0

CONSIGNA:

Dada la siguiente tabla de datos:

X	1,2	2,1	2,8	3,1	3,5	4,1	4,4	4,9	5,6	5,9	6,2	6,5
Y	1,06	2,14	3,23	3,8	4,7	6,3	7,33	9,48	13,98	16,56	20,23	25,45

Se pide:

a) Graficar la nube de puntos.

b) Hallar la Recta de Mínimos cuadrados.

c) Hallar la Parábola de Mínimos cuadrados.

d) Ajustar al modelo $y = a\, x / (b+x)$

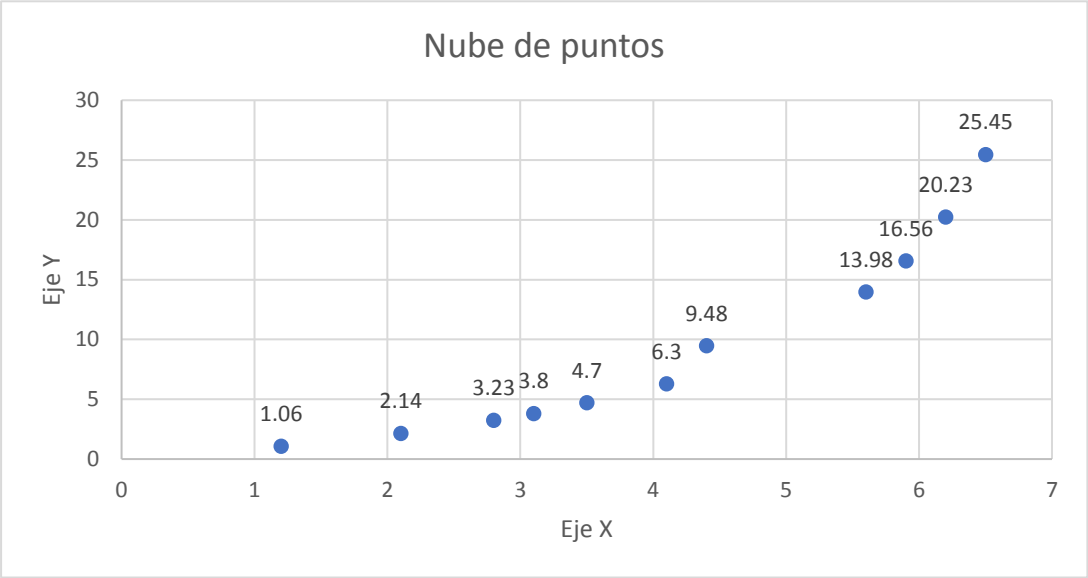
e) Ajustar a un modelo $y = a\, b^x$

f) Comparar y decidir la mejor aproximación.

Nota: en todos los casos deben indicar las sumas para plantear el Sistema de Ecuaciones Normales, y si corresponde aclarar el cambio de variables. Dicho sistema se puede resolver como se desee pero debe indicarlo. Y luego debe graficarse la función hallada junto a la nube de puntos.

1

a)



B

Proponemos como expression aproximante: P(X) = ax+b

$$\begin{cases} a\sum x_i^2 + b\sum x_i = \sum x_i y_i \\ a\sum x_i + bn = \sum y_i \end{cases}$$

i	x_i	y_i	X_i^2	$X_i . Y_i$
0	1,2	1,06	1,44	1,272
1	2,1	2,14	4,41	4,494
2	2,8	3,23	7,84	9,044
3	3,1	3,8	9,61	11,78
4	3,5	4,7	12,25	16,45
5	4,1	6,3	16,81	25,83
6	4,4	7,33	19,36	32,252
7	4,9	9,48	24,01	46,452
8	5,6	13,98	31,36	78,288
9	5,9	16,56	34,81	97,704
10	6,2	20,23	38,44	125,426
11	6,5	25,45	42,25	165,425
	50,3	**114,26**	**242,59**	**614,417**

