Osman Yücel - 2305803 Kaan Tütek - 2375954

EE464 Homework #1

Introduction

Parallel to Hebelsan A.Ş.'s order of a 24W, 16V/12V, 50kHz DC/DC converter with an output voltage ripple of 2%, three converter topologies (buck-boost, Cuk, and SEPIC) will be compared. As evaluation criteria, voltage and current ratings of the LC elements and semiconductor devices will be compared in addition to the pricing per 1000 units.

Converter specifications	
Power	24W
Switching frequency	50kHz
Input voltage	16V
Output voltage	12V
Input current (implicit)	1.5A
Output current (implicit)	2A
Output resistance (implicit)	6Ω

Table 1. Converter specifications.

Procedure

Q1. Inverting Buck-Boost Converter

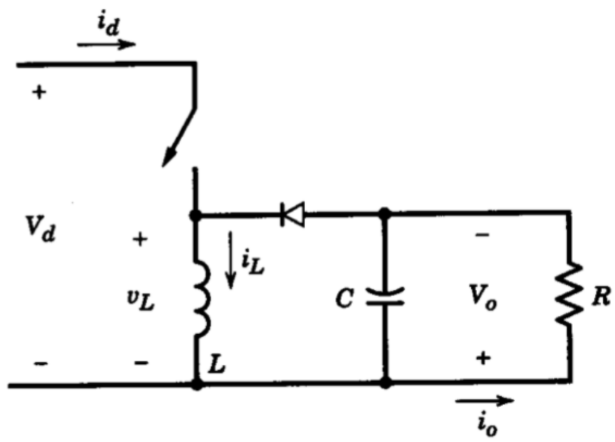


Figure 1. Inverting buck-boost converter schematic.

Accordingly, as per a boost converter, the ON cycle will mainly be comprised of the input charging the inductor and the output capacitor supplying the load current. The diode will remain reverse biased during the ON cycle due to the inverted sign of the output with respect to the input regardless of their magnitudes. Upon entering the OFF cycle, the inductor's contact with the input will cease and the inductor will supply the output current and charge the capacitor back with the charge lost during the ON cycle.

a)

Assuming a lossless operation, the converter specifications and ratings will be used as denoted in Table 1.

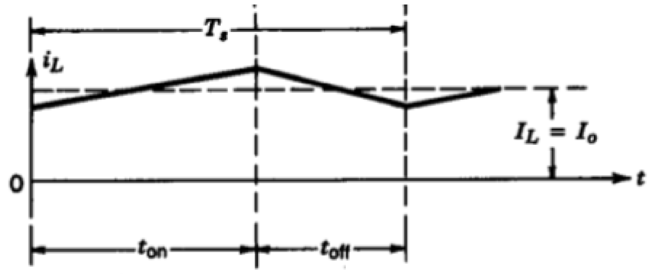


Figure 1. Inductor current waveform over a period.

The transfer function for all of the topologies to be investigated are:

$$\frac{V_{out}}{V_{in}} = \frac{D}{1-D}$$

Plugging in the relevant input-output voltages, we obtain a common D = 3/7 for all topologies.

During the ON cycle, the inductor current will equal the input current, with an average of 1.5A. However, it is important to consider that the input current is zero over the entire OFF cycle. To obtain $i_{L,ave}$, we need to find the average of the input current throughout the ON cycle:

$$i_{L,ave} = i_{in,ave(ON)} = i_{in,ave} / D$$
$$i_{L,ave} = 3/2A \ / \ 3/7 = 3.5A$$

For a 10% ripple rating around the average, the peak-to-peak ripple will correspond to 0.35A.

Considering $v_L = L\frac{di_L}{dt}$, and that the inductor voltage is equal to the input voltage,

$$v_L = L\frac{\Delta i_L}{\Delta t} \Rightarrow \Delta i_L = \frac{v_L \Delta t}{L} = 0.35A$$
$$L = \frac{v_L \Delta t}{\Delta i_L} = \frac{16V \times 3}{7 \times 50kHz \times 0.35A} \simeq 0.392mH$$

b)

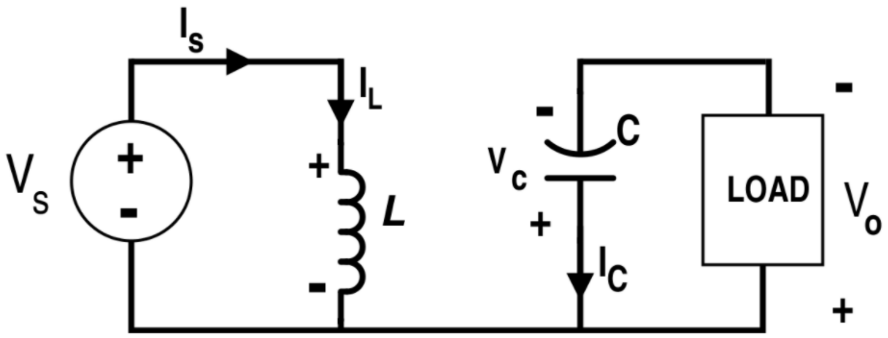


Figure 2. The ON cycle schematic for an inverting buck-boost converter.

With the capacitor supplying the output throughout this cycle, upon discharging to produce current, its voltage decreases. It will then be charged by the inductor during the OFF cycle. It is required that the output voltage ripple be less than 2%. Assuming a constant current of discharge at 2A (the average output current implicitly implied by the converter power rating denoted in Table 1),

$$i_{out} = C\frac{\Delta V_{out}}{\Delta t} \Rightarrow 2A = C \times \frac{0.02 \times 12V \times 50kHz}{\frac{3}{7}} \Rightarrow C = \frac{2A\times 3}{7 \times 50kHz \times 0.24V} \simeq 71.43\mu F$$

c)

During component selection, voltage limits are taken as the maxima (in the absolute sense) to prevent overvoltage breakdowns (which only concern peak voltages). Current limits will be based on average currents being carried, for currents are usually evaluated in not the peak sense, but the average sense as convention. Some devices will not be carrying current continuously, meaning thermal limits will not be exceeded thanks to off times. Moreover, ripples of continuously current carrying elements (inductors) have low ripples that help stay within the tolerance lines of these products.

