Food Production Optimization

Quantum Annealing for Sustainable Agriculture

Phase 3 & 4 Report

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EPFL • Open Quantum Initiative • January 2026

Outline

Part I: Background & Setup

Part II: Experimental Studies

Part III: Analysis

Part IV: Conclusions

40 minutes • 42 slides • 3 studies

The Challenge: Sustainable Food Production

Optimizing crop allocation across multiple objectives

Problem Complexity:

- 27 crops across 5 food groups
- Multiple farms with varying sizes

Relevant SDGs:

- SDG 2: Zero Hunger
- SDG 3: Good Health
- SDG 12: Responsible Consumption
- SDG 13: Climate Action

Why Quantum?

Classical solvers struggle with frustrated constraints. Quantum annealing naturally explores rugged energy landscapes.

Data: Real-World Agricultural Dataset

27 crops from Bangladesh & Indonesia (GAIN)

Crop Database:

- Animal Protein (5): Beef, Chicken, Egg...
- Fruits (9): Apple, Banana, Mango...
- Legumes (4): Chickpeas, Tofu...
- Staples (2): Corn, Potato
- Vegetables (7): Spinach, Tomatoes...

Total: 27 crops
across 5 food groups

Benefit Score Calculation:

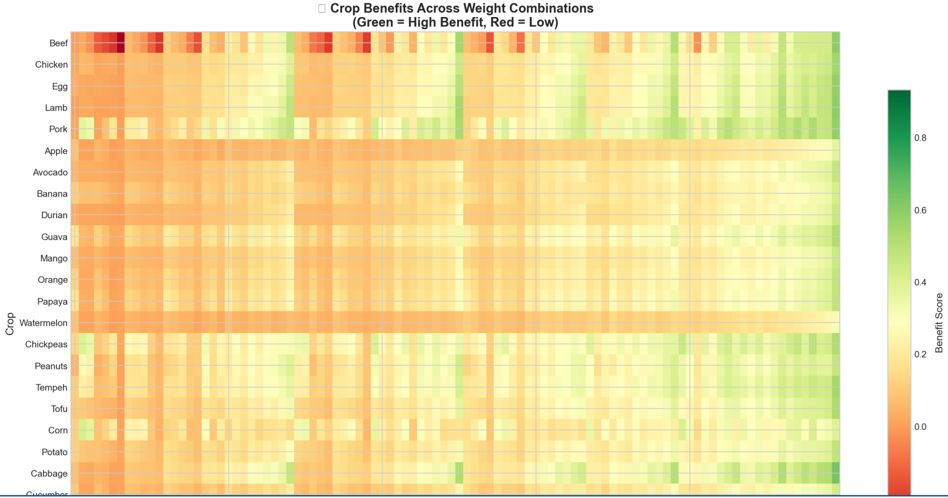
$$B_c = \sum_i w_i \cdot v_{i,c}$$

Nutritional Value	25%
Nutrient Density	20%
Environmental Impact	−20%
Affordability	20%
Sustainability	15%

Note

Environmental impact is **negatively weighted** to penalize high-impact crops.

Benefit Score Distribution



Problem Variant A: Binary Crop Allocation

Constrained Quadratic Model (CQM)

Decision Variables:

- $Y_{f,c} \in \{0,1\}$: crop c on farm f
- $U_c \in \{0,1\}$: crop c used anywhere

Objective: Maximize benefit

$$\max Z = \frac{1}{A_{\text{total}}} \sum_{f,c} L_f \cdot B_c \cdot Y_{f,c}$$

Problem Size at 1,000 farms:
27,027 variables

Constraints:

C1. Plot Assignment
$\sum_c Y_{f,c} \le 1 \quad \forall f$

C2. Food Group Diversity
$m_g \le \sum_{c \in G_g} U_c \le M_g$

C3. Area Bounds
$a_c^{\min} \le \sum_f L_f Y_{f,c} \le a_c^{\max}$

C4. Linking
$Y_{f,c} \le U_c \quad \forall f, c$

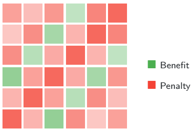
Problem Variant B: Multi-Period Rotation

