

# Causal inference in social network data

Oleg Sofrygin, Mark J. van der Laan

(UC Berkeley)

June 30, 2016

# Outline

- I. Background
- II. Network-dependent data as high-dimensional data problem
- III. Simulations & Estimation in **R** (`simcausal`/`tm1enet`)
- IV. Simulation study

# Outline

- 1 Background
- 2 Network-dependent data as high-dimensional data problem
- 3 Estimation with `tmle`net / Simulations with `simcausal`
- 4 Simulation Study

# Background

- This talk is not about statistical modeling of network formation
- Data has been gathered on individual people that are known to be connected by a social network
- The field has been gaining interest:
  - ▶ New ways of gathering data (online social networks, mobile fitness sensors)
- Want to know estimate an effect of some intervention among these people
- We hypothesize that network plays a role in the way the personal-level data was generated
  - ▶ The intervention might propagate amongst people
  - ▶ May induce dependence among units

# Background

- Christakis and Fowler (2007, 2008, 2009, 2010, 2011, 2012) initiated a wave of interest
- Widely publicized results with significant peer effects for **obesity, smoking, alcohol consumption, sleep habits**, etc.
- Criticized for ignoring the dependent nature of the data and for making unrealistic modeling assumptions
- Two problems for traditional (independent data) inference:
  - ▶ CLT may not hold
  - ▶ Not taking into account for dependence may result in S.E.s that are too small (anti-conservative)

# Main Goals

- Framework for estimation and inference in such data
- Software for simulation of synthetic population data under network dependence (`simcausal`)
- Software for estimation of effects in network-dependent data (`tm1enet`)
- Correct inference (a good estimate of the variance of our estimate)
- Side note: the issues discussed here are applicable to non-network (independent) high dimensional data

# Outline

- 1 Background
- 2 Network-dependent data as high-dimensional data problem
- 3 Estimation with `tmlenet` / Simulations with `simcausal`
- 4 Simulation Study

# Classical causal framework with IID setting

- Suppose we have  $N$  individuals (units) enrolled in a study
- $O_i = (W_i, A_i, Y_i)$  denotes the data collected on each unit, for  $i = 1, \dots, N$ :
  - ▶  $W_i$  - are the baseline covariates
  - ▶  $A_i$  - exposure (0/1)
  - ▶  $Y_i$  - outcome
- Want to estimate the ATE:

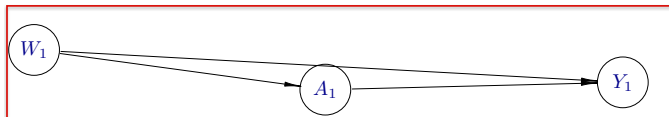
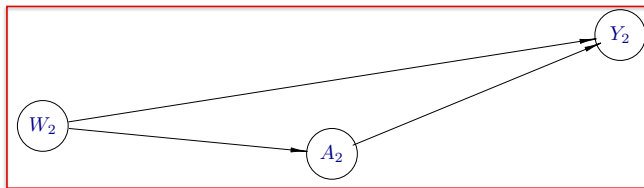
$$E_W [E(Y_i | A_i = 1, W_i) - E(Y_i | A_i = 0, W_i)]$$

- This parameter is a function of the true distribution of the data,  $P_0$
- It has causal interpretation under additional assumptions
- It is **interpretable** even when we don't believe in these assumptions!
- We can use state-of-the-art machine learning without ever losing this interpretability



## Two independent units with DAGs

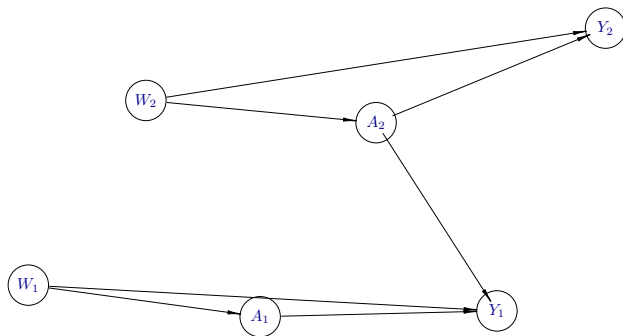
- Consider a typical causal DAG for two i.i.d. observations (**1** & **2**) with treatment  $A$ , baseline covariates  $W$  and outcome  $Y$ :



- Now these two units are also “connected” by a network (set of friends  $F_1$  and  $F_2$  that was also measured)

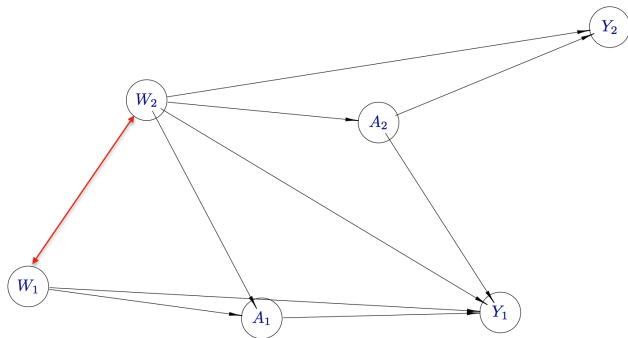
# What we mean by a “network”?

