Algorithm 3.1 (Wave Elevation and Wave Slope)

1. Divide the spectral density function $S(\omega)$ into N intervals with length $\Delta\omega$, see Figure 3.1.
2. Pick a random frequency ω_i in each of the frequency intervals and compute $S(\omega_i)$.
3. Compute the wave amplitude $A_i = \sqrt{2S(\omega_i)\,\Delta\omega}$ and the wave number $k_i = \omega_i^2/g$ for $(i = 1...N)$.
4. Compute ζ_i and s_i by applying Formulas (3.70) and (3.71).

3.3 Wind

Wind forces and moments on a vessel can usually be described in terms of a mean wind speed in combination with a wind spectrum describing the variation of the wind speed (gusting). We will first describe some standard wind spectra for this purpose and then relate the wind speed and direction to the forces and moments acting on the vehicle.

3.3.1 Standard Wind Spectra

One the most used spectral formulations for wind gust is the Davenport (1961) spectrum:

$$S_w(\omega) = k\,\frac{916700\,\omega}{[1 + (191\,\omega/V_w(10))^2]^{4/3}} \tag{3.78}$$

where

k	=	0.05 (turbulence factor)
$V_w(10)$	=	average wind speed at a level of 10 m above the water surface (knots)
ω	=	frequency of the wind oscillations (rad/s)

Another attractive spectral formulation is the so-called Harris (1971) spectrum which is written:

$$S_w(\omega) = k\,\frac{5286\,V_w(10)}{[1 + (286\,\omega/V_w(10))^2]^{5/6}} \tag{3.79}$$

These spectra are based on land-based measurements. More recently Ochi and Shin (1988) presented a spectral formulation relying on wind speed measurements carried out at sea. This spectrum is written in non-dimensional form according to:

$$S(f_*) = \begin{cases} 583\,f_* & \text{for} \quad 0 \le f_* < 0.003 \\ \frac{420\,f_*^{0.70}}{(1+f_*^{0.35})^{11.5}} & \text{for} \quad 0.003 \le f_* \le 0.1 \\ \frac{838\,f_*}{(1+f_*^{0.35})^{11.5}} & \text{for} \quad f_* > 0.1 \end{cases} \tag{3.80}$$

where

$$
\begin{aligned}
f_* &= 10\, f/V_w(10) \\
S(f_*) &= f \cdot S(f)/C_{10} \cdot V_w^2(10) \\
f &= \text{frequency of oscillation (Hz)} \\
C_{10} &= \text{surface drag coefficient, see Ochi and Shin (1988)} \\
S(f) &= \text{spectral density}
\end{aligned}
$$

Other useful spectral formulations are Hino (1971), Kaimal et al. (1972), Simiu and Leigh (1983) and Kareem (1985); see the 10th ISSC (1988) pp. 15–18 and references therein.

Linear Approximation to the Harris Spectrum

The above wind spectra are nonlinear approximations. A linear 1st-order approximation for the Harris spectrum is:

$$h(s) = \frac{K}{1 + Ts} \tag{3.81}$$

which implies that:

$$S_w(\omega) \approx |h(j\omega)|^2 = \frac{K^2}{1 + (\omega T)^2} \tag{3.82}$$

Hence, we can choose the time and gain constant according to:

$$K = \sqrt{5286\, k\, V_w(10)}; \qquad T = \sqrt{286/V_w(10)} \tag{3.83}$$

Wind Velocity Profile

In order to determine the local velocity z (m) above the sea surface we can use the boundary-layer profile (see Bretschneider 1969):

$$V_w(z) = V_w(10) \cdot (z/10)^{1/7} \tag{3.84}$$

where $V_w(10)$ is the relative wind velocity 10 (m) above the sea surface.

3.3.2 Wind Forces and Moments

As mentioned in the previous section the total wind speed will contain a slowly-varying component (average wind speed) and a high-frequency component (wind gust). The resultant wind forces and moment acting on a surface vessel are usually defined in terms of relative wind speed V_R (knots) and angle γ_R (deg) according to:

$$V_R = \sqrt{u_R^2 + v_R^2} \qquad \gamma_R = \tan^{-1}(v_R/u_R) \tag{3.85}$$

where the components of V_R in the x- and y-directions are:

$$u_R = V_w \cos(\gamma_R) - u + u_c \tag{3.86}$$
$$v_R = V_w \sin(\gamma_R) - v + v_c \tag{3.87}$$

Here (u, v) and (u_c, v_c) are the ship and current velocity components while $\gamma_R = \psi_w - \psi$ is the angle of relative wind of the ship bow, see Figure 3.8.

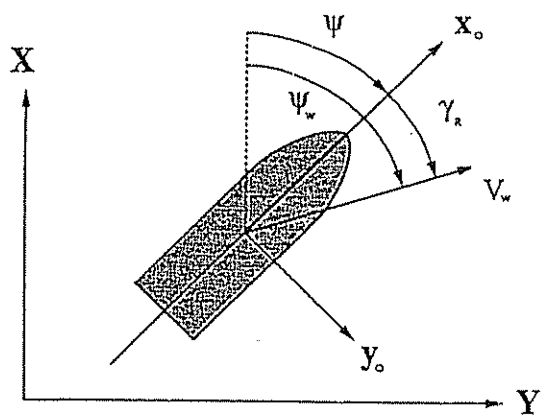

Figure 3.8: Definition of wind speed and direction.

We can simulate time-series for V_w and ψ_w by adding a *mean* and a *turbulent* component according to:

$$\begin{aligned} \dot{x}_1 &= w_1 & \qquad \dot{x}_3 &= w_3 \\ \dot{x}_2 &= -\tfrac{1}{T}(x_2 - K\, w_2) & \qquad \dot{x}_4 &= -\tfrac{1}{T}(x_4 - K\, w_4) \\ V_w &= x_1 + x_2 & \qquad \psi_w &= x_3 + x_4 \end{aligned}$$

where w_i (i = 1...4) are zero-mean Gaussian white noise processes and T and K are the time and gain constants of the Harris spectrum, for instance.

For most ships the wind gust cannot be compensated for by the control system since the dynamics of the ship is too slow compared with the gusts. However, slowly-varying wind forces can be fed forward to the controller by measuring the average wind speed and direction. This requires the wind force and moment coefficients to be known with sufficient accuracy. We will now describe two attractive methods for computation of the wind force and moment vector:

$$\tau_{\text{wind}} = [X_{\text{wind}}, Y_{\text{wind}}, N_{\text{wind}}]^T \tag{3.88}$$

acting on a surface ship.