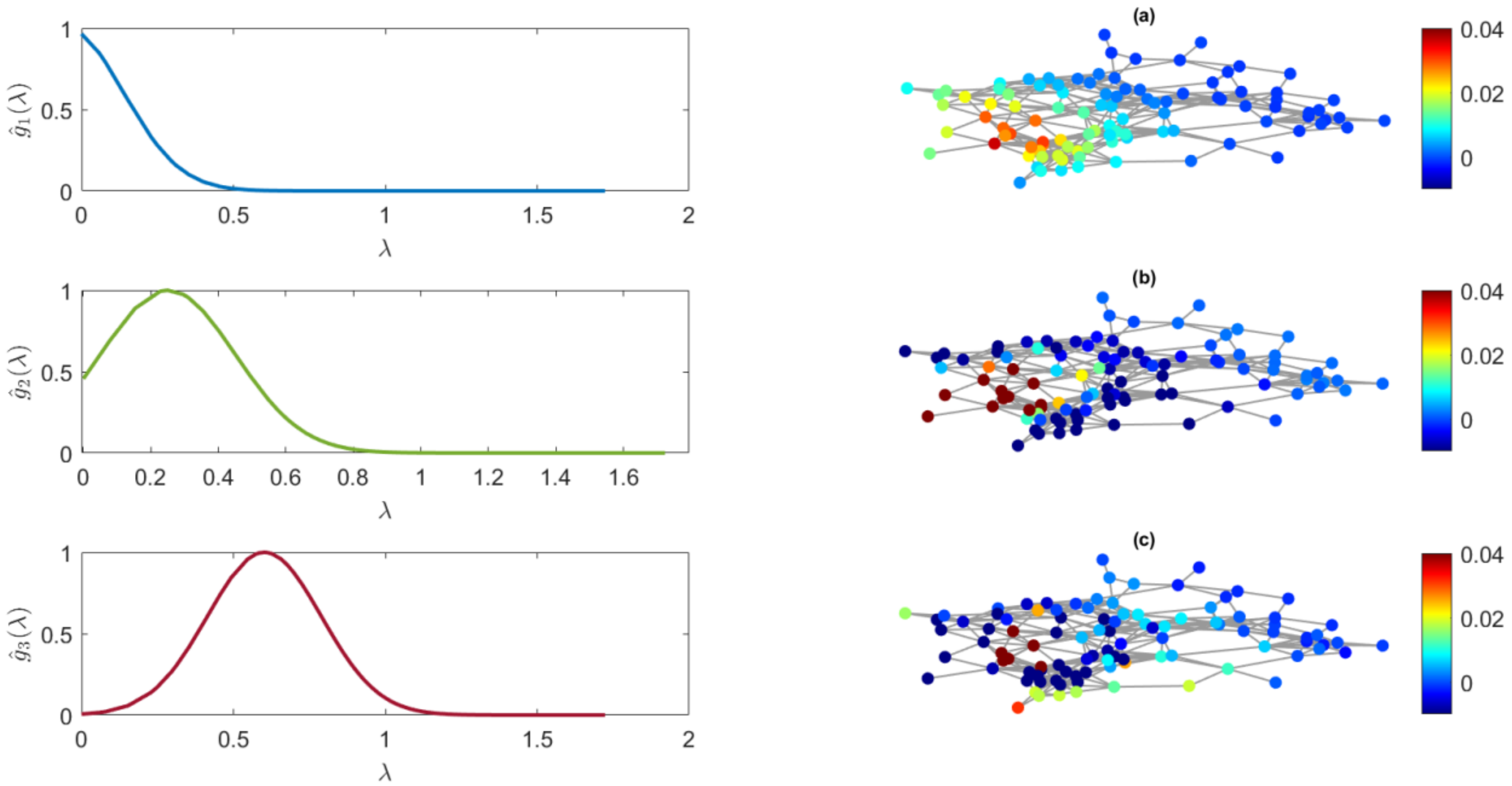
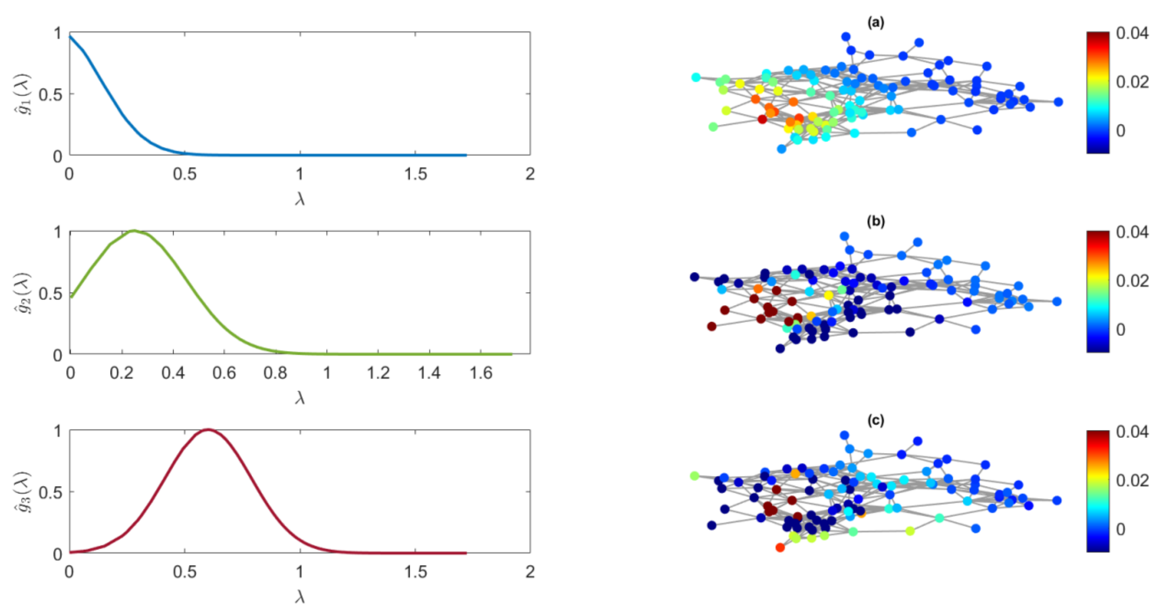
Introduction: Graph Signal Processing



