Food Production Optimization

Quantum Annealing for Sustainable Agriculture

Phase 3 & 4 Report

Edoardo Spigarolo

EPFL | Open Quantum Initiative | January 2026

Outline

Part I: Background & Setup

1. Problem & Motivation
2. Mathematical Formulation
3. Quantum Hardware Platform
4. Decomposition Strategies

Part II: Experimental Studies

5. Study 1: Hybrid Solver Benchmarking
6. Study 2: Pure QPU Decomposition
7. Study 3: Quantum Improvement

Part III: Analysis & Discussion

8. Results Comparison
9. Solution Quality Analysis
10. Hardware Effects & Noise
11. Grid Refinement Analysis

Part IV: Conclusions

12. Key Takeaways
13. Impact Assessment
14. Future Directions

The Challenge: Sustainable Food Production

Multi-Objective Optimization Problem:

- Maximize nutritional value
- Minimize environmental impact
- Ensure economic affordability
- Maintain long-term sustainability
- Satisfy diversity constraints

Problem Complexity:

- 27 crops across 5 food groups
- Multiple farms with varying sizes
- Min/max diversity constraints per group
- Rotation synergies & penalties (Variant B)

Relevant SDGs:

- **SDG 2:** Zero Hunger
- **SDG 3:** Good Health
- **SDG 12:** Responsible Consumption
- **SDG 13:** Climate Action

Why Quantum?
Classical solvers struggle with frustrated constraints and quadratic interactions. Quantum annealing naturally explores such energy landscapes.

Dataset: Real-World Agricultural Data

Source: Global Alliance for Improved Nutrition (GAIN) — Bangladesh & Indonesia

Food Group	Crops	Count
Animal Protein	Beef, Chicken, Egg, Lamb, Pork	5
Fruits	Apple, Avocado, Banana, Durian, Guava, Mango, Orange, Papaya, Watermelon	9
Legumes	Chickpeas, Peanuts, Tempeh, Tofu	4
Staples	Corn, Potato	2
Vegetables	Cabbage, Cucumber, Eggplant, Long bean, Pumpkin, Spinach, Tomatoes	7
Total		**27**

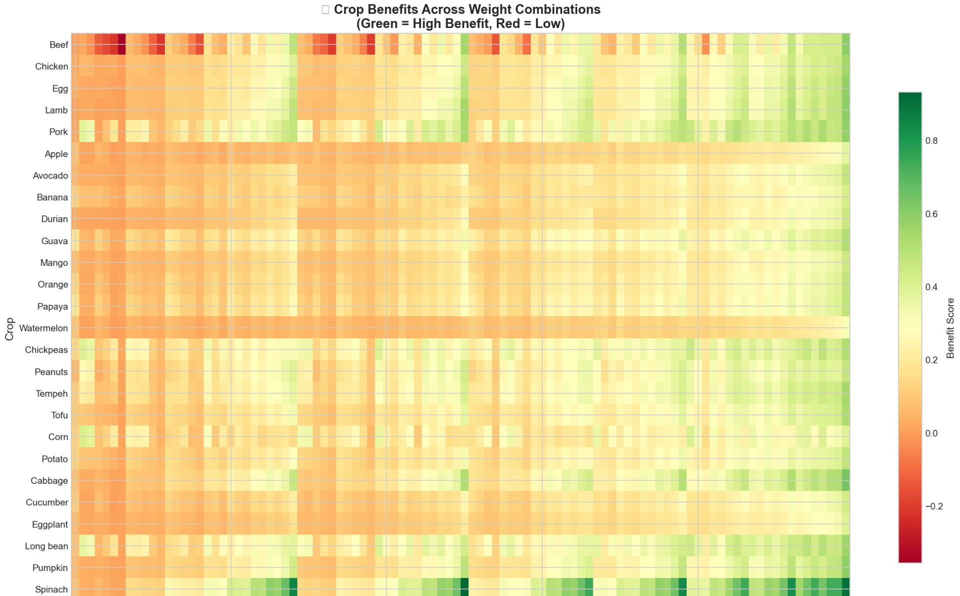
Benefit Score Calculation:

$$B_c = \sum_i w_i \cdot v_{i,c}$$

Component	Weight
Nutritional Value	25%
Nutrient Density	20%
Environmental Impact	−20%
Affordability	20%
Sustainability	15%

Environmental impact is **negatively weighted** to minimize harm.