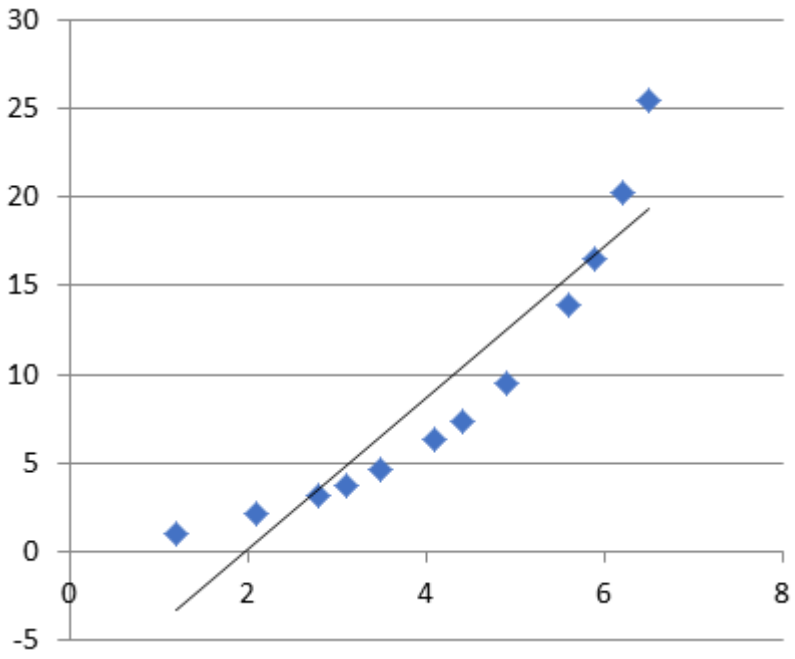
$$\begin{cases} a_{242,59} + b_{50,3} = 614,417 \\ a_{50,3} + b_{12} = 114,26 \end{cases}$$

Resolviendo el sistema queda:

a =4,26723

b =-8,364335

Y = 4,26723*x -8,364335 (es la mejor recta de ajuste de los datos)



c) proponemos : $P(x) = ax^2+bx+c$

$$\begin{cases} an + b\sum x_i + c\sum x_i^2 = \sum y_i \\ a\sum x_i + b\sum x_i^2 + c\sum x_i^3 = \sum x_i y_i \\ a\sum x_i^2 + v\sum x_i^3 + c\sum x_i^4 = \sum x_i^2 y_i \end{cases}$$

i	x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i . y_i$	$x_i^2 . y_i$
0	1,2	1,06	1,44	1,728	2,0736	1,272	1,5264
1	2,1	2,14	4,41	9,261	19,4481	4,494	9,4374
2	2,8	3,23	7,84	21,952	61,4656	9,044	25,3232
3	3,1	3,8	9,61	29,791	92,3521	11,78	36,518
4	3,5	4,7	12,25	42,875	150,0625	16,45	57,575
5	4,1	6,3	16,81	68,921	282,5761	25,83	105,903
6	4,4	7,33	19,36	85,184	374,8096	32,252	141,9088
7	4,9	9,48	24,01	117,649	576,4801	46,452	227,6148
8	5,6	13,98	31,36	175,616	983,4496	78,288	438,4128
9	5,9	16,56	34,81	205,379	1211,7361	97,704	576,4536
10	6,2	20,23	38,44	238,328	1477,6336	125,426	777,6412
11	6,5	25,45	42,25	274,625	1785,0625	165,425	1075,2625
∑	50,3	114,26	242,59	1271,309	7017,1495	614,417	3473,5767

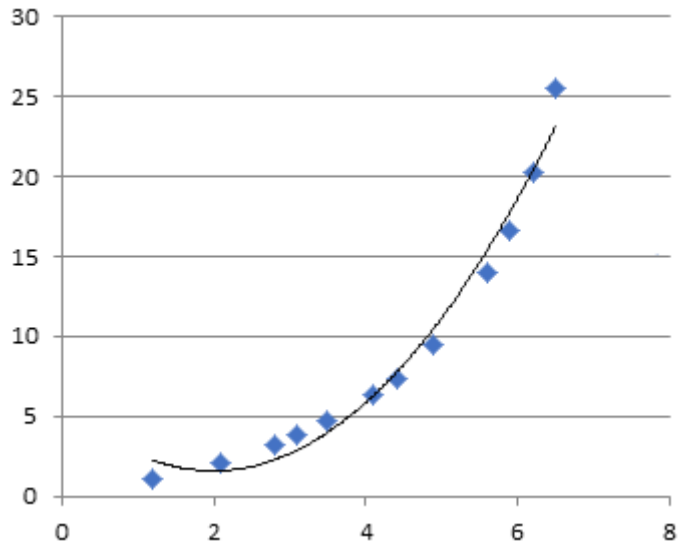
$$\begin{cases} a12 + b50{,}3 + c242{,}59 = 114{,}26 \\ a50{,}3 + b242{,}59 + c1271{,}309 = 614{,}417 \\ a242{,}59 + b1271{,}309 + c7017{,}1495 = 3473{,}5767 \end{cases}$$