For inductors and capacitors, current/voltage limits should be considered to allow a safe operation. However, considering ripples, their nominal inductance and capacitance values will be prioritized as long as their limits allow operation at our rated currents/voltages. Our output capacitance is large, implying the voltage limit should be much higher than 12V. An alternative might be to connect many capacitors in parallel (and thus reducing equivalent ESR), but since no spacing constraint has been given, the more convenient approach of choosing a large C will be followed.

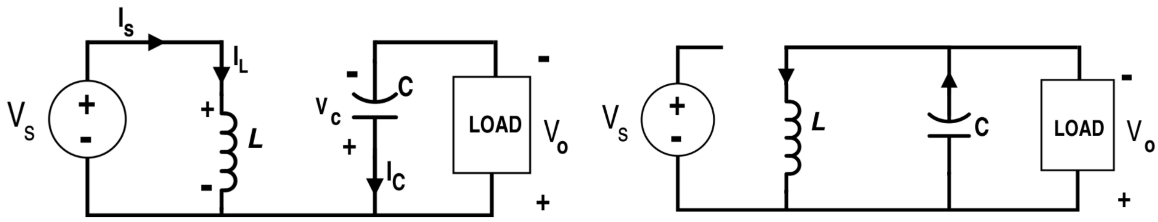


Figure 3. The ON (left) and OFF (right) cycles schematic for an inverting buck-boost converter.

Here, the following observations can be made:

- The diode should be able to withstand -28V reverse voltage during the ON cycle. (-12V - (+16V = -28V))
- The switch should be able to withstand +16V - (-12V) = 28V across its terminals during the OFF cycle.
- The switch needs to allow an average current of 1.5A (as it is in series with the input) on average.
- The diode needs to be able to carry 2A current. Calculating the trapezoidal nonzero area for the diode (and hence integrating the inductor current waveform for the OFF cycle and from there calculating $i_{D,ave}$):

$$i_{D,ave} = \frac{(i_{Lmin} + i_{Lmax})}{2} \times t_{off} \times \frac{1}{T_{switching}} = \frac{3.5A - 0.35/2\,A + 3.5A + 0.35/2A}{2} \times \frac{4}{7} = 2A$$

- The inductor should be rated at 3.5A average current.
- The capacitor should allow 12V + 0.01% V = 12.12V peak.

Device		Voltage rating	Current Rating	ESR	C	L	total price
Diode [1]	SBR3M30P1-7	30V	3A	-	-	-	$0.24310
Switch [2]	RSL020P03FRATR	30V	2A	-	-	-	$0.24684
Capacitor [3]	C4AQCBW5750A3NJ	650V	-	3.1mΩ	75μF	-	$6.30600
Inductor [4]	33470C	-	3.5A (± 15% tolerance)	48mΩ	-	0.470mH	$1.43640
						total:	**$8.232**

Table 2. Component selection for the buck-boost converter.

d)

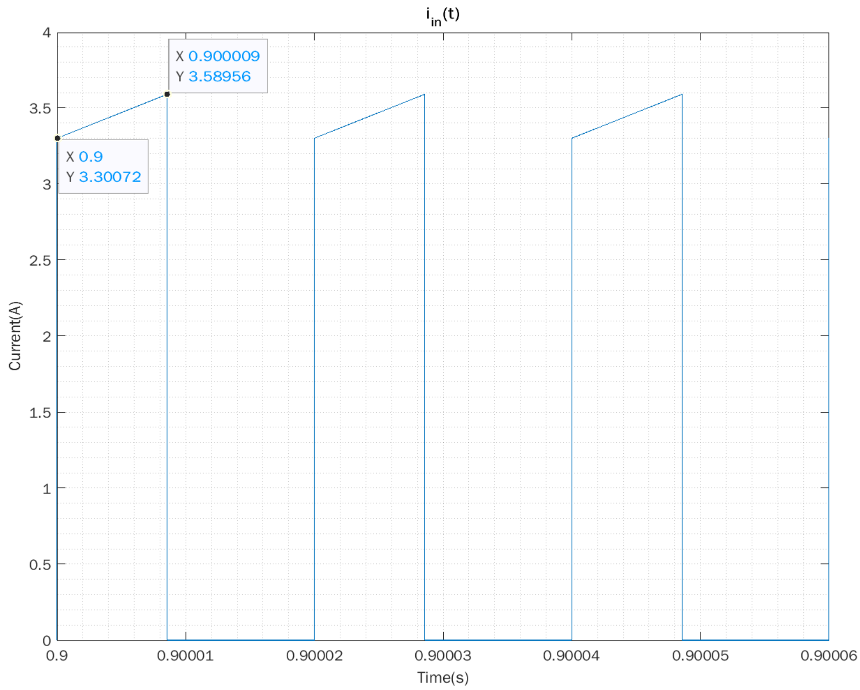


Figure 4. The input current waveform for a practical circuit.

The average input current appears at 1.378A according to Simulink.

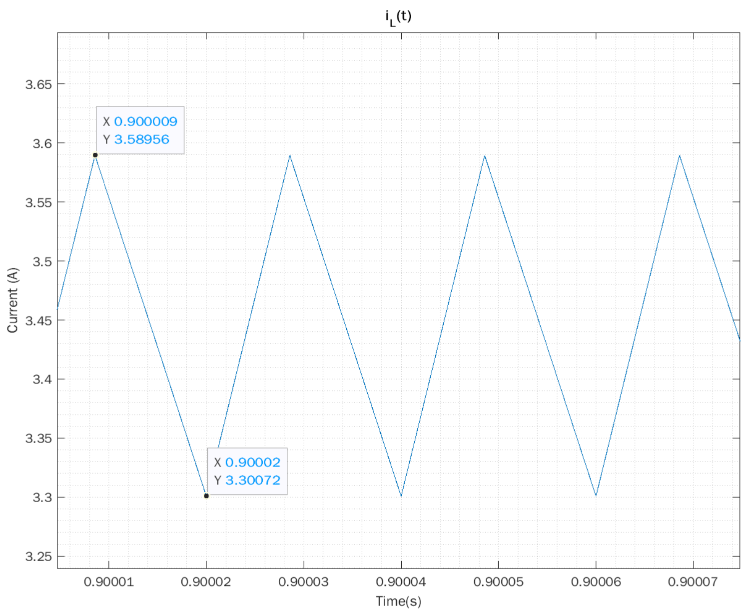


Figure 5. The inductor current waveform for a practical circuit.