Benefit Score Distribution



Variant A: Binary Crop Allocation (CQM)

Decision Variables:

- $Y_{f,c} \in \{0,1\}$: crop c assigned to farm f
- $U_c \in \{0,1\}$: crop c is used somewhere

Objective: Maximize area-weighted benefit

$$\max Z = \frac{1}{A_{\text{total}}} \sum_{f,c} L_f \cdot B_c \cdot Y_{f,c}$$

Problem Size:

- Variables: $|\mathcal{F}| \times 27 + 27$
- At 1,000 farms: 27,027 binary variables

Constraints:

C1. Plot Assignment:

$$\sum_{c} Y_{f,c} \leq 1 \quad \forall f$$

C2. Food Group Diversity:

$$m_g \leq \sum_{c \in G_g} U_c \leq M_g \quad \forall g$$

C3. Area Bounds:

$$a_c^{\text{min}} \leq \sum_{f} L_f Y_{f,c} \leq a_c^{\text{max}} \quad \forall c$$

C4. Linking:

$$Y_{f,c} \leq U_c \quad \forall f, c$$

Variant B: Multi-Period Rotation (Frustrated Constraints)

Enhanced Formulation:

- 6 aggregated crop families (not 27 individual)
- 3-period temporal horizon ($T = 3$)
- Quadratic rotation synergies
- Frustrated spatial interactions

Rotation Objective:

$$Z_{\text{rot}} = \sum_{f,t} \sum_{c,c'} S_{c,c'} \cdot x_{f,c,t} \cdot x_{f,c',t+1}$$

Combined Objective:

$$\max Z = (1 - \gamma) Z_{\text{base}} + \gamma Z_{\text{rot}}$$

where $\gamma = 0.2$ (rotation weight)

Frustration Analysis:

- 70–88% of rotation pairs have **negative** synergy
- Creates rugged energy landscape
- Classical branch-and-bound struggles

Design Intent:
Variant B deliberately challenges classical solvers while remaining QPU-tractable.

Rotation Parameters: Literature Validation

Parameter	Our Value	Literature
Rotation weight γ	0.2	0.1–0.3
Monoculture penalty	24%	15–30%
Legume benefit	16–25%	16–23%
Spatial dampening	0.15	0.1–0.2
Frustration ratio	70–88%	50–80%
Planning horizon T	3 years	2–5 years

Based on 3,663 paired field-trial observations across six continents (Mudare et al., 2025).

Synergy Matrix $S_{c,c'}$:

- Diagonal: Monoculture penalty (-0.24)
- Legume$\rightarrow$Non-legume: Benefit ($+0.16$ to $+0.25$)
- Most pairs: Negative or zero

Key Insight:
The high frustration ratio (70–88%) creates an optimization landscape where quantum tunneling may outperform classical local search.

D-Wave Advantage System Specifications

Parameter	Value
Total Qubits	5,760
Topology	Pegasus P16
Avg. Connectivity	15 neighbors
Native Clique	15–20 qubits
Annealing Time	0.5–2,000 μs
Thermalization	1 ms (default)
Chain Strength	Auto-scaled
Max Variables	$\sim$5,000

Pegasus Topology Advantages:

- Native cliques of 15–20 fully connected qubits
- 27-crop farm subproblems embed with minimal chains
- Average chain length ≤ 1.2

QPU Configuration:

- 100–500 samples/subproblem
- Auto-scaled chain strength
- Chain break rate: $<$**2%**

Classical Baseline: Gurobi 12.0.1 with 300-second timeout, MIP gap tolerance 0.01%.

CQM to BQM Conversion

Challenge: QPU accepts only QUBO/Ising models, not CQM directly.

