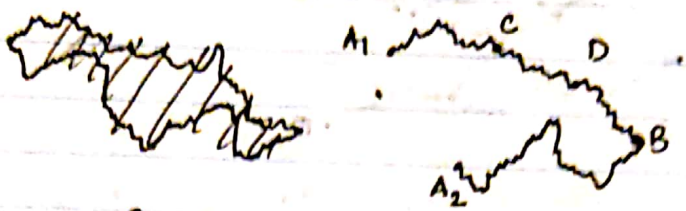
MATH ASIDE (contd...)

(8)



Let Y be a RV. s.t. $\mathrm{Cov}(Y, X_{A_1A_2}) = \zeta \neq 0$

$\mathrm{Cov}(X_{CD}, Y) = ?$

$$X_{A_1B} = \mathcal{N}\left(\frac{t_{A_1B}\, X_{A_1A_2}}{t_{A_1B}+t_{A_2B}}, \frac{t_{A_1B}\, t_{A_2B}}{t_{A_1B}+t_{A_2B}} \right).$$

$$X_{CD} = B_{t_{A_1D}} - B_{t_{A_1C}} - \frac{t_{CD}}{t_{A_1B}} \left(B_{t_{A_1B}} - X_{A_1B} \right)$$

$$\mathrm{Cov}(X_{CD}, Y) = E[Y X_{CD}] - E[Y]E[X_{CD}]$$

But $E[Y X_{CD}] = E\left[E[Y X_{CD} \mid (Y, X_{A_1B})] \right]$

$$= E\left[Y\, E[X_{CD} \mid (Y, X_{A_1B})] \right]$$

$$= E\left[Y \frac{t_{CD}}{t_{A_1B}} X_{A_1B} \right] = \frac{t_{CD}}{t_{A_1B}} E[Y X_{A_1B}]$$

$$= \frac{t_{CD}}{t_{A_1B}} E\left[E[Y X_{A_1B} \mid (Y, X_{A_1A_2})] \right] = \frac{t_{CD}}{t_{A_1B}} E\left[Y \frac{t_{A_1B}}{t_{A_1B}+t_{A_2B}} X_{A_1A_2} \right]$$

$$= \frac{t_{CD}}{t_{A_1B}+t_{A_2B}} E[Y X_{A_1A_2}]$$

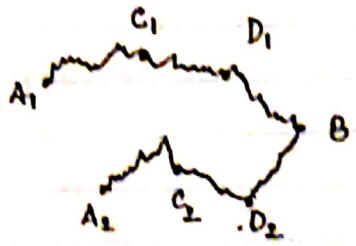
and $E[X_{CD}] = E[E[X_{CD} \mid X_{A_1B}]] = \frac{t_{CD}}{t_{A_1B}} E[X_{A_1B}] = \frac{t_{CD}}{t_{A_1B}} E\left[E[X_{A_1B} \mid X_{A_1A_2}] \right]$

$$= \frac{t_{CD}}{t_{A_1B}\, t_{A_2B}} E[X_{A_1A_2}]$$

$\therefore \mathrm{Cov}(X_{CD}, Y) = \frac{t_{CD}}{t_{A_1B}+t_{A_2B}} \left(E[Y X_{A_1A_2}] - E[Y]E[X_{A_1A_2}] \right)$

$$\Rightarrow \boxed{\mathrm{Cov}(X_{CD}, Y) = \frac{t_{CD}}{t_{A_1B}+t_{A_2B}} \mathrm{Cov}(Y, X_{A_1A_2}).}$$

(9.)



$Cov(C_1D_1, C_2D_2) = ?$

$$X_{A_1B} \sim \mathcal{N}\left(\frac{t_{A_1B}\, X_{A_1A_2}}{t_{A_1B} + t_{A_2B}} , \frac{t_{A_1B}\, t_{A_2B}}{t_{A_1B} + t_{A_2B}} \right)$$

$$X_{A_2B} \sim \mathcal{N}\left(\frac{t_{A_2B}\, X_{A_2A_1}}{t_{A_1B} + t_{A_2B}} , \frac{t_{A_1B}\, t_{A_2B}}{t_{A_1B} + t_{A_2B}} \right) \quad \text{Note: } X_{A_2A_1} = -X_{A_1A_2}$$

$$X_{C_1D_1} = B^{(1)}_{t_{A_1D_1}} - B^{(1)}_{t_{A_1C_1}} - \frac{t_{C_1D_1}}{t_{A_1B}} \left(B^{(1)}_{t_{A_1B}} - X_{A_1B} \right)$$

$$X_{C_2D_2} = B^{(2)}_{t_{A_2D_2}} - B^{(2)}_{t_{A_2C_2}} - \frac{t_{C_2D_2}}{t_{A_2B}} \left(B^{(2)}_{t_{A_2B}} - X_{A_2B} \right)$$

Further $X_{A_1B} - X_{A_2B} = X_{A_1A_2}$

$$Cov(X_{C_1D_1}, X_{C_2D_2}) = \frac{t_{C_1D_1}\, t_{C_2D_2}}{t_{A_1B}\, t_{A_2B}} Cov(X_{A_1B}, X_{A_2B})$$