The average inductor current appears at 3.453A according to Simulink. The peak to peak current is 0.2888A. Owing to the increase in the inductance from 0.392mH to 0.470mH, ripple has decreased. Moreover, since the voltage the inductor sees decreases with its increase in current due to the presence of an ESR, there is a slight decrease in the average current as well as the ripple. The new ripple percentage is 8.364%, less than the limit of 10%.

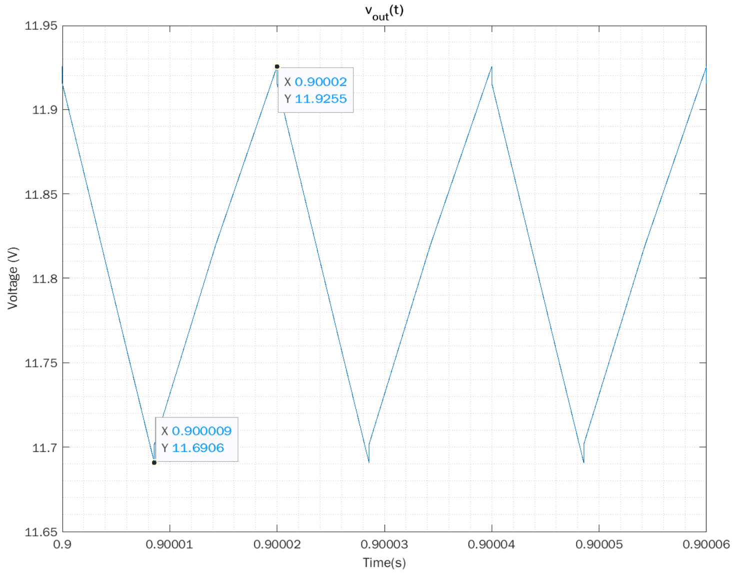


Figure 6. The inductor current waveform for a practical circuit.

The average voltage is 11.80V. The peak-to-peak voltage is 0.2349V. With a load of 6 Ohms, the output power decreases to 23.207W. With the ESR comes a voltage division of the ESR and the load resistance which may explain why we observe a decrease in the output voltage. The new ripple percentage is 1.99%, comparable to the limit of 2%. With a more appropriate duty cycle, the output voltage can be brought back up to the desired 12V.

Q2. Ćuk Converter

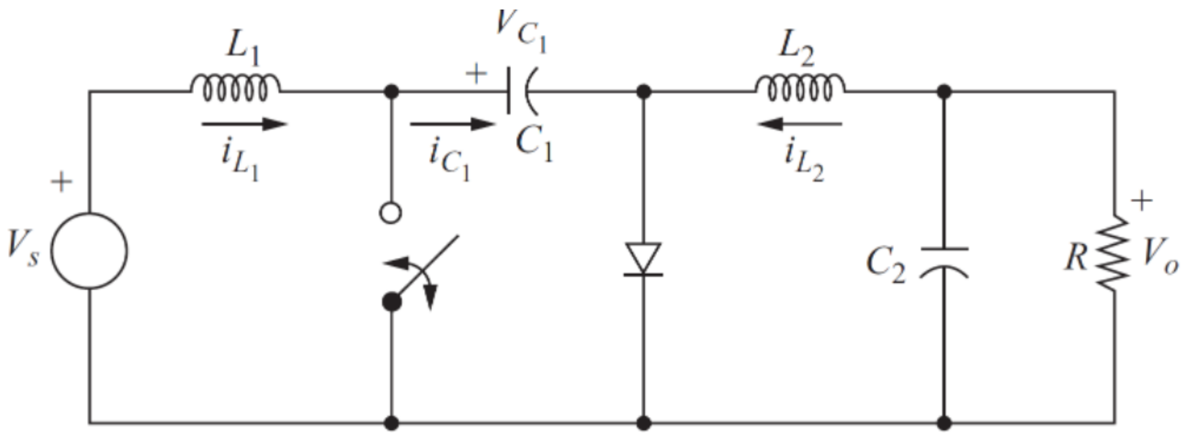


Figure 7. Ćuk Converter Schematic

$$\frac{V_O}{V_{IN}} = \frac{D}{(1-D)}$$

$$\Rightarrow D = \frac{3}{7} = 0.428$$

a)

$$I_{L1,AVG} = I_{IN,AVG} = \frac{24W}{16V} = 1.5A$$

For 10% current ripple, $\Delta I_{L1} = 0.15A$.

For on position of switch $V_{L1} = V_{IN}$.

$$\Delta I_{L1} = \frac{V_{IN}*D*T_s}{L_1}$$

$$0.15A = \frac{16V*3}{7*50kHz*L_1}$$

$$\Rightarrow L_1 = 0.914mH$$

$$I_{L2,AVG} = I_{O,AVG} = \frac{24W}{12V} = 2A$$

For 10% current ripple, $\Delta I_{L2} = 0.2A$ and for switch OFF position of switch $V_{L2} = V_O$.

$$\Delta I_{L2} = \frac{V_O*(1-D)*T_s}{L_2}$$

$$0.2A = \frac{12V*4}{7*50kHz*L_2}$$

$$\Rightarrow L_2 = 0.685mH$$

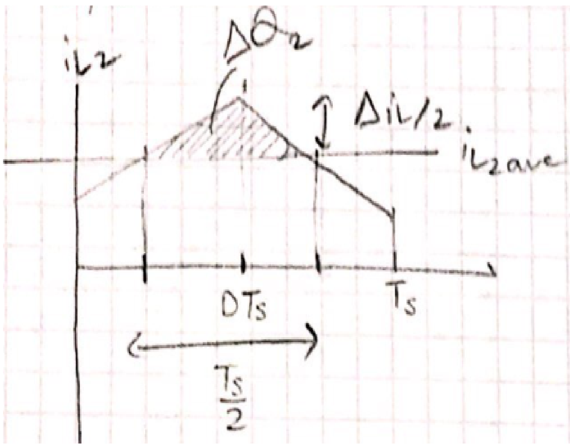


Figure 8. Charging area for C2 with the current of L2

$$V_{C2,AVG} = V_{O,AVG} = 12V$$

For 2% voltage ripple, $\Delta V_{C2} = 0.24V$.

