

Last name(s)

Name

ID

Midterm EDA exam

Length: 2.5 hours

07/11/2016

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- The exam has 4 sheets, 8 sides and 4 problems.
  - Write your full name and ID on every sheet.
  - Write your answers to all problems in the exam sheets within the reserved space.
  - Unless otherwise indicated, all your answers must be justified.
- 

**Problem 1**

**(2 points)**

In this problem you do not need to justify your answers.

- (a) (0.8 pts.) Fill the gaps in the following table (except for the cell marked with **Do not fill**) with the costs in time for sorting a vector of integers of size  $n$  using the indicated algorithms. Assume uniform probability in the average case.

|                                       | Best case | Average case       | Worst case |
|---------------------------------------|-----------|--------------------|------------|
| Quicksort<br>(with Hoare's partition) |           |                    |            |
| Mergesort                             |           |                    |            |
| Insertion                             |           | <b>Do not fill</b> |            |

- (b) (0.2 pts.) The solution to the recurrence  $T(n) = 2T(n/4) + \Theta(1)$  is

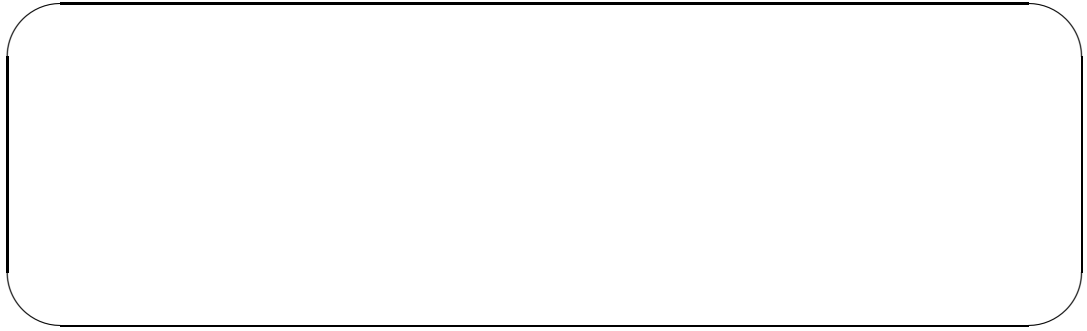
- (c) (0.2 pts.) The solution to the recurrence  $T(n) = 2T(n/4) + \Theta(\sqrt{n})$  is

- (d) (0.2 pts.) The solution to the recurrence  $T(n) = 2T(n/4) + \Theta(n)$  is

- (e) (0.3 pts.) What does Karatsuba's algorithm compute? What is its cost?

(it continues on the back)

(f) (0.3 pts.) What does Strassen's algorithm compute? What is its cost?

A large, empty rounded rectangular box with a thin black border, intended for the user to write their answer to the question.

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**Problem 2**

**(3 points)**

Given  $n \geq 2$ , we say that a sequence of  $n$  integers  $a_0, \dots, a_{n-1}$  is *bi-increasing* if  $a_{n-1} < a_0$  and there exists an index  $t$  (with  $0 \leq t < n$ ) that satisfies the following conditions:

- $a_0 \leq \dots \leq a_{t-1} \leq a_t$
- $a_{t+1} \leq a_{t+2} \leq \dots \leq a_{n-1}$

For example, the sequence 12, 12, 15, 20, 1, 3, 3, 5, 9 is bi-increasing (take  $t = 3$ ).

(a) (2 pts.) Implement in C++ a function

**bool** *search*(**const** *vector*<**int**>& *a*, **int** *x*);

which, given a vector  $a$  that contains a bi-increasing sequence and an integer  $x$ , returns whether  $x$  appears in the sequence or not. If you use auxiliary functions that are not part of the C++ standard library, implement them too. The solution must have cost  $\Theta(\log(n))$  in time in the worst case.

(it continues on the back)

- (b) (1 pt.) Justify that the cost in time in the worst case of your function *search* is  $\Theta(\log(n))$ . When does this worst case take place?

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**Problem 3**

**(2 points)**

Consider the following function:

```
int mystery(int m, int n) {  
    int p = 1;  
    int x = m;  
    int y = n;  
    while (y  $\neq$  0) {  
        if (y % 2 == 0) {  
            x *= x;  
            y /= 2;  
        }  
        else {  
            y -= 1;  
            p *= x;  
        }  
    }  
    return p;  
}
```

- (a) (1 pt.) Given two integers  $m, n \geq 0$ , what does  $mystery(m, n)$  compute? You do not need to justify your answer.

- (b) (1 pt.) Analyse the cost in time in the worst case as a function of  $n$  of  $mystery(m, n)$ .

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**Problem 4**

**(3 points)**

The *Fibonacci sequence* is defined by the recurrence  $F(0) = 0$ ,  $F(1) = 1$  and

$$F(n) = F(n-1) + F(n-2) \quad \text{if } n \geq 2.$$

- (a) (0.5 pts.) Let  $\phi = \frac{\sqrt{5}+1}{2}$  be the so-called *golden number*. Prove that  $\phi^{-1} = \phi - 1$ .

- (b) (1.5 pts.) Prove that, for any  $n \geq 0$ , we have

$$F(n) = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}}$$

*(it continues on the back)*

(c) (1 pt.) Prove that  $F(n) = \Theta(\phi^n)$ .