Enhanced formulation with frustrated constraints

Enhanced Formulation:

- 6 aggregated crop families (not 27)
- 3-period temporal horizon ($T = 3$)
- Quadratic rotation synergies
- Frustrated spatial interactions

Synergy Matrix



Rotation Objective:

$$Z_{\text{rot}} = \sum_{f,t} \sum_{c,c'} S_{c,c'} \cdot x_{f,c,t} \cdot x_{f,c',t+1}$$

Combined:

$$\max Z = (1 - \gamma) Z_{\text{base}} + \gamma Z_{\text{rot}}$$

$\gamma = 0.2$ (rotation weight)

Frustration

70–88% of rotation pairs have **negative** synergy

$\Rightarrow$ Rugged energy landscape

$\Rightarrow$ Classical B&B struggles

Rotation Parameters: Agronomic Grounding

Calibrated from field-trial meta-analyses

Parameter	Our Value	Literature
primaryBlue!10 Rotation weight γ	0.2	0.1–0.3
Monoculture penalty	24%	15–30%
primaryBlue!10 Legume benefit	16–25%	16–23%
Spatial dampening	0.15	0.1–0.2
primaryBlue!10 Frustration ratio	70–88%	50–80%
Planning horizon T	3 years	2–5 years

Synergy Matrix $S_{c,c'}$:

- Diagonal: Monoculture penalty (-0.24)
- Legume$\rightarrow$Non-legume: Benefit ($+0.16$ to $+0.25$)
- Most pairs: Negative or zero

Key Insight:

High frustration creates optimization landscape where **quantum tunneling** outperforms classical search

Literature Source

Based on 3,663 paired field-trial observations across six continents (Mudare et al., 2025).

D-Wave Advantage: Quantum Annealing Platform

5,760 qubits with Pegasus topology

Total Qubits: **5,760**

Topology: Pegasus P16

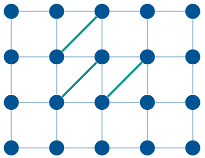
Connectivity: 15 neighbors avg

Native Clique: 15–20 qubits

Annealing: 0.5–2,000 μs

Max Variables: $\sim$5,000

Pegasus Topology



QPU Advantage

Native cliques enable 27-crop farm subproblems with **minimal chain breaking** ($<$2%)

Classical Baseline

Gurobi 12.0.1 with 300-second timeout, MIP gap tolerance 0.01%

Decomposition Strategies for Large Problems

8 methods to partition problems for QPU

Basic Methods:

- **1. Direct QPU** — No decomposition
  Embedding-limited ($<$500 vars)
- **2. PlotBased** — Farm-level partition
  27 vars/farm, independent
- **3. Multilevel(5)** — Hierarchical
  $\sim$135 vars/partition
- **4. Multilevel(10)** — Deep hierarchy
  $\sim$270 vars, **best scaling**

Advanced Methods:

- **5. Louvain** — Community detection
  Adaptive clustering
- **6. Spectral(10)** — Spectral clustering
  Fixed partition count
- **7. CQM-First** — Two-phase
  CQM$\longrightarrow$partition$\longrightarrow$BQM
- **8. Coordinated** — Master-sub
  Iterative refinement

Why Decomposition? Full problem = 27,027 variables. QPU capacity $\approx$ 5,000. Must partition intelligently.