$$\Delta V_{C2} = \frac{\Delta Q_{C2}}{C_2}$$

$$\Delta Q_{C2} = \frac{\Delta I_{L2}*T_s}{2*2*2}$$

$$\Delta Q_{C2} = \frac{0.2A}{50kHz*8} = 0.5\mu A/Hz$$

$$0.24V = \frac{0.5\mu}{C_2}$$

$$\Rightarrow C_2 = 2.083\mu F$$

b)

$$V_{C1,AVG} = \frac{V_{IN}}{(1-D)} = \frac{16*7}{4} = 28V$$

For 10% voltage ripple, $\Delta V_{C1} = 2.8V$ and for off position of switch $I_{C2} = I_{IN}$.

$$\Delta V_{C1} = \frac{I_{IN}*(1-D)*T_s}{C_1}$$

$$2.8V = \frac{1.5A*4}{7*50kHz*C_1}$$

$$\Rightarrow C_1 = 6.12\mu F$$

c)

For component selection, the considired features are also used in this topology.

Here, the following observations can be made:

- The diode should be able to withstand -29.6V reverse voltage during the ON cycle. ($V_{C1} + \Delta V_{C1}/2$)
- The switch should be able to withstand +16V across its terminals during the OFF cycle.
- The switch needs to allow a peak current of 1.65A and an average current of 0.7A .
- The diode needs to be able to carry 3A current.
- L1 should be rated at 1.65A peak current.
- L2 should be rated at 2.2A peak current.
- C1 should allow 28V + 0.5% V = 29.6V peak.
- C2 should allow 12V + 0.01% V = 12.12V peak.

Device		Voltage rating	Current Rating	ESR	C	L	total price
Diode [5]	SBRT4U30LP-7	30V	4A	-	-	-	\$0.27280
Switch [6]	STT4P3LLH6	30V	4A	-	-	-	\$0.27438
C1 [7]	865080540003	35V	-	220mΩ	6.8µF	-	\$0.18
C2 [8]	EEF-CD1C2R2R	16V	-	110mΩ	2.2µF	-	\$0.72
L1 [9]	2100HT-102-V-RC	-	1.6A	430mΩ	-	1mH	\$2.504
L2 [10]	5713-RC	-	2.25A	420Ω	-	700µH	\$6.8
						total:	**\$10.76**

d)