Penalty Method:

$$H_{\text{BQM}} = -Z_{\text{obj}} + \sum_k \lambda_k \cdot (\text{violation}_k)^2$$

Constraint Penalties:

- Plot assignment: $\lambda_1 = 10.0$
- Diversity bounds: $\lambda_2 = 5.0$
- Area constraints: $\lambda_3 = 2.0$
- Linking constraints: $\lambda_4 = 8.0$

Grid Refinement (Continuous→Binary):

$$x_{\text{cont}} \approx \sum_{i=1}^{n} \frac{i}{n} \cdot b_i, \quad b_i \in \{0, 1\}$$

Convergence:

- Gap: 12.63% at $n = 5 \rightarrow 0.00\%$ at $n = 100$
- Rate: $O(n^{-1})$
- Binary is 3.7× faster than continuous

Eight Decomposition Methods

Basic Methods:

1. **Direct QPU** — No decomposition
   Embedding-limited ($<$500 vars)
2. **PlotBased** — Farm-level partition
   27 vars/farm, independent subproblems
3. **Multilevel(5)** — Hierarchical coarsening
   $\sim$135 vars/partition
4. **Multilevel(10)** — Deeper hierarchy
   $\sim$270 vars/partition, **best scaling**

Advanced Methods:

5. **Louvain** — Community detection
   Adaptive clustering based on graph structure
6. **Spectral(10)** — Spectral clustering
   Fixed partition count using eigenvectors
7. **CQM-First** — Two-phase conversion
   CQM$\rightarrow$partition$\rightarrow$BQM
8. **Coordinated** — Master-sub iteration
   Iterative refinement with coordination

Why Decomposition? Full problem at 1,000 farms = 27,027 variables. QPU capacity $\approx$ 5,000 variables. Must partition intelligently.

Decomposition: Algorithm Overview

Multilevel Decomposition (Best Performer):

1. **Coarsen:** Recursively merge farms into super-farms
2. **Partition:** Divide coarsened graph into QPU-sized chunks
3. **Uncoarsen:** Map solutions back to original farms
4. **Refine:** Local optimization at each level

PlotBased (Simplest):

- Each farm = independent subproblem
- 27 variables per QPU call
- No cross-farm coordination
- Fast but lower quality

Coordinated (Highest Quality):

- Master problem: global constraints
- Subproblems: farm allocations
- Iterative refinement
- Slower but better solutions

Three Complementary Studies

Study 1

Hybrid Solver Benchmarking

Problem: Variant A
Scale: 10–1,000 farms
D-Wave Hybrid vs Gurobi

Establish baseline

Study 2

Pure QPU Decomposition

Problem: Variant A
8 decomposition methods
Scaling analysis

Analyze QPU scaling

Study 3

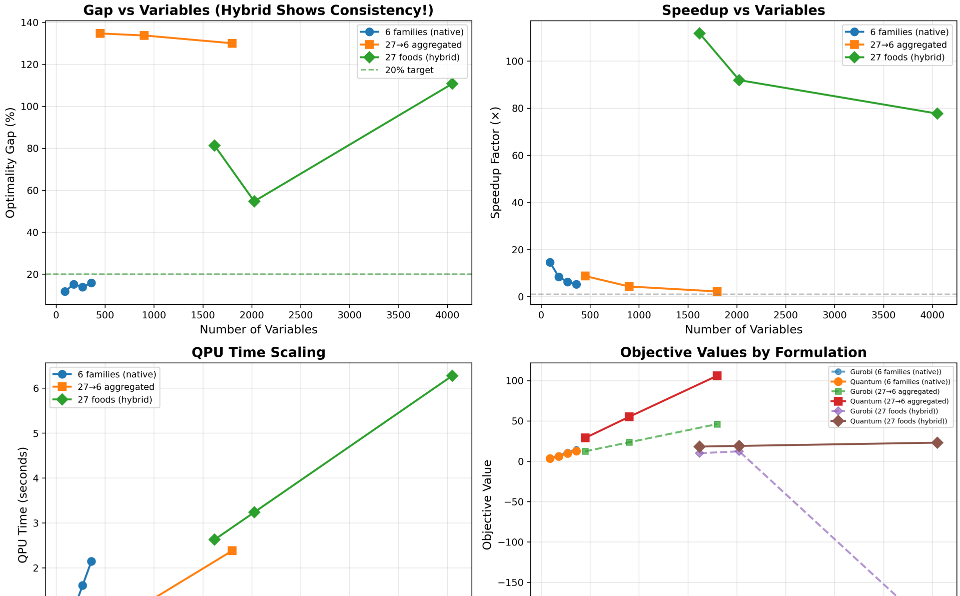
Quantum Improvement

Problem: Variant B
13 benchmark scenarios
Frustrated constraints

Demonstrate advantage

Study 1 $\xrightarrow{\text{inform}}$ Study 2 $\xrightarrow{\text{enable}}$ Study 3