- Suppose unit 1 lists unit 2 is her “friend” (but not vice versa),
  - ▶ Allow **spillover**:  $Y_1$  depends on the treatment assignment of unit 2,  $A_2$ .



# What we mean by a “network”?

- $Y_1$  depends on  $W_2$  (baseline covariates of unit 1); and
- May allow  $W_1$  and  $W_2$  to be dependent (correlated) if units 1 and 2 are friends
- We may also assume  $A_1$  depends on  $W_2$  (in addition to  $W_1$ )



# Analogue to ATE in a network setting

- The ATE in IID data:

$$E_W [E(Y_i|A_i = 1, W_i) - E(Y_i|A_i = 0, W_i)]$$

- Network:

- ▶ Want to know the effect of setting  $A_j$  for  $j \in F_i$  on  $Y_i$
- ▶ All  $W_j$ , for  $j \in F_i$  are all confounders - need to adjust for them

- The ATE analogue in “networked” data:

$$\begin{aligned} & \frac{1}{N} \sum_i E(Y_i|A_i = 1, \mathbf{A}_{F_i} = \mathbf{1}^{|F_i|}, W_i, W_j : j \in F_i) \\ - & \frac{1}{N} \sum_i E(Y_i|A_i = 0, \mathbf{A}_{F_i} = \mathbf{0}^{|F_i|}, W_i, W_j : j \in F_i) \end{aligned}$$

# Network curse of dimensionality

- Suppose that  $i$  has 100 friends ( $|F_i| = 100$ )
- Have to adjust for  $W_i$  plus additional 100 ( $W_j : j \in F_i$ )
- Have to fit a model for the effect of  $A_i$  on  $Y_i$  plus the effect of 100 additional exposures ( $A_j : j \in F_i$ ) on  $Y_i$
- To have any hope of fitting the outcome model we have to assume some common model for  $N$  observations
  - ▶ But  $i$  and  $j$  can have different number of friends! How can we even have a common model?
- Ways around it:
  - ▶ Assume same number of friends for everybody
  - ▶ Assume very small number of friends (a most 2) - only household members
  - ▶ Clearly this is not a good representation of real data

# Network curse of dimensionality: network summaries

- Assume that my outcome ( $Y_i$ ) depends only on some functions (**network summaries**):

$$W_i^s := w_i^s(\mathbf{W}_{F_i}, W_i) \text{ and } A_i^s := a_i^s(\mathbf{A}_{F_i}, A_i)$$

- They have the same and **fixed** dimension for all  $i$  and are otherwise arbitrary
- Assume:

- Conditional probability  $P(A_i | \cdot)$  is only a function of summary  $w_i^s(\mathbf{W}_{F_i}, W_i)$
- Conditional density  $P(Y_i | \cdot)$  is only a function of  $w_i^s(\mathbf{W}_{F_i}, W_i)$  and  $a_i^s(\mathbf{A}_{F_i}, A_i)$

- Simplifies the notation:

- ▶ Data on  $N$  units can be represented:

$$O_i^s = (W_i^s, A_i^s, Y_i), \text{ for } i = 1, \dots, N$$

- ▶ Our estimand (ATE):

$$\frac{1}{N} \sum_{i=1}^N [E(Y_i | A_i^s = a_i^s(\mathbf{o}), W_i^s) - E(Y_i | A_i^s = a_i^s(\mathbf{1}), W_i^s)]$$

# Outline

- 1 Background
- 2 Network-dependent data as high-dimensional data problem
- 3 Estimation with `tmle`net / Simulations with `simcausal`
- 4 Simulation Study

# Syntax for network summaries in R (tmlenet and simcausal)

- Define network baseline summaries / features  $W^s$  with function **def\_sW**:

```
def_sW(netW1W2 = sum(W1[[1:Kmax]]*W2[[1:Kmax]]))
```

- Define network exposure summaries / features  $A^s$  with function **def\_sA**:

```
def_sA(A, sum.net.A = (sum(A[[1:Kmax]])))
```