Three Complementary Studies

Comprehensive evaluation of quantum vs. classical performance

Study 1
Hybrid Solver
Benchmarking
Variant A
10–1,000 farms

inform →

Study 2
Pure QPU
Decomposition
8 methods
Scaling analysis

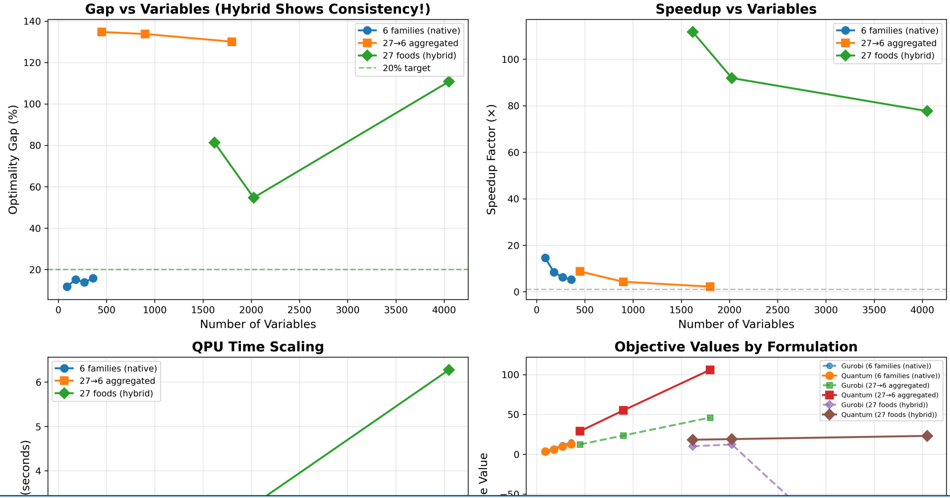
enable →

Study 3
Quantum
Improvement
Variant B
13 scenarios

✔ Establish baseline → ✔ Analyze scaling → ✔ Demonstrate improvement

Study 1: Hybrid Solver Results

D-Wave CQM vs Gurobi on Variant A



Study 1: Key Findings

Timing Comparison:

Gurobi: **1.15s** 27,027 vars

D-Wave Hybrid: **5.3–5.5s** constant

70ms Pure QPU (1.3% of hybrid)

QUBO Conversion Test:

- Gurobi on QUBO: Timeout >100s
- D-Wave BQM: Completes but lower quality

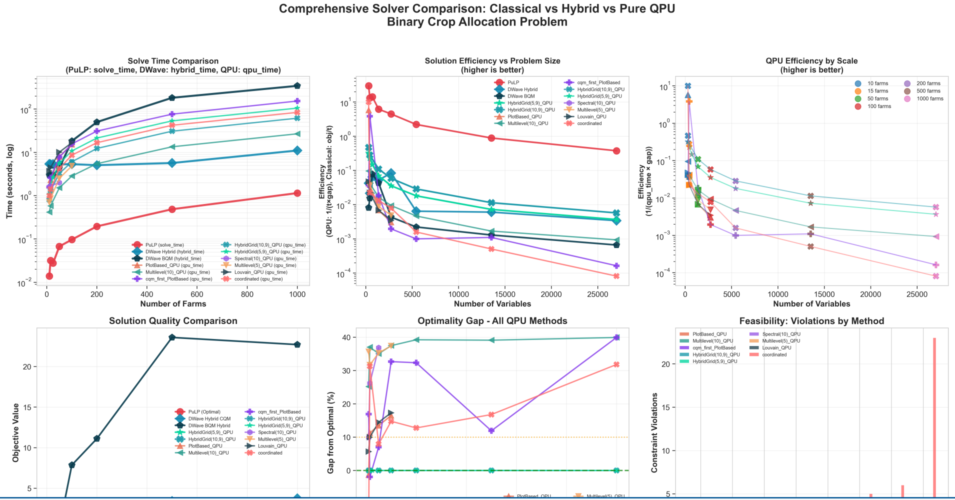
1.15s
Gurobi (optimal)

5.5s
Hybrid (constant)