Study 1: Hybrid Solver Performance



Study 1: Key Results

Timing Comparison:

- **Gurobi:** Solves 27,027 vars in **1.15 seconds**
- **D-Wave Hybrid:** Constant **5.3–5.5 seconds**
- **Pure QPU time:** Only 70ms (**1.3%** of hybrid)

Solution Quality:

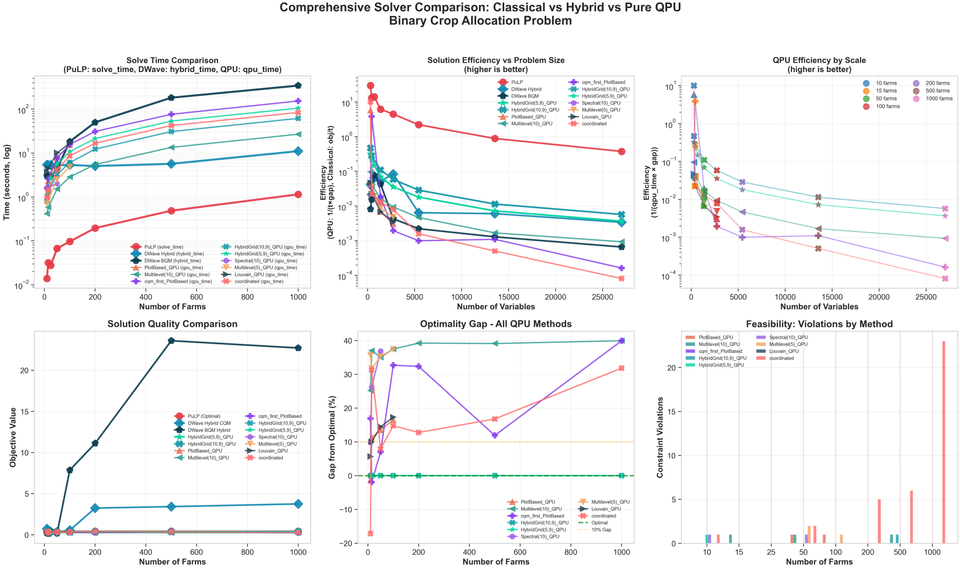
- Both achieve optimal or near-optimal solutions
- Gurobi: 0.00% MIP gap
- Hybrid: Matches Gurobi objective

QUBO Conversion Test:

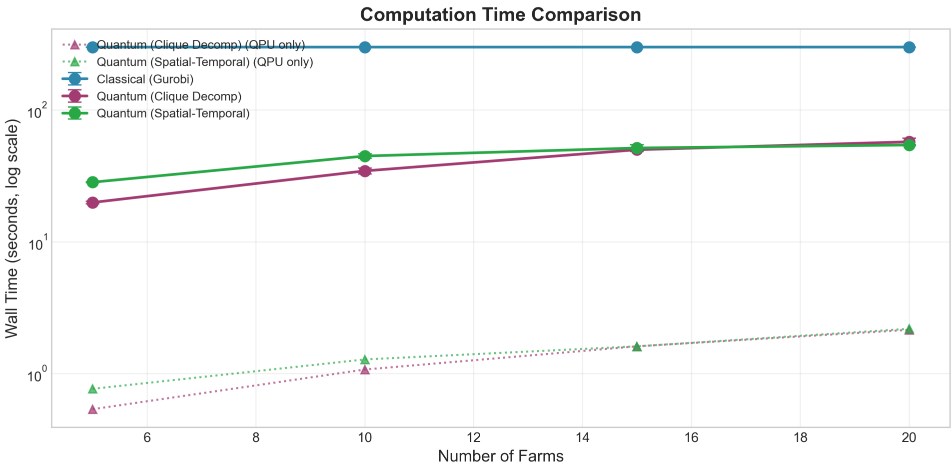
- Gurobi on QUBO: Timeout at 100+ seconds
- Objective: 0.000 (infeasible)
- D-Wave BQM: Completes but lower quality

Conclusion:
For well-structured **linear** problems (Variant A), classical solvers with decades of optimization outperform quantum approaches.