- Implements 3 estimators for network data
- **IPW**: Inverse Probability Weighted Estimator
  - ▶ Re-weights the outcomes  $Y_i$  by the inverse probability of receiving the network exposure summary (the effective exposure)
- **GCOMP**: G-Computation Estimator
  - ▶ Directly fit the outcome model:  $(E(Y_i|A_i^s, W_i^s))$
- **TMLE**: Targeted Maximum Likelihood Estimator
  - ▶ Combines IPW and GCOMP into a single estimator to take advantage of both
  - ▶ Involves only a single additional modeling step (at low computational cost)
  - ▶ Recovers the CLT for the estimator (allows ML)
  - ▶ Provides asymptotically valid confidence intervals
- `tmlenet` will work with independent data just as well (no network)
- For network data, `tmlenet` implements two approaches for estimating variance that adjusts for dependence

- Defining “effective” exposure  $A_i^s$  created another problem:
  - ▶ Even when  $A_i \in \{0, 1\}$ , the summary  $A_i^s$  is likely to be continuous
- The “effective” exposure model is now a **multivariate conditional** density rather than a binary classification problem:  $p_{A_i^s|W_i^s}(a^s|w^s)$
- tmlenet implements conditional histogram density estimator for  $p_{A_i^s|W_i^s}$ 
  - ▶ Discretize range of  $A_i^s$  by splitting it into intervals (bins)
  - ▶ Fit a separate binary classification/regression for each bin as a function of the baseline summaries  $W_i^s$
  - ▶ Automatically detects the type of the exposure summary and then decides how to fit it
- tmlenet allows for stochastic interventions, among others:
  - ▶ **Stochastic Intervention**: covered a random 40% of the community?
  - ▶ **Targeted Intervention**: covered only the top 10% most connected community members?
  - ▶ **Network intervention**: remove or add a new friend?

# Network simulation with `simcausal` - example

- `simcausal`:

- ▶ Simulates synthetic datasets to test statistical methods applied in causal inference
- ▶ Time-varying (longitudinal data) and network-dependent data
- ▶ Single pipeline for conducting a “typical” simulation study in causal inference
- ▶ Supports arbitrary univariate and multivariate (conditional) distributions

```
node("Y", distr = "rbern", prob = plogis(0.5*W - 0.35*A - 0.5*sum(A[[1:Kmax]])))
```

- Above defined  $P(Y_i = 1|\cdot)$  as logit-linear function of:
  - ▶ Baseline covariate (**W**), exposure (**A**), and
  - ▶ Sum of friends' exposures (**sum(A[[1:Kmax]])**)
- Note: **Kmax** is a special constant - maximum number of friends and is evaluated automatically by `simcausal`

# Estimation with *tmlenet* - example

- Define baseline summaries / features  $W^s$  with function **def\_sW**:

```
sW <- def_sW(W1, W2) +  
  def_sW(netW1W2 = sum(W1[[1:Kmax]]*W2[[1:Kmax]]),  
        nF.PA = sum(PA[[1:Kmax]]),  
        nFPAeq0.PAeq1 = (nF.PA < 1) * (PA == 1),  
        replaceNAw0 = TRUE)
```

- Define exposure summaries / features  $A^s$  with function **def\_sA**:

```
sA <- def_sA(A, A.PAeq0 = A * (PA == 0)) +  
  def_sA(sum.net.A = (sum(A[[1:Kmax]])*(HUB==0) +  
                    sum((W1[[1:Kmax]] > 4)*A[[1:Kmax]])*(HUB==1)),  
        replaceNAw0 = TRUE)
```

# Estimation with *tmlenet* - example

- Define interventions with function **def\_new\_sA**:

```
intervene_1 <- def_new_sA(A = 0)
intervene_2 <- def_new_sA(A = 1 - A)
intervene_stoch <- def_new_sA(A = rbinom(n = length(A), size = 1, prob = 0.35))
intervene_dyn <- def_new_sA(A = rbinom(n = length(A), size = 1,
                                         prob = ifelse(nF >= 20, 0.9, 0.1)))
```

# Estimation with *tmlenet* - example

- Function **tmlenet** performs estimation (also requires the network matrix and the input data):

```
# REGRESSION FORMULAS
Qform <- "Y ~ nF.PA + A.PAeq0 + nFPAeq0.PAeq1 + sum.net.A + PA + W1 + W2"
hform.g0 <- "A + sum.net.A ~ HUB + PA + nF.PA + nFPAeq0.PAeq1"
# EFFECT ESTIMATION
res <- tmlenet(data = sim_dat, sW = sW, sA = sA,
  NETIDmat = NetInd_mat,
  Kmax = ncol(NetInd_mat),
  intervene1.sA = intervene_stoch,
  Qform = Qform,
  hform.g0 = hform.g0,
  hform.gstar = hform.g0,
  optPars = list(
    bootstrap.var = FALSE)
)
```