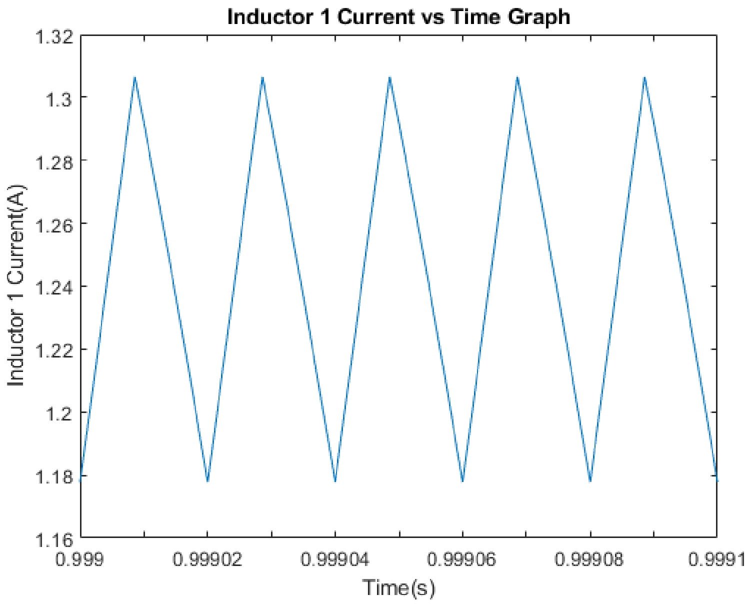


Figure 9. Current vs Time graph of the inductor 1

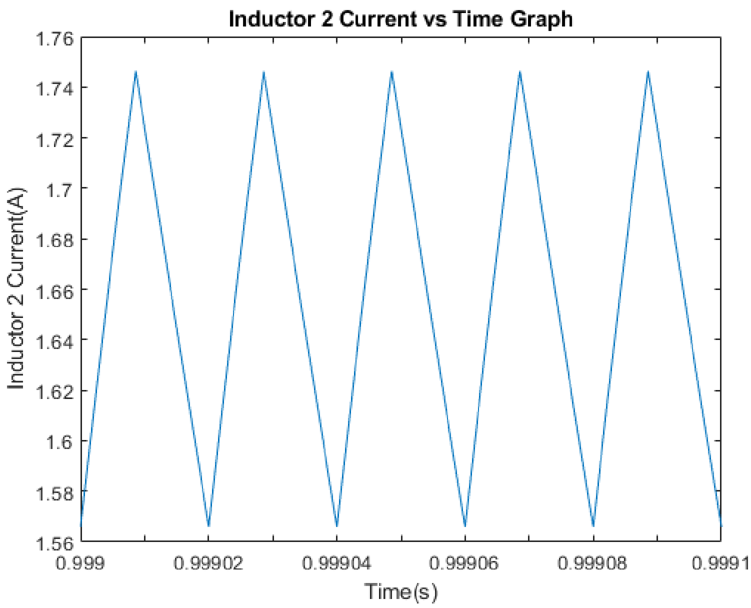


Figure 10. Current vs Time graph of the inductor 2

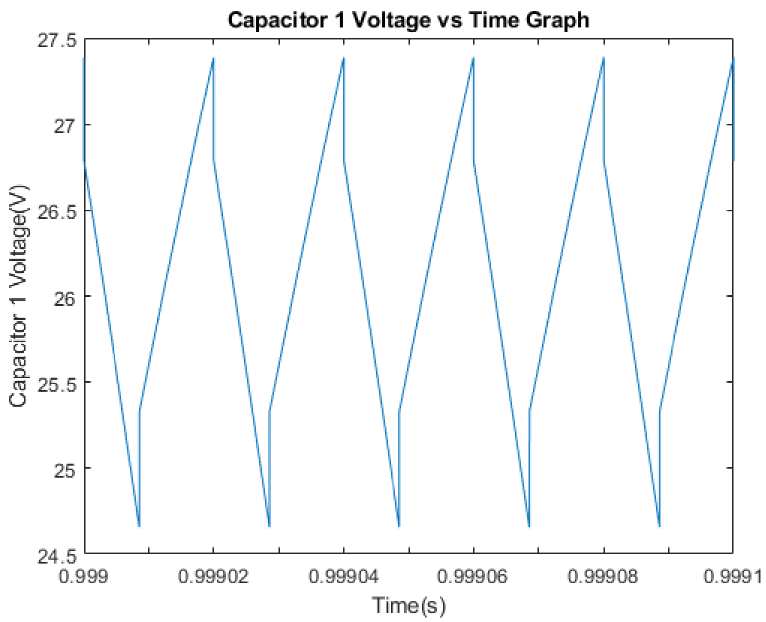


Figure 11. Voltage vs Time graph of the capacitor 1

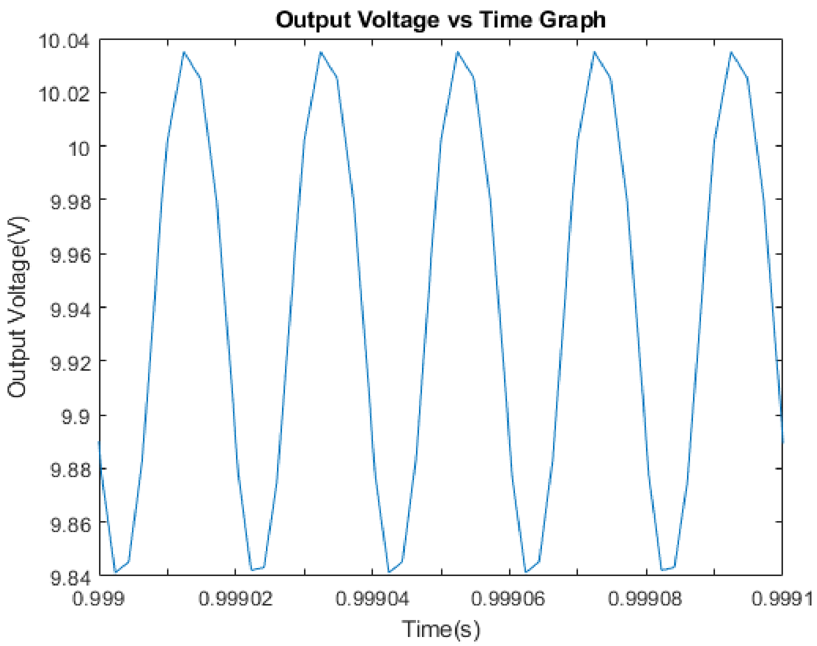


Figure 12. Output Volatge vs Time graph

The calculations are made to reach 12V output, here, in figure 12, we can see the output voltage is not the desired one. This is the result of non-ideal components. The ESRs and DSRs are causing a voltage drop like the $R_{DS(ON)}$. The voltage drop on the C1 and the current drops on the inductors are also result from same reasons.

Q3. Single Ended Primary Inductor Converter (SEPIC)

a)

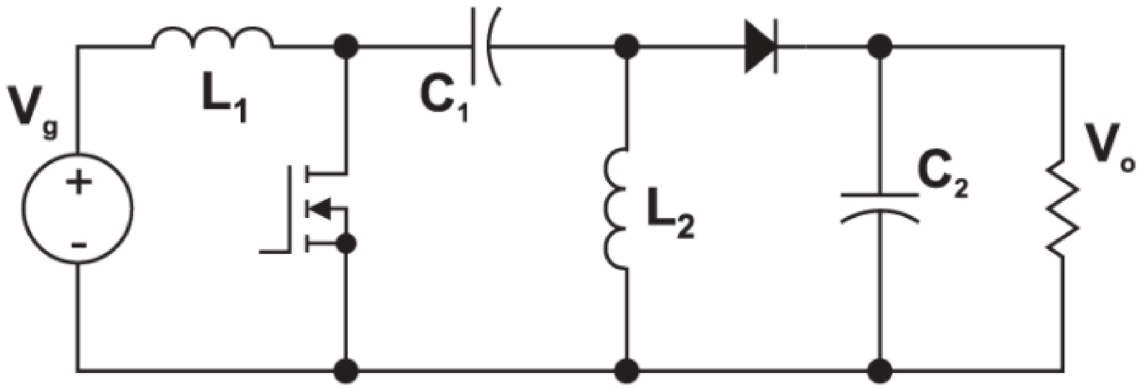


Figure 13. SEPIC schematic.

Since the transfer function remains identical to that of a buck-boost or a Ćuk converter, the duty cycle is still 3/7.

During the ON cycle, L1 will charge with the input voltage. Since L1 is in series with the input, it should have an average current of 1.5A. With a 10% ripple rating, the ripple should be 0.15A at most.

$$v_{L1} = L_1 \frac{\Delta i_{L1}}{\Delta t} \Rightarrow \Delta i_{L1} = \frac{v_{L1}\Delta t}{L_1} = 0.15A$$

$$L_1 = \frac{v_{L1}\Delta t}{\Delta i_{L1}} = \frac{16V \times 3}{7 \times 50kHz \times 0.15A} \simeq 0.914mH$$

In a SEPIC, each of the switching devices carry a total current of $i_{in,ave} + i_{out,ave}$. This implies the L2 current average is 2A. In the ON cycle, L2 will charge via C1, whose average voltage (assumed constant) is 16V (Vg in Figure 13). With a 10% ripple, the ripple should be 0.2A.

$$L_2 = \frac{v_{L2}\Delta t}{\Delta i_{L2}} = \frac{16V \times 3}{7 \times 50kHz \times 0.2A} \simeq 0.686mH\ ,$$