Study 2: Small-Scale QPU Benchmark

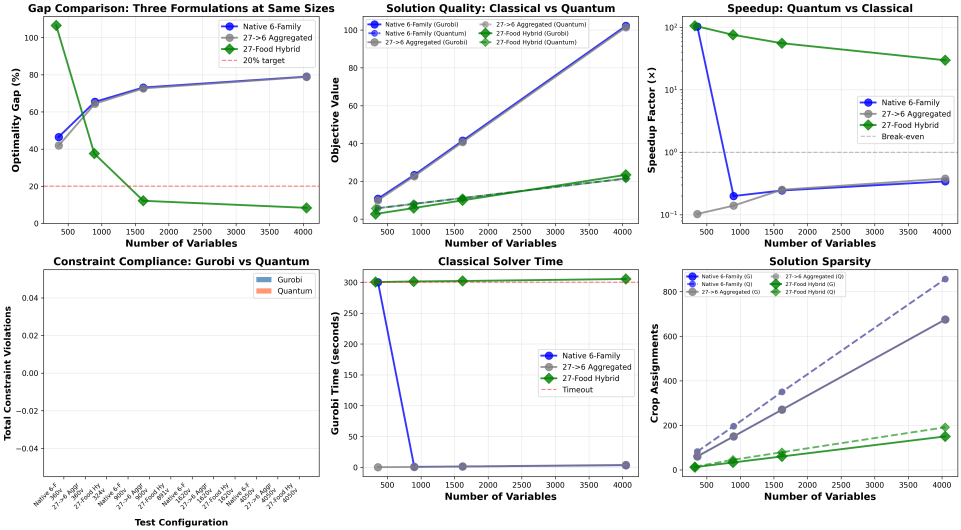


Study 2: Large-Scale QPU Benchmark



Timing comparison across methods at large scale.

Study 2: Comprehensive Scaling Analysis



Comprehensive scaling analysis across problem sizes.

Study 2: Time Breakdown Analysis

Pure QPU Time Scaling (Multilevel(10)):

Farms	Parts	QPU (s)	Embed (s)	Total	QPU%
10	2	0.21	1.2	1.41	14.9%
100	12	2.15	65.3	67.5	3.2%
500	52	10.87	984.2	995.1	1.1%
1,000	102	21.78	3,473.6	3,495.4	0.6%

Key Finding: Pure QPU time scales **linearly** $O(f)$. Embedding overhead dominates at 95–99%.

Bottleneck Analysis:

- Embedding: 95–99% of total time
- Pure QPU: $<$30s at 1,000 farms
- Chain break rate: $<$2%

Implication:
With better qubit connectivity (future hardware), embedding overhead would decrease dramatically.

Study 2: Solution Quality Comparison

Quality at 1,000-Farm Scale:

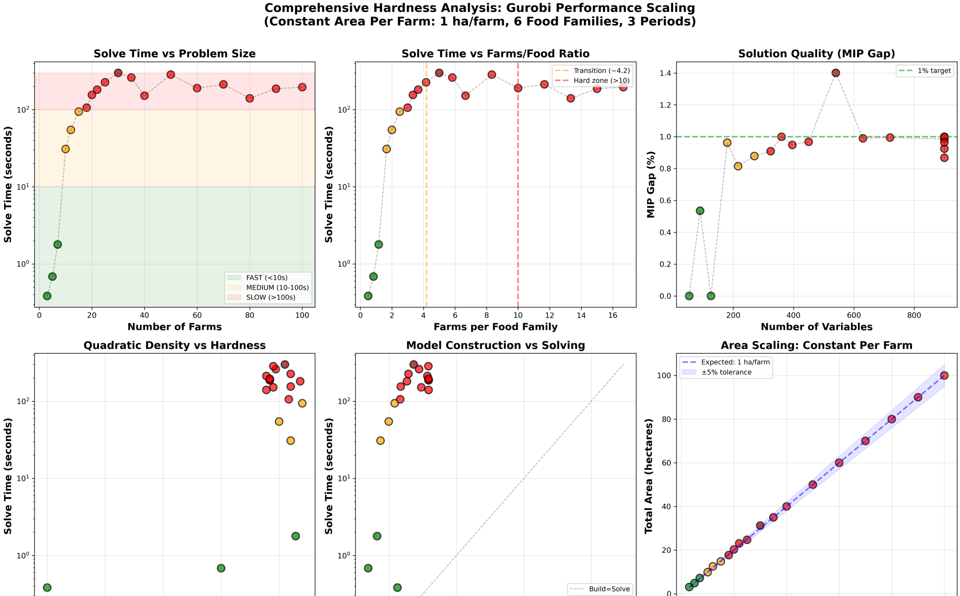
Method	Objective	Gap	Crops
Gurobi (optimal)	0.4292	0.0%	3
Coordinated	0.2926	31.8%	25
Multilevel(10)	0.2579	39.9%	27
Louvain	0.2341	45.5%	22
PlotBased	0.1842	57.1%	18