a =1,059591

b =-4,308264

c=5,784906

y = 1,059591 x^2 + -4,208264 x + 5,784906



d)y =a/b+x vamos a linealizar

1/y =b/a + 1/a.x

Y=B +AX siendo A=1/a y B=b/a

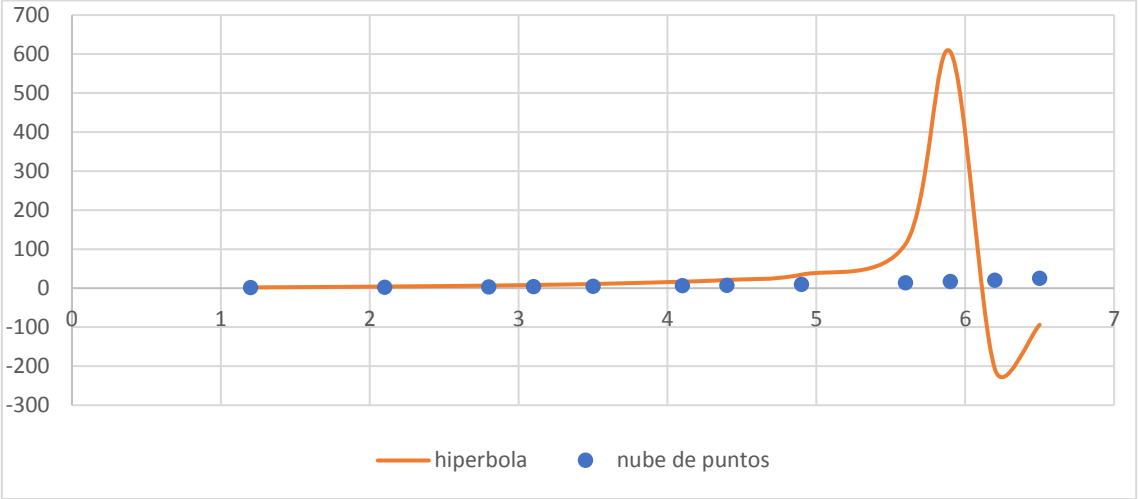
$$\begin{cases} A\sum x_i^2 + B\sum x_i = \sum x_i Y_i \\ A\sum x_i + Bn = \sum Yi \end{cases}$$

i	x_i	y_i	Yi	xiYi	x_i^2
0	1.2	1.06	0.943396226	1.132075472	1,44
1	2.1	2.14	0.46728972	0.981308411	4,41
2	2.8	3.23	0.309597523	0.866873065	7,84
3	3.1	3.8	0.263157895	0.815789474	9,61
4	3.5	4.7	0.212765957	0.744680851	12,25
5	4.1	6.3	0.158730159	0.650793651	16,81
6	4.4	7.33	0.136425648	0.600272851	19,36
7	4.9	9.48	0.105485232	0.516877637	24,01
8	5.6	13.98	0.071530758	0.400572246	31,36
9	5.9	16.56	0.060386473	0.356280193	34,81
10	6.2	20.23	0.049431537	0.306475531	38,44
11	6.5	25.45	0.039292731	0.25540275	42,25
∑	50,3	114.26	2.81748986	7.627402133	242,59

$$\begin{cases} A(242.59) + B(\mathbf{50.3}) = \mathbf{7.627402} \\ A\mathbf{50,3} + B12 = \mathbf{2.817490} \end{cases}$$

A= -0,131738 B=0,786993 a=-7,590824 b=-5,973925

y = $\frac{-7{,}590824x}{x-5{,}973925}$



e) $\begin{cases} A\sum(x_i)^2 + B\sum x_i = \sum x_i Ln(y_i) \\ A\sum x_i + Bn = \sum Ln(y_i) \end{cases}$

i	x_i	y_i	$Ln(y_i)$	$x_i * Ln(y_i)$	X_i^2
0	1,2	1,06	0.058268908	0.06992269	1,44
1	2,1	2,14	0.760805829	1.59769224	4,41
2	2,8	3,23	1.172482137	3.28294998	7,84
3	3,1	3,8	1.335001067	4.13850331	9,61
4	3,5	4,7	1.547562509	5.41646878	12,25
5	4,1	6,3	1.840549633	7.54625350	16,81
6	4,4	7,33	1.991975516	8.76469227	19,36
7	4,9	9,48	2.249184316	11.0210032	24,01
8	5,6	13,98	2.637627737	14.7707153	31,36
9	5,9	16,56	2.806990149	16.5612419	34,81
10	6,2	20,23	3.007166651	18.6444332	38,44
11	6,5	25,45	3.236715743	21.0386523	42,25
∑	**50,3**	**114,26**	**22.6443302**	**112.852529**	**242.59**