C2 will supply the output with its current during the OFF cycle. Consequently, through discharge, the output voltage decreases. Assuming a ripple of 2% and a constant discharge current of average output current (2A),

$$i_{out} = C_2 \frac{\Delta V_{out}}{\Delta t} \Rightarrow 2A \Rightarrow C_2 = \frac{2A \times 3}{7 \times 50kHz \times 0.24V} \simeq 71.43\mu F$$

b)

As mentioned in (a), with C1 supplying L2 in the on cycle, it discharges. Now assuming an average current of discharge, C1 with a voltage ripple of 10% may be found as follows:

$$i_{C1} = C_1 \frac{\Delta V_{C1}}{\Delta t} \Rightarrow C_1 = \frac{2A \times 3}{7 \times 50kHz \times 1.6V} \simeq 10.71\mu F$$

c)

Here, the following observations can be made:

- The diode should be able to withstand -28V reverse voltage during the ON cycle. (-16V (the C1 voltage) - (+12V) (the output voltage) = -28V)
- The switch should be able to withstand +16V + 12V = 28V across its terminals during the OFF cycle.
- The switch needs to allow an average current of $i_{in,ave} + i_{out,ave} = 3.5A$ on average.
- The diode needs to be able to carry an average current of $i_{in,ave} + i_{out,ave} = 3.5A$ per the switch.
- L1 should be rated at 1.5A average current as it is in series with the input and have an inductance of 0.914mH at least.
- L2 should be rated at 2A average current and have an inductance of 0.686mH at least.
- C1 should be able to withstand 16V + 0.05% = 16.8V at least and have a capacitance of 10.71μF.
- C2 should be able to withstand 12V + 0.01% V = 12.12V at least and have a capacitance of 71.43μF.

Device		Voltage rating	Current Rating	ESR	C	L	total price
Diode [5]	SBRT4U30LP-7	30V	4A	-	-	-	\$0.27280
Switch [6]	STT4P3LLH6	30V	4A	-	-	-	\$0.27438
C1 [11]	EZP-V80116LTA	800V	-	10.6mΩ	11μF	-	\$3.75820
C2 [3]	C4AQCBW5750A3NJ	650V	-	3.1mΩ	75μF	-	\$6.30600
L1 [9]	2100HT-102-V-RC	-	1.6A	430mΩ	-	1mH	\$2.504
L2 [10]	5713-RC	-	2.25A	420mΩ	-	700μH	\$6.8
						total:	**\$19.91**

Table 4. Component selection for the SEPIC.

Inductors L1 and L2 were chosen minding the current ratings denoted in Digikey.

Capacitors C1 and C2 were chosen according to their capacitances. Since the capacitances are high, the voltage ratings were expected to well exceed the minimum limit of 12V.

The diode and MOSFET were determined according to the current and voltage limits, allowing a safety margin of +2V (slightly below the 10% limit of 2.8V) and +0.5A (above 10% of 3.5A).

d)

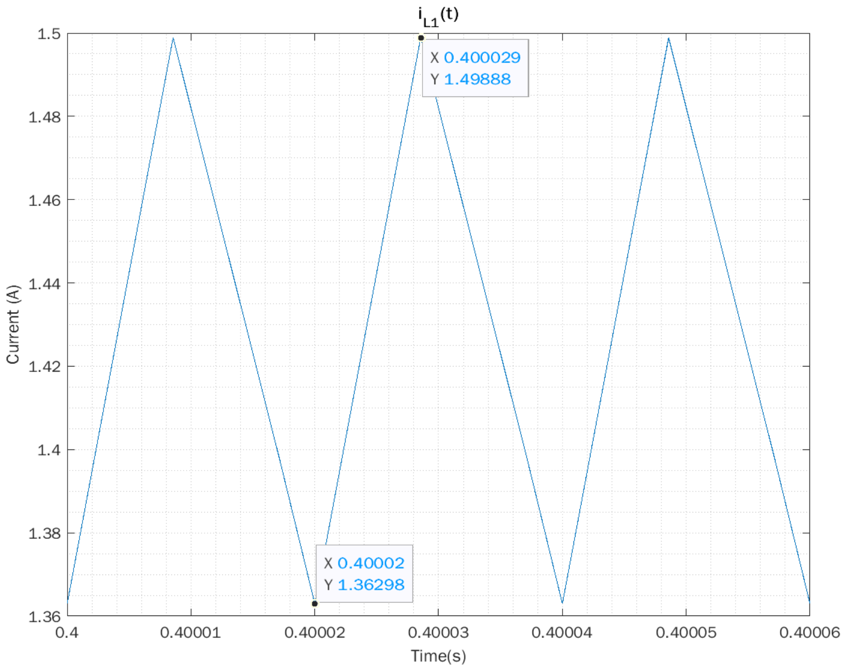
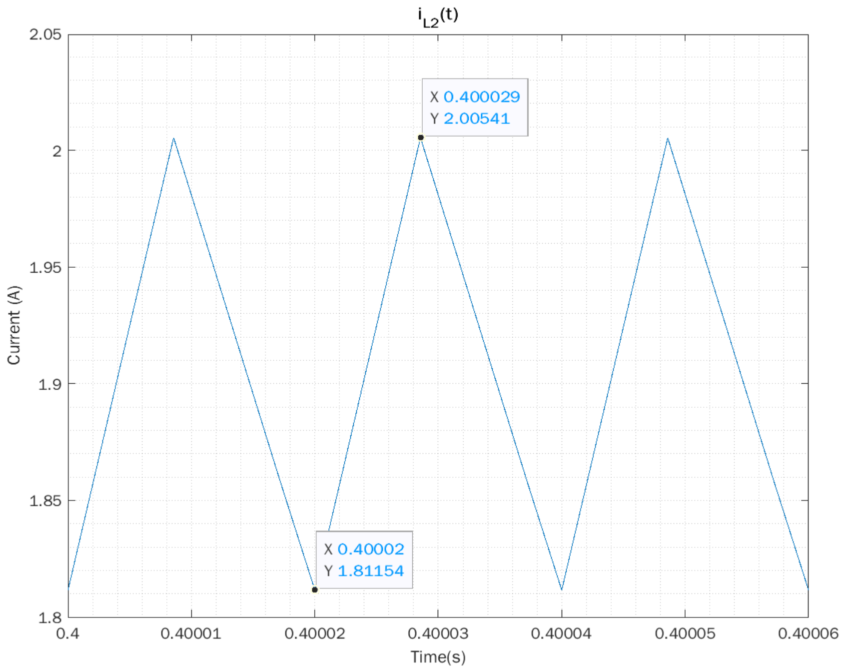


Figure 14. Current vs Time graph of L1.