But $Cov(X_{A_1B}, X_{A_2B}) = Cov(X_{A_1B}, X_{A_1B}) - Cov(X_{A_1B}, X_{A_1A_2})$

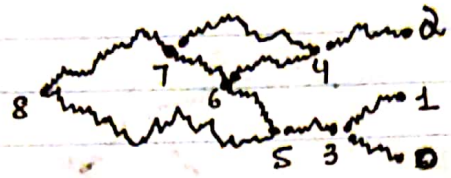
$$= Var\, X_{A_1B} - \frac{t_{A_1B}}{t_{A_1B}+t_{A_2B}} Var\, X_{A_1A_2}$$

$$= \frac{t_{A_1B}\, t_{A_2B}}{t_{A_1B}+t_{A_2B}} + \frac{t^2_{A_1B}}{(t_{A_1B}+t_{A_2B})^2} Var\, X_{A_1A_2} - \frac{t_{A_1B}}{t_{A_1B}+t_{A_2B}} Var\, X_{A_1A_2}$$

$$= \frac{t_{A_1B}\, t_{A_2B}}{t_{A_1B}+t_{A_2B}} - \frac{t_{A_1B}\, t_{A_2B}}{(t_{A_1B}+t_{A_2B})^2} Var\, X_{A_1A_2}$$

$$Cov(X_{C_1D_1}, X_{C_2D_2}) = \frac{t_{C_1D_1}\, t_{C_2D_2}}{t_{A_1B}+t_{A_2B}} - \frac{t_{C_1D_1}\, t_{C_2D_2}}{(t_{A_1B}+t_{A_2B})^2} Var\, X_{A_1A_2}$$

Opt 1



$\{$ $\text{Var } X_{80} = t_{80}$ $\text{Var } X_{81} = t_{81}$ $\text{Var } X_{42} = t_{42}$ $\text{Var } X_{53} = t_{53}$

$\text{Var } X_{85} = \frac{t_{85}}{2}$; $\text{Cov}(X_{87}, X_{85}) = \frac{t_{87}}{2}$; $\text{Cov}(X_{76}, X_{85}) = \frac{t_{76}}{2}$; $\text{Cov}(X_{65}, X_{85}) = \frac{t_{65}}{2}$

$\text{Var } X_{87} = \frac{t_{87}(2t_{85} - t_{87})}{2t_{85}}$; $\text{Var } X_{76} = \frac{t_{76}(2t_{85} - t_{87})}{2t_{85}}$; $\text{Var } X_{65} = \frac{t_{65}(2t_{85} - t_{65})}{2t_{85}}$

$\text{Cov}(X_{87}, X_{76}) = \frac{-t_{87}t_{76}}{t_{87}+t_{76}} 2t_{85}$; $\text{Cov}(X_{87}, X_{65}) = \frac{-t_{87}t_{65}}{2t_{85}}$; $\text{Cov}(X_{76}, X_{65}) = \frac{-t_{76}t_{65}}{2t_{85}}$

$\text{Var } X_{74} = \frac{t_{74}t_{64}}{t_{74}+t_{64}} + \frac{t_{74}^2}{(t_{74}+t_{64})^2} \text{Var } X_{76}$; $\text{Var } X_{64} = \frac{t_{74}t_{64}}{t_{74}+t_{64}} + \frac{t_{64}^2}{(t_{74}+t_{64})^2} \text{Var } X_{67}$

$\text{Cov}(X_{74}, X_{64}) = \frac{t_{74}t_{64}}{t_{74}+t_{64}} - \frac{t_{74}t_{64}}{(t_{74}+t_{64})^2} \text{Var } X_{76}$

$\text{Cov}(X_{74}, X_{76}) = \frac{t_{74}}{t_{74}+t_{64}} \text{Var } X_{76}$; $\text{Cov}(X_{64}, X_{76}) = -\text{Cov}(X_{64}, X_{67}) = \frac{-t_{64}}{t_{74}+t_{64}} \text{Var } X_{76}$

$\text{Cov}(X_{87}, X_{74}) = \frac{t_{74}}{t_{74}+t_{64}} \text{Cov}(X_{87}, X_{76})$; $\text{Cov}(X_{65}, X_{74}) = \frac{t_{74}}{t_{74}+t_{64}} \text{Cov}(X_{65}, X_{76})$

$\text{Cov}(X_{85}, X_{74}) = \frac{t_{74}}{t_{74}+t_{64}} \text{Cov}(X_{85}, X_{76})$

$\text{Cov}(X_{87}, X_{64}) = \frac{-t_{64}}{t_{74}+t_{64}} \text{Cov}(X_{87}, X_{76})$; $\text{Cov}(X_{65}, X_{64}) = \frac{-t_{64}}{t_{74}+t_{64}} \text{Cov}(X_{65}, X_{76})$

$\text{Cov}(X_{85}, X_{64}) = \frac{-t_{64}}{t_{74}+t_{64}} \text{Cov}(X_{85}, X_{76})$.