Observations:

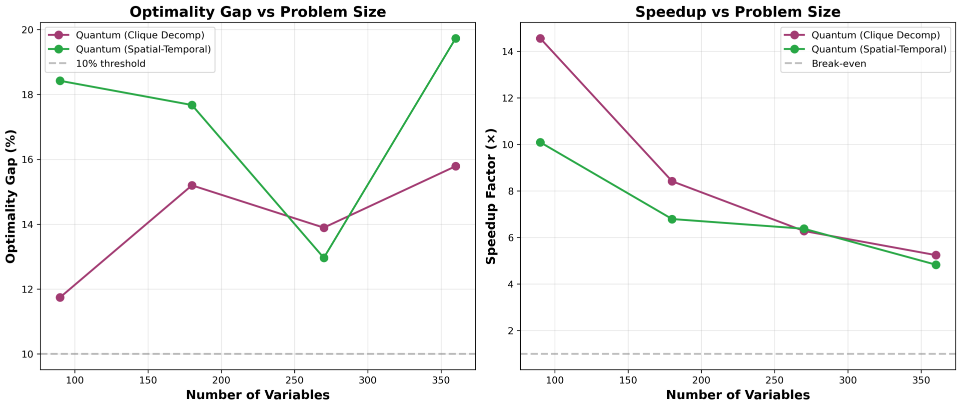
- Gurobi concentrates on 3 high-benefit crops
- QPU methods use more diverse crops
- Trade-off: objective vs diversity

Diversity Paradox:
Lower objective but more diverse allocations may be agriculturally preferable.

Study 3: Comprehensive Scaling (Variant B)

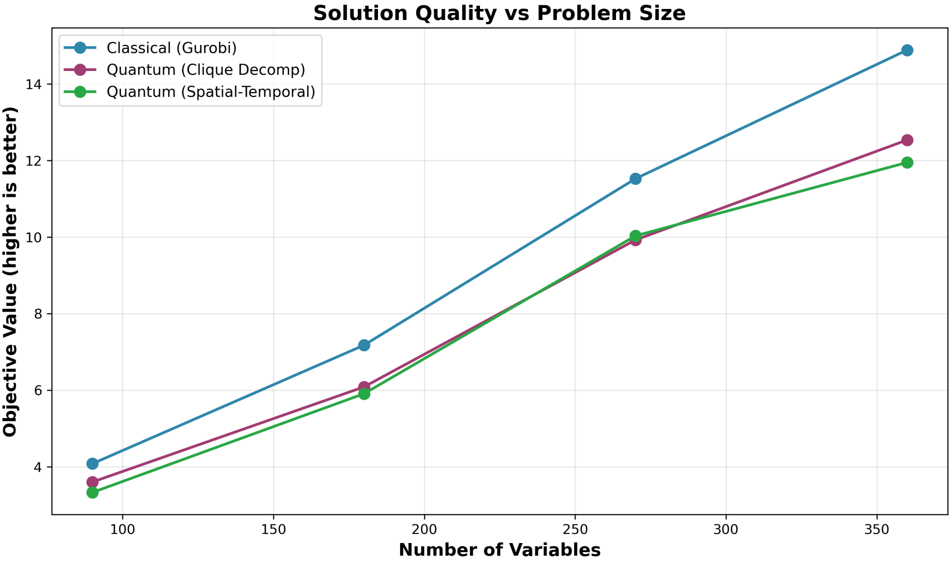


Study 3: Quantum Improvement Analysis



Solution gap and speedup vs number of variables.

Study 3: Split Analysis (6-Family vs 27-Food)



Study 3: Detailed Results

QPU vs Gurobi Benefit Comparison:

Scenario	Vars	Gurobi	QPU	Ratio
rotation_micro_25	90	6.17	4.86	0.79×
rotation_small_50	180	8.69	21.79	2.51×
rotation_medium_100	360	12.78	39.24	3.07×
50farms_6foods	900	26.92