Aim

- Estimation of partially observed graph signals
  - Narrowband Spectral Graph Kernels
  - Spectral Graph Dictionaries



Signal Model and Notation

$$\mathcal{G}^m = (\mathcal{V}^m, \mathcal{E}^m, W^m)$$

$$L^m = (\mathcal{D}^m)^{-1/2}(\mathcal{D}^m - W^m)(\mathcal{D}^m)^{-1/2}$$

$$L^m = U^m \Lambda^m (U^m)^T$$

$$D_j^m = U^m \hat{g}_j(\Lambda^m)(U^m)^T \in \mathbb{R}^{N^m \times N^m}$$

$$\hat{g}_j(\lambda) = \exp\left(-\frac{\|\lambda - \mu_j\|^2}{s_j^2}\right)$$

$$D^m = \begin{bmatrix} D_1^m & D_2^m & \cdots & D_J^m \end{bmatrix} \in \mathbb{R}^{N^m \times JN^m}$$

$$y_i^m = D^m x_i^m + w_i^m$$

Proposed Method

Two Step Minimization.

Spectral Kernel Parameters

Sparse Representation over Graph Dictionary

$$
\begin{aligned}
\min_{\{X^m\},\psi} \; & \overbrace{\sum_{j=1}^{J} (\mu_j)^2 + \eta_s \sum_{j=1}^{J} (s_j - s_0)^2}^{\text{Spectral Kernel Parameters}} + \eta_x \sum_{m=1}^{M} \|X^m\|_1 \\
& + \eta_w \sum_{m=1}^{M} \sum_{i=1}^{K^m} \left\| S^{m,i} y_i^m - S^{m,i} D^m x_i^m \right\|^2 \\
& + \underbrace{\eta_y \sum_{m=1}^{M} \mathrm{tr}((X^m)^T (D^m)^T L^m D^m X^m)}_{\text{Smoothly Varying Reconstructed Signal}} + \eta_c \sum_{m=1}^{M} \mathrm{tr}((X^m) \widetilde{L}^m (X^m)^T)
\end{aligned}
$$