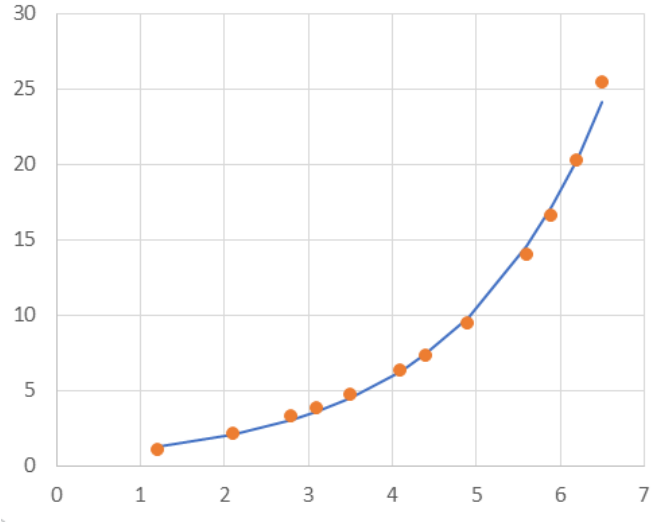
$\begin{cases} A(242.59) + B(\mathbf{50.3}) = \mathbf{112.852529} \\ A\mathbf{50,3} + B12 = \mathbf{22.6443302} \end{cases}$

y = ab^x

Ln(y) = Ln(a) + xLn(b)

Con Y = Ln(y), A=Ln(b), B=Ln(a)

A = 0.562281 a = 0.62508
B = -0.46987 b = 1.75467
y = $0.62508 * 1.75467^x$



f)para calcular cual aproximación ajusta mejor a los datos, se calcula la suma de los cuadrados de las distancia

para la recta : Y = 4,26723*x -8,364335

x_i	y_i	recta y(x)	di^2
1.2	1.06	-3.243659	18.52148079
2.1	2.14	0.596848	2.381318095
2.8	3.23	3.583909	0.12525158
3.1	3.8	4.864078	1.13226199
3.5	4.7	6.57097	3.500528741
4.1	6.3	9.131308	8.016304991
4.4	7.33	10.411477	9.495500502
4.9	9.48	12.545092	9.394788968
5.6	13.98	15.532153	2.409178935
5.9	16.56	16.812322	0.063666392
6.2	20.23	18.092491	4.568944725
6.5	25.45	19.37266	36.93406148
			96.54328718

Para la parábola y = 1,059591 x^2 + -4,208264 x + 5,784906

x_i	y_i	parabola y(x)	di^2
1.2	1.06	2.26080024	1.441921216
2.1	2.14	1.62034791	0.270038295
2.8	3.23	2.30896024	0.84831424
3.1	3.8	2.92195711	0.770959317
3.5	4.7	4.03597175	0.440933517
4.1	6.3	6.34274831	0.001827418
4.4	7.33	7.78222616	0.2045085
4.9	9.48	10.60519231	1.266057734
5.6	13.98	15.44740136	2.153266751
5.9	16.56	17.84051111	1.639708703
6.2	20.23	20.42434724	0.03777085
6.5	25.45	23.19890975	5.067407314
			14.14271385

Para la hipérbola

y = $\frac{-7{,}590824x}{x-5{,}973925}$

x_i	y_i	hiperbola y(x)	di^2
1.2	1.06	1.908071199	0.719224759
2.1	2.14	4.114878424	3.900144791
2.8	3.23	6.696537316	12.01688096
3.1	3.8	8.187950068	19.2541058
3.5	4.7	10.73916307	36.4714906
4.1	6.3	16.6081238	106.2574164
4.4	7.33	21.22059539	192.9486403
4.9	9.48	34.63466965	632.757405
5.6	13.98	113.682194	9940.527493
5.9	16.56	605.8283612	347237.2015
6.2	20.23	-208.1747597	52168.73426
6.5	25.45	-93.78958514	14218.07866
			424568.8672

Para la exponencial $y = 0.62508 * 1.75467^{x}$

x_i	y_i	exponencial y(x)	di^2
1.2	1.06	1.227354854	0.028007647
2.1	2.14	2.035851285	0.010846955
2.8	3.23	3.017747272	0.04505122
3.1	3.8	3.572247174	0.05187135
3.5	4.7	4.473214064	0.051431861
4.1	6.3	6.268114948	0.001016657
4.4	7.33	7.419857891	0.00807444
4.9	9.48	9.828637392	0.121548031
5.6	13.98	14.56901293	0.346936226
5.9	16.56	17.24601517	0.470616818
6.2	20.23	20.41490669	0.034190482
6.5	25.45	24.16607029	1.648475491
			2.818067179

La que tiene menos suma de cuadrados de las distancias es la función exponencial

Por lo cual es la que mejor ajusta a los datos