# Outline

- 1 Background
- 2 Network-dependent data as high-dimensional data problem
- 3 Estimation with `tmlenet` / Simulations with `simcausal`
- 4 Simulation Study

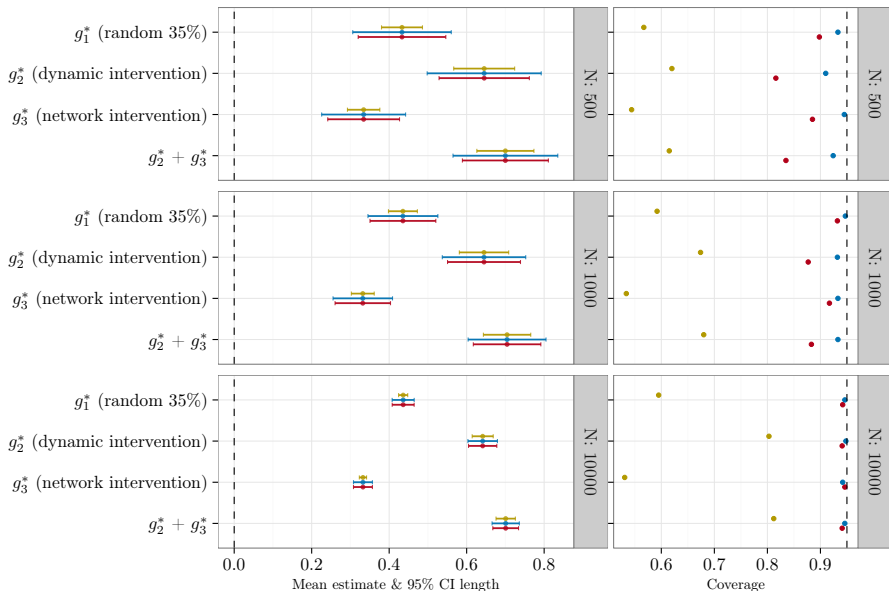
# Simulation - Peer Effects of Exercise

- Simulated a small world network
- Study designed to increase the levels of physical activity in a highly-connected community
- Individuals randomly received vouchers to attend a local gym
- Outcome is a binary indicator of maintaining gym membership
- Estimated the effects of:
  - ▶ Assigning exposure to random 35%
  - ▶ Targeted exposure assignment to top 10% most connected units
  - ▶ Effect of combining the exposure with network interventions (additional physically active friend for each units with  $<10$  friends)



# Simulation Results - Small World Network

CI.type ● dependent IC Var ● bootstrap Var ● iid Var



# Concluding remarks

- **tmlenet** solves some estimation challenges in network-dependent data
- Allows continuous exposures & arbitrary stochastic interventions
- Flexible interface for defining arbitrary summaries/features of network covariates
- Two ways of doing inference while adjusting for dependence
- Ongoing work, new features are being added (e.g., networks over multiple time-points)
- See **simcausal** vignette on CRAN and JSS paper to appear  
<https://cran.r-project.org/web/packages/simcausal>
- Github:
  - ▶ **simcausal**: <https://github.com/osofr/simcausal>
  - ▶ **tmlenet**: <https://github.com/osofr/tmlenet>
  - ▶ **stremr** (most recent expansion of **tmlenet** code into longitudinal IID data, estimation with h2o ML libraries): <https://github.com/osofr/stremr>

# REFERENCES

- ① Sofrygin, O and van der Laan, M J, "Semi-Parametric Estimation and Inference for the Mean Outcome of the Single Time-Point Intervention in a Causally Connected Population" (December 2015). *U.C. Berkeley Division of Biostatistics Working Paper Series*. Working Paper 344.
- ② Sofrygin, O. and van der Laan, M. J. (2015). simcausal R Package: Conducting Transparent and Reproducible Simulation Studies of Causal Effect Estimation with Complex Longitudinal Data. *Submitted to J of Stat Soft*.
- ③ Sofrygin, O. and van der Laan, M. J. (2015). tmlenet: Targeted Maximum Likelihood Estimation for Network Data. R package version 0.1.0.
- ④ van der Laan, M. J. (2014). Causal Inference for a Population of Causally Connected Units. *Journal of Causal Inference*, 2(1):1–62.

## ● FUNDING ACKNOWLEDGEMENT:

- ▶ This work was partially supported through an NIH grant (R01 AI074345-07) and a Patient-Centered Outcomes Research Institute (PCORI) Award (ME-1403-12506).