Coherency with partial observations

Smoothly Varying Reconstructed Signal

Similar Reconstructed Signals with Similar Dictionary Atoms

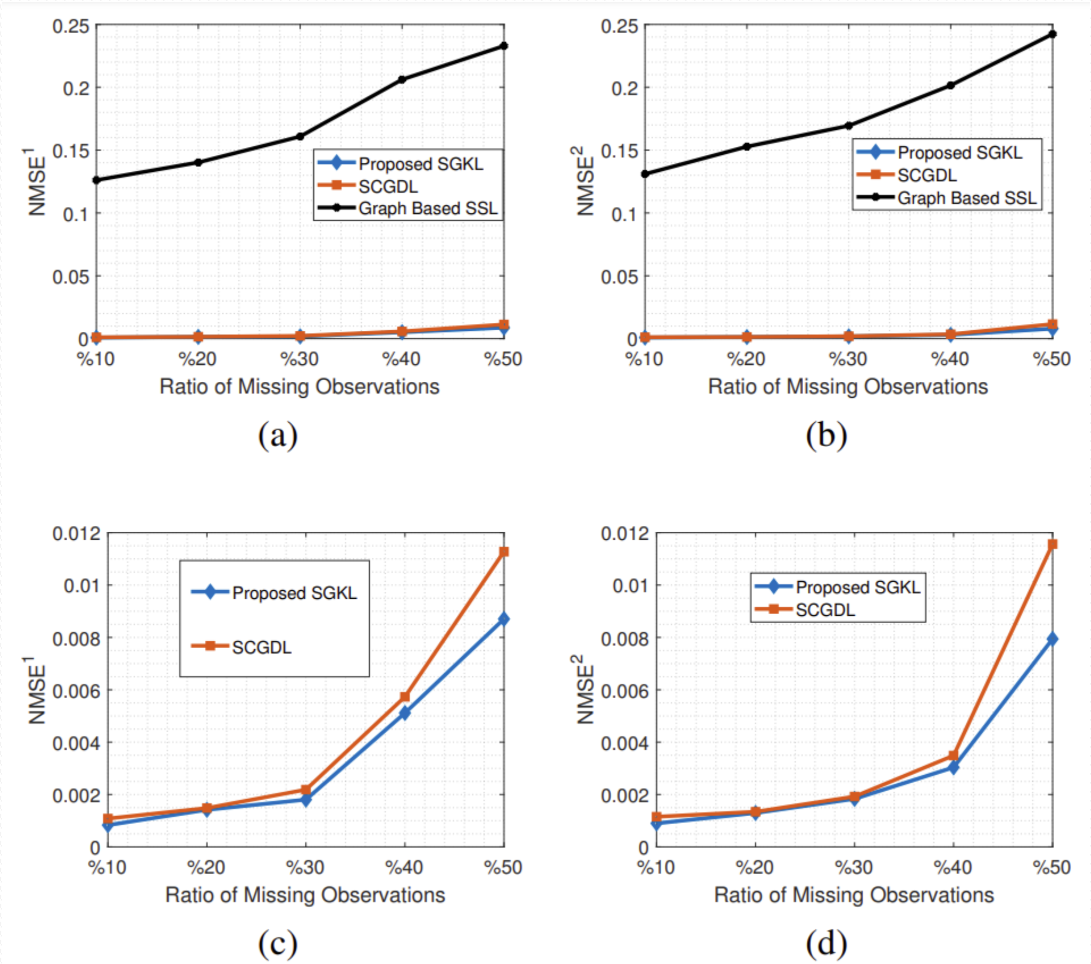
Experiments: Data Set

Synthetic Data Set:

- G^1 and G^2 10-NN graphs with 100 Nodes.
- J=4 Spectral Kernels
- K^1 = 200 and K^2 = 400 Signals

Experiments: Results

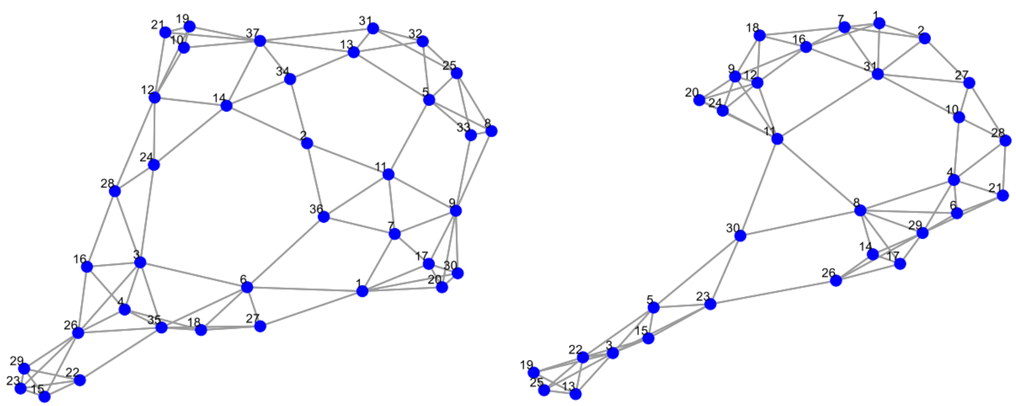
$$NMSE^m = \left\| Y_u^m - \tilde{Y}_u^m \right\|^2 / \|Y_u^m\|^2$$



(a) G^1, (b) G^2 . In panels (c: G^1) and (d: G^2) the results are replotted only for the proposed SGKL and the SCGDL methods for visual clarity.