The mean appears to be 1.424A in Simulink. The ripple is 0.1359A. This corresponds to a ratio of 9.54%, below the acceptable limit. The input power is less than 24W, at 22.78W. As per the case in the buck-boost converter, a lesser voltage headroom is allocated for L1 when its ESR has a higher voltage drop on it with increasing current, explaining the slightly lesser average current.



$i_{L2}(t)$
Current (A)
Time(s)
X 0.400029
Y 2.00541
X 0.40002
Y 1.81154
2.05
2
1.95
1.9
1.85
1.8
0.4
0.40001
0.40002
0.40003
0.40004
0.40005
0.40006

Figure 15. Current vs Time graph of L2.

The mean appears to be 1.899A in Simulink. The ripple is 0.1939A. This corresponds to a ratio of 10.21%, slightly above the acceptable limit.

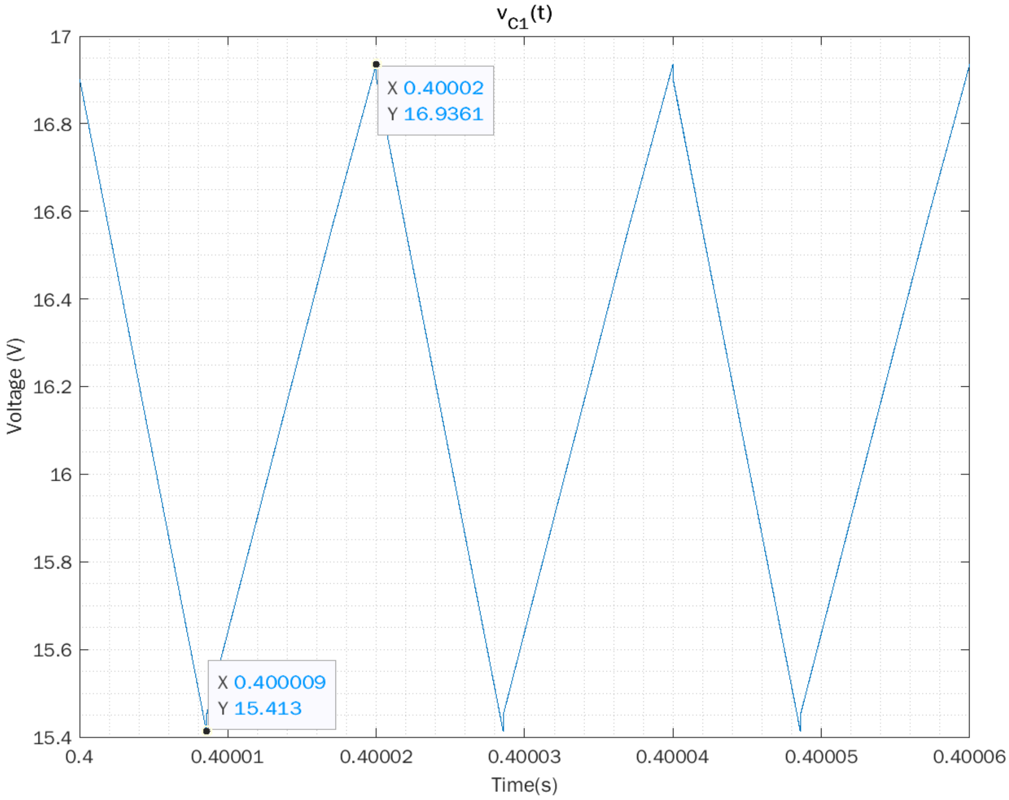


Figure 16. Voltage vs Time graph of C1.

The mean appears to be 16.26V in Simulink. The ripple is 1.523V. This corresponds to a ratio of 9.37%, below the acceptable limit.

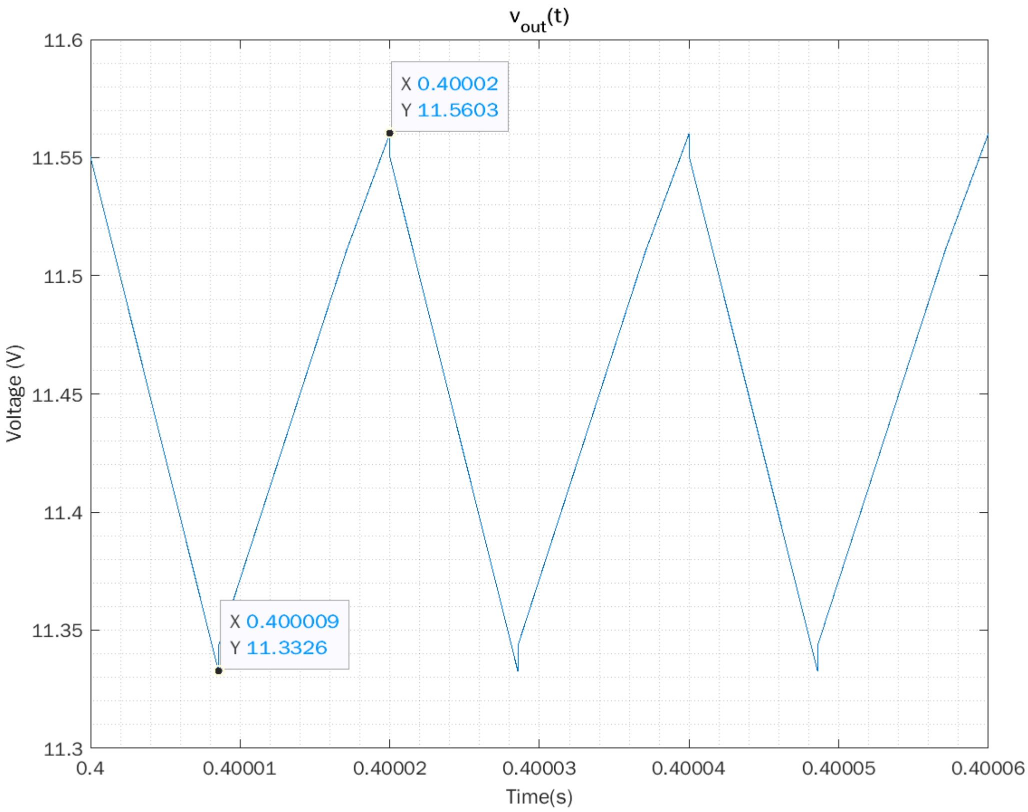


Figure 16. Voltage vs Time graph of C2.