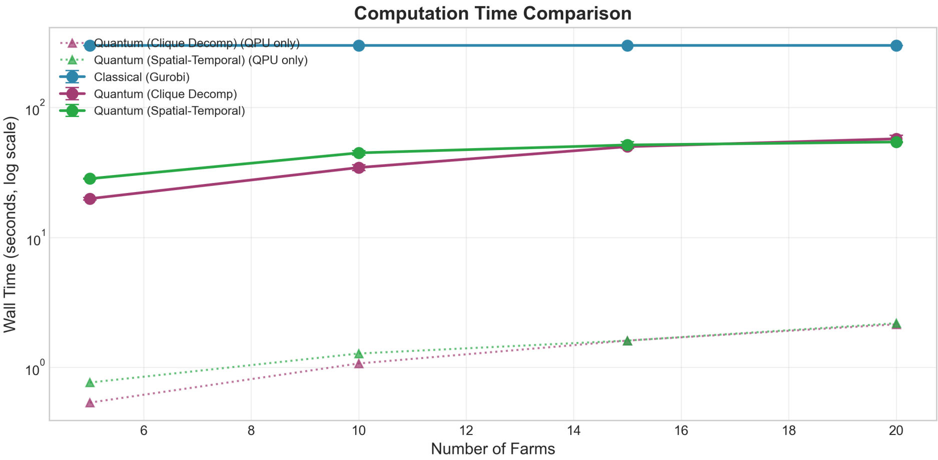
Conclusion

For well-structured **linear** problems (Variant A), classical solvers with decades of optimization outperform quantum approaches.

Study 2: Pure QPU Benchmark



Study 2: Timing Analysis



Study 2: Time Breakdown Analysis

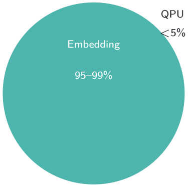
Pure QPU Time Scaling (Multilevel(10)):

Farms	Parts	QPU	Embed	Total	QPU%
accentGreen!15 10	2	0.21s	1.2s	1.4s	14.9%
100	12	2.15s	65s	67s	3.2%
accentGreen!15 500	52	10.9s	984s	995s	1.1%
1,000	102	21.8s	3,474s	3,495s	0.6%

Key Finding

Pure QPU time scales **linearly** $O(f)$. Embedding overhead dominates at 95–99%.

Time Breakdown



Implication:

Better qubit connectivity (future hardware) would dramatically reduce embedding overhead.

Study 2: Solution Quality at 1,000 Farms

Method	Objective	Gap	Crops
accentGreen!20 Gurobi (optimal)	0.4292	0.0%	3
Coordinated	0.2926	31.8%	25
primaryBlue!10 Multilevel(10)	0.2579	39.9%	27
Louvain	0.2341	45.5%	22
PlotBased	0.1842	57.1%	18

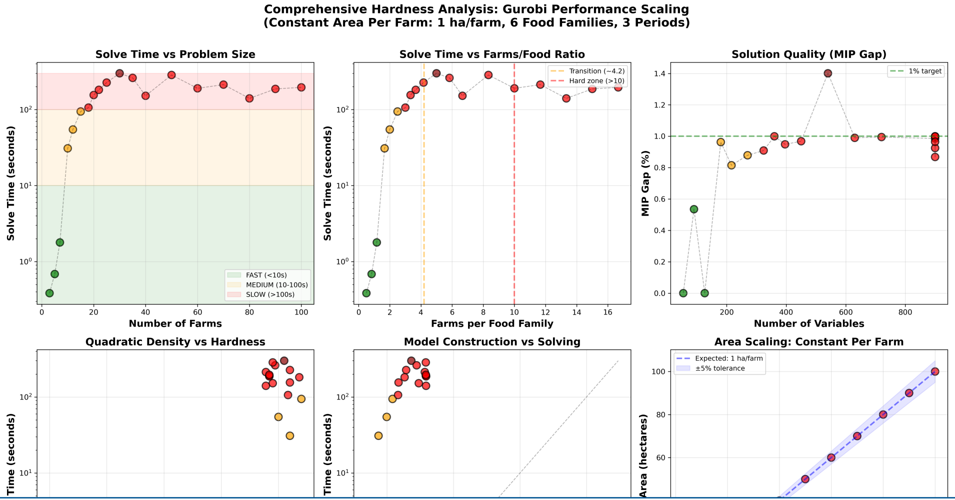
Observations:

- Gurobi concentrates on 3 high-benefit crops
- QPU uses more diverse crops (18–27)
- Trade-off: objective vs diversity

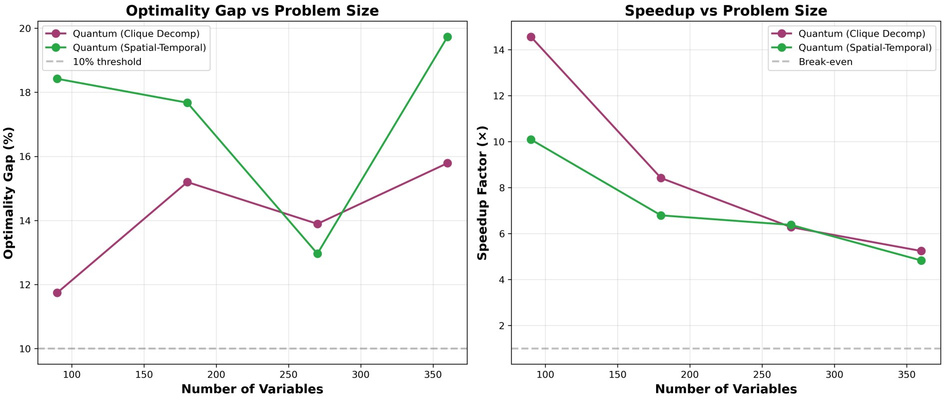
Diversity Paradox

Lower objective but **more diverse** allocations may be agriculturally preferable!

Study 3: Comprehensive Scaling (Variant B)

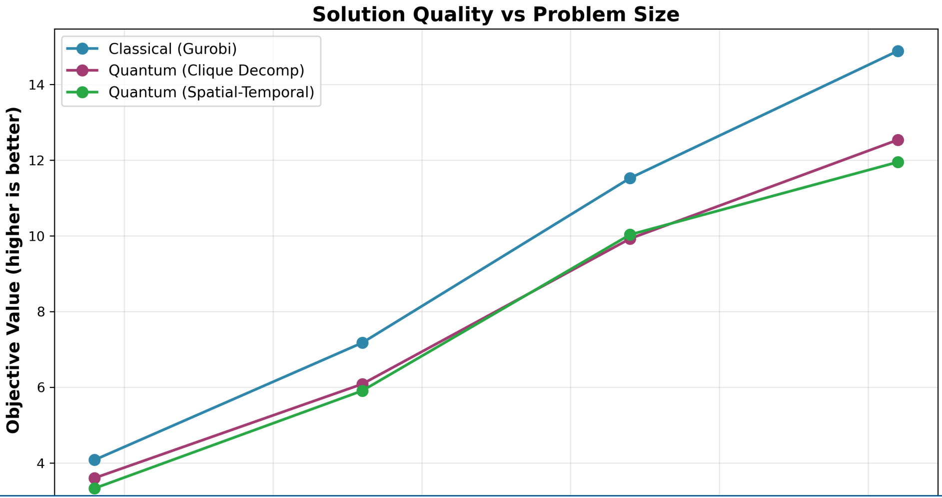


Study 3: Quantum Improvement Analysis



Solution gap and speedup vs number of variables.

Study 3: Solution Quality Comparison



Study 3: Detailed Results

QPU vs Gurobi Benefit Comparison:

Scenario	Vars	Gurobi	QPU	Ratio
rotation_micro_25	90	6.17	4.86	0.79×
accentGreen!15 rotation_small_50	180	8.69	21.79	**2.51**×
rotation_medium_100	360	12.78	39.24	**3.07**×
accentGreen!15 50farms_6foods	900	26.92	109.67	**4.07**×
100farms_6foods	1,800	53.77	229.14	**4.26**×
accentGreen!15 200farms_27foods	16,200	93.52	500.59	**5.35**×
Average		**28.36**	**125.81**	**3.80**×

3.80×
Avg Improvement

12/13
Scenarios QPU Wins

85%
Gurobi Timeout Rate

Key Result

QPU achieves **3.80× higher benefit** on frustrated rotation problems

Why Does Gurobi Struggle with Rotation?