Experiments: Data Set

Molene Data Set:
Released by the French national meteorological service which consists of temperature and wind speed measurements taken at different locations in the Brittany region of France.

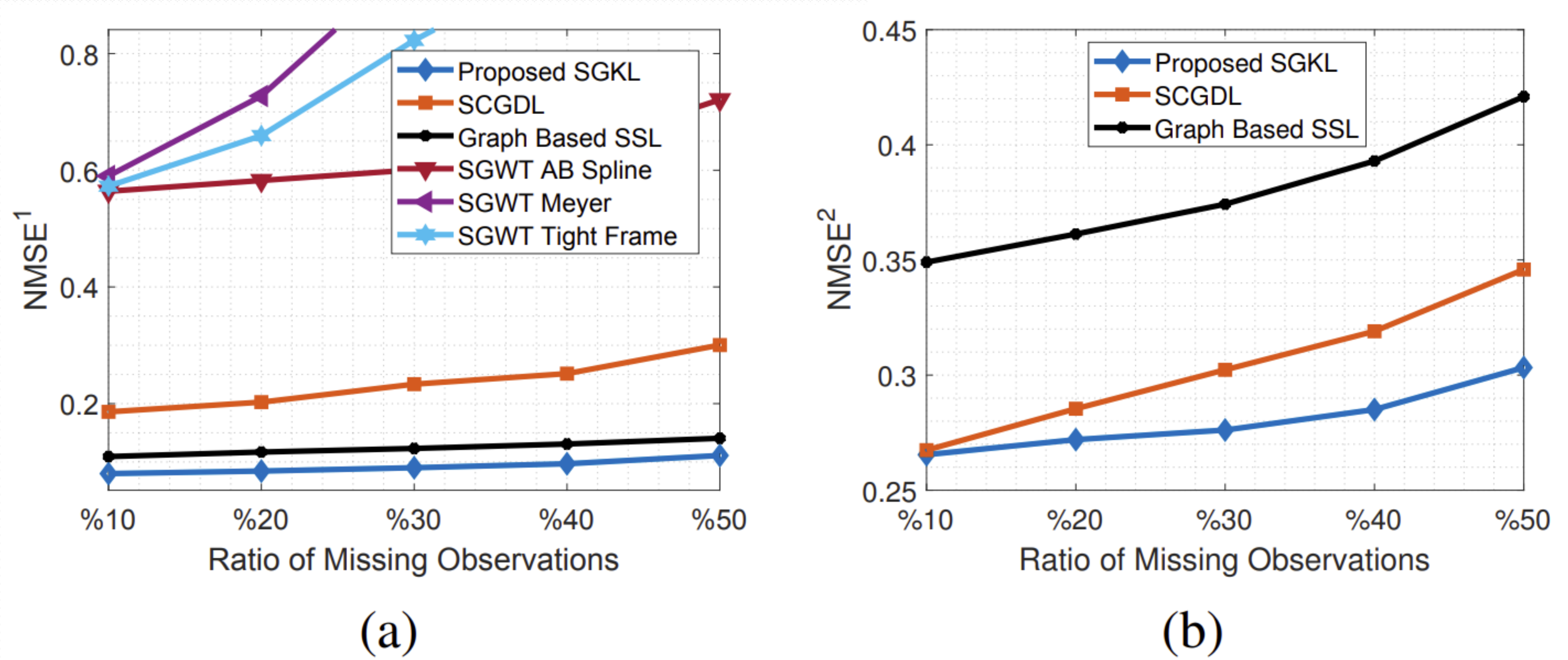


(a) (b)

(a) Temperature (b) Wind-Speed

Experiments: Results

$$NMSE^m = \left\| Y_u^m - \tilde{Y}_u^m \right\|^2 / \| Y_u^m \|^2$$



(a) G^1-Temperature (b) G^2-Wind-Speed

Conclusion

- In this study, we have proposed a graph signal model based on representations using narrowband graph kernel prototypes. Our algorithm takes a set of partially observed graph signals as input, and jointly optimizes the signal representations along with the parameters of a set of graph kernels.
- The initially unknown observations of the signals are then estimated based on this learnt model.
- Experiments on synthetic and real graph signal sets show that the proposed method provides promising signal estimation performance compared to baseline solutions.

Thank You

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