The mean appears to be 11.46V, (0.44V off) in Simulink. The ripple is 0.2277V. This corresponds to a ratio of 1.96%, just below the acceptable limit. The output power, however, is less than 12W, at 21.89W. The lesser output voltage may be explained with the addition of ESR's, and how the ESR impacts the transfer function. A different duty than the idealized case should be used to still supply 12V at the output. The efficiency is 96.09%.

Q4. Verdict

a)

Buck Boost Converter

Device		Voltage rating	Current Rating	ESR	C	L	total price
Diode [1]	SBR3M30P1-7	30V	3A	-	-	-	$0.24310
Switch [2]	RSL020P03FRATR	30V	2A	-	-	-	$0.24684
Capacitor [3]	C4AQCBW5750A3NJ	650V	-	3.1mΩ	75μF	-	$6.30600
Inductor [4]	33470C	-	3.5A (± 15% tolerance)	48mΩ	-	0.470mH	$1.43640

total:	**$8.232**

Cuk Convereter

Device		Voltage rating	Current Rating	ESR	C	L	total price
Diode [5]	SBRT4U30LP-7	30V	4A	-	-	-	$0.27280
Switch [6]	STT4P3LLH6	30V	4A	-	-	-	$0.27438
C1 [7]	865080540003	35V	-	220mΩ	6.8μF	-	$0.18
C2 [8]	EEF-CD1C2R2R	16V	-	110mΩ	2.2μF	-	$0.72
L1 [9]	2100HT-102-V-RC	-	1.6A	430mΩ	-	1mH	$2.504
L2 [10]	5713-RC	-	2.25A	420mΩ	-	700μH	$6.8
						total:	**$10.76**

SEPIC

Device		Voltage rating	Current Rating	ESR	C	L	total price
Diode [5]	SBRT4U30LP-7	30V	4A	-	-	-	$0.27280
Switch [6]	STT4P3LLH6	30V	4A	-	-	-	$0.27438
C1 [11]	EZP-V80116LTA	800V	-	10.6mΩ	11μF	-	$3.75820
C2 [3]	C4AQCBW5750A3NJ	650V	-	3.1mΩ	75μF	-	$6.30600
L1 [9]	2100HT-102-V-RC	-	1.6A	430mΩ	-	1mH	$2.504
L2 [10]	5713-RC	-	2.25A	420mΩ	-	700μH	$6.8
						total:	**$19.91**

b)

- The Buck-Boost and Cuk Converter supplies negative voltage with respect to the input, whereas a SEPIC supplies one in the same polarity.
- The Cuk and SEPIC filter the input and output current and render it continuous, hence, the EMI is reduced. There should not be any ringing or oscillations at the switching devices thanks to this change from the buck-boost converter.
- A SEPIC may shut down completely and safely during the OFF cycle once C1 is fully charged, contributing to its immunity against EMI.
- SEPICs and Cuk converters have twice the number of first order elements compared to a first order circuit and these elements should be chosen carefully minding ESRs. With large L and C values, the converter prices also rise. Practically, they have fourth order transfer functions.
- SEPICs may offer single core inductors.
- The Buck- Boost converter is cheaper because of component number and it is smaller.
- The Buck- Boost converter is more efficient and easier to control because there is less ESR, stemming from a lower number of first order elements. [12]

c)

We choose the Cuk Converter, because its output voltage and input current ripple is less than buck-boost converter and its cheaper than SEPIC.

We would choose the Buck Boost if the budget is the most important.

We would choose the SEPIC if the filtering and the polarity of the output is more important.

Components

[1] Buck-boost diode:
https://www.digikey.com/en/products/detail/diodes-incorporated/SBR3M30P1-7/1629593
[2] Buck-boost MOSFET:
https://www.digikey.com/en/products/detail/rohm-semiconductor/RSL020P03FRATR/8028499
[3] Buck-boost capacitor and SEPIC C2:
https://www.digikey.com/en/products/detail/kemet/C4AQCBW5750A3NJ/8345970
[4] Buck-boost inductor:
https://www.digikey.com/en/products/detail/murata-power-solutions-inc/33470C/1924622
[5] SEPIC&Cuk diode:
https://www.digikey.com/en/products/detail/diodes-incorporated/SBRT4U30LP-7/5371882
[6] SEPIC&Cuk switch:
https://www.digikey.com/en/products/detail/stmicroelectronics/STT4P3LLH6/5244879
[7] Cuk C1:
https://www.digikey.com/en/products/detail/w%C3%BCrth-elektronik/865080540003/5728097
[8] Cuk C2:
https://www.digikey.com/en/products/detail/panasonic-electronic-components/eef-cd1c8r2cr/9471543
[9] Cuk L1 and SEPIC L1;
https://www.digikey.com/en/products/detail/bourns-inc/2100HT-102-V-RC/2534597?s=N4IgTCBcDa4lwAYEAkAqBaRZ0DV0CUBhEAXQF8g
[10] Cuk L2 and SEPIC L2;
https://www.digikey.com/en/products/detail/bourns-inc/5713-RC/775258?s=N4IgTCBcDaIKwHYCMBmAtAJwMYgLoF8g
[11] SEPIC C1:
https://www.digikey.com/en/products/detail/panasonic-electronic-components/EZP-V80116LTA/13984908
[12] “DC/DC Converters Continued” Keysan, O. (2022)
http://keysan.me/presentations/ee464_cuk_sepic.html#1