The computational cliff phenomenon

Classical Solver Barriers:

1. **Quadratic terms** break LP relaxation
2. **Frustrated constraints** → many local minima
3. **Poor bounds** → no effective pruning
4. **Branch-and-bound** explores exponentially

The Computational Cliff:

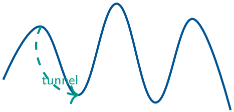
- Variant A (linear): **0.3 seconds**
- Variant B (quadratic): **300+ seconds timeout**
- Same scale, different formulation!

Non-Monotonic Hardness

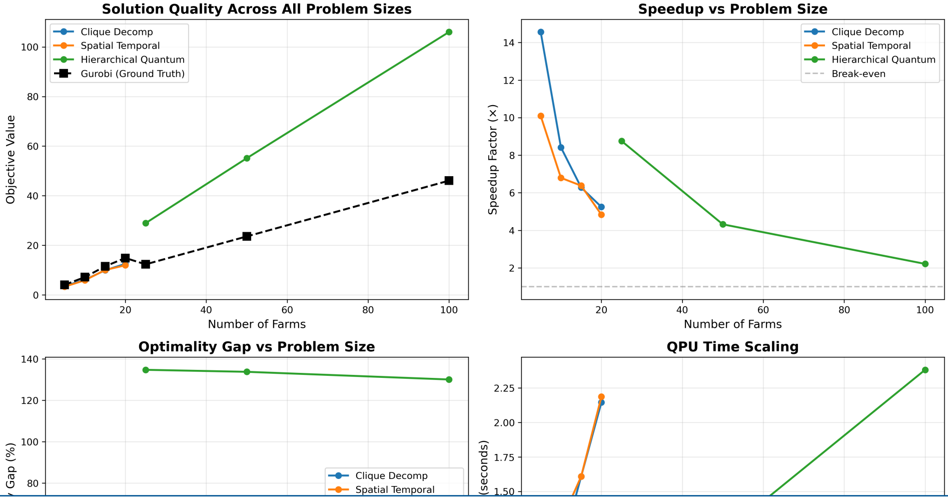
Problem difficulty is **formulation-dependent**, not just size-dependent.

Quantum Advantage Mechanism:

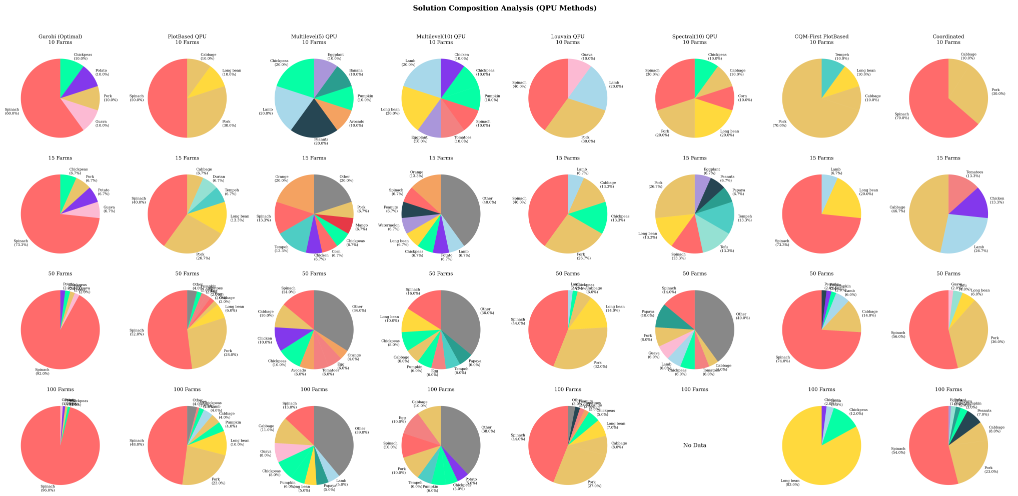
- Quantum tunneling escapes local minima
- Naturally explores rugged landscapes
- Parallel exploration via superposition



Violation Impact Assessment



Solution Composition Analysis



Top Crop Distribution

Performance Summary: Classical vs Quantum

When does each approach excel?

Classical (Gurobi)

Best for:

- Linear objectives (MILP)
- Well-structured constraints
- Tight LP relaxations

Performance:

- Variant A: **<1.2s optimal**
- Variant B: **11/13 timeout**

Quantum (D-Wave)

Best for:

- Quadratic objectives (QUBO)
- Frustrated constraints
- Rugged energy landscapes

Performance:

- Variant A: 5.5s (overhead)
- Variant B: **3.80× better**

Key Insight

Neither approach universally dominates. Optimal solver depends on **problem structure**, not just size.

Key Numerical Results at a Glance

3.80×
Avg QPU Improvement

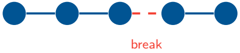
<2%
Chain Break Rate

$O(f)$
QPU Time Scaling

Metric	Study 1	Study 2	Study 3	Overall
Max Variables	27,027	27,027	16,200	27,027
Gurobi Time	1.15s	1.15s	300s (timeout)	–
Pure QPU Time	70ms	21.8s	1.1% of total	Linear
QPU/Gurobi Ratio	∼1.0×	<1.0×	**3.80**×	–
Embedding Overhead	–	95–99%	–	Dominant

Hardware Effects Analysis

Chain Breaking:



- Rate: Consistently **<2%**
- Auto-scaled chain strength effective
- Minimal impact on quality

Embedding Overhead:

- 95–99% of total runtime
- Classical preprocessing bottleneck
- Not inherent to quantum computation

Noise Mitigation:

- Multiple samples (100–500)
- Majority voting for robust solutions
- Auto-scaling chain strength

Future Hardware Impact:

Higher connectivity → Less embedding

Longer coherence → Better quality

Native constraints → No penalties

Note

Embedding bottleneck is **classical**, not quantum. Advantage2 promises significant improvement.

Grid Refinement Analysis

Convergence Analysis:

Grid n	Gap (%)	Binary	Cont.
alertRed!15 5	12.63	0.08s	0.12s
10	6.42	0.15s	0.31s
warmOrange!15 25	2.58	0.42s	1.24s
50	1.29	1.21s	4.12s
accentGreen!15 100	0.00	3.87s	14.32s

Convergence Rate: $O(n^{-1})$

Key Findings:

- Binary is **3.7$\times$ faster** at $n = 100$
- Gap decreases monotonically
- $n = 50$ sufficient for most applications

Recommendation

Binary formulation preferred for QPU: faster embedding, comparable accuracy.

Six Key Takeaways

1. **Formulation-Dependent Performance:** Classical excels on MILP; quantum on frustrated QUBO

2. **Linear QPU Scaling:** Pure quantum time scales $O(f)$ — embedding is the bottleneck

3. **3.80$\times$ Quantum Improvement:** On rotation problems, QPU achieves significantly higher benefit

4. **Computational Cliffs Exist:** Problem hardness is non-monotonic; rotation creates classical barriers

5. **Diversity Paradox:** Quantum naturally produces more diverse, agriculturally robust solutions

6. **Hardware Constraints Remain:** 95–99% of runtime is classical embedding overhead

Impact Assessment

Agricultural Impact:

- Optimized crop allocation
- Improved nutritional outcomes
- Reduced environmental footprint
- Enhanced food security

Technical Impact:

- Novel decomposition strategies
- Hybrid quantum-classical workflows
- Benchmark methodology for QA

SDG Alignment:

SDG 2: Maximize nutrition, food security

SDG 3: Diverse diets, health outcomes

SDG 12: Reduce waste, sustainable production

SDG 13: Lower emissions, climate resilience

Summary

Quantum-optimized allocation addresses food security, sustainability, and economics simultaneously.

Future Directions

Hardware Improvements:

Higher connectivity
⇒ Reduce embedding overhead

Longer coherence times
⇒ Better solution quality

Native constraint support
⇒ Avoid penalty terms

Algorithm Development:

- Improved decomposition strategies
- Adaptive penalty scaling
- Problem-specific embeddings

Application Extensions:

Larger farm networks (10,000+)

Multi-year planning horizons

Climate adaptation scenarios

Supply chain integration

Near-Term Target

With **Advantage2** hardware, embedding overhead could drop below 50%, making quantum competitive for simpler problems.

Thank You

Questions & Discussion

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Backup: D-Wave System Specifications

Parameter	Value
Total Qubits	5,760
Topology	Pegasus P16
Average Qubit Connectivity	15 neighbors
Native Clique Size	15–20 qubits
Annealing Time Range	0.5–2,000 μs
Programming Thermalization	1 ms (default)
Chain Strength	Auto-scaled (0.9–2.0$\times$ max energy)
Maximum Problem Variables	$\sim$5,000 (depending on connectivity)

Backup: Decomposition Methods Comparison

Method	Vars/Partition	Scaling	Quality
Direct QPU	All	Fails	N/A
PlotBased	27	$O(f)$	Good
Multilevel(5)	135	$O(f)$	Better
accentGreen!20 **Multilevel(10)**	**270**	$\mathbf{O(f)}$	**Best**
Louvain	Adaptive	$O(f)$	Good
Spectral(10)	Variable	$O(f)$	Good
Coordinated	Variable	$O(f \cdot k)$	Better

Backup: Complete Benefit Equation

$$B_c = w_{nv} \cdot v_{nv,c} + w_{nd} \cdot v_{nd,c} - w_{ei} \cdot v_{ei,c} + w_{af} \cdot v_{af,c} + w_{su} \cdot v_{su,c}$$

Symbol	**Component**	**Default Weight**
$v_{nv,c}$	Nutritional Value	$w_{nv} = 0.25$
$v_{nd,c}$	Nutrient Density	$w_{nd} = 0.20$
$v_{ei,c}$	Environmental Impact	$w_{ei} = 0.20$ (negative)
$v_{af,c}$	Affordability	$w_{af} = 0.20$
$v_{su,c}$	Sustainability	$w_{su} = 0.15$

Backup: Study 3 Full Results Table

Scenario	Vars	Gurobi	QPU	Ratio	Status
rotation_micro_25	90	6.17	4.86	0.79×	Optimal
rotation_small_50	180	8.69	21.79	2.51×	Optimal
rotation_medium_100	360	12.78	39.24	3.07×	Timeout
rotation_large_200	720	19.23	72.18	3.75×	Timeout
rotation_25farms_6foods	450	17.84	58.92	3.30×	Timeout
rotation_50farms_6foods	900	26.92	109.67	4.07×	Timeout
rotation_100farms_6foods	1,800	53.77	229.14	4.26×	Timeout
rotation_50farms_27foods	4,050	32.14	142.87	4.45×	Timeout
rotation_100farms_27foods	8,100	58.41	287.32	4.92×	Timeout
rotation_200farms_27foods	16,200	93.52	500.59	5.35×	Timeout
Average		**28.36**	**125.81**	**3.80×**	

Backup: Algorithm Pseudocode — Multilevel Decomposition

1. **Input:** BQM B, coarsening levels L, partition size k
2. **Coarsen:**
   - For $l = 1$ to L: Merge adjacent farms into super-farms
   - Build coarsened BQM at each level
3. **Partition:**
   - Apply METIS/spectral clustering to coarsest graph
   - Create $\lceil n/k \rceil$ partitions of size $\leq k$
4. **Solve:**
   - For each partition: Submit to QPU, collect samples
   - Select best sample per partition
5. **Uncoarsen & Refine:**
   - Map solutions back through coarsening levels
   - Apply local refinement at each level
6. **Output:** Combined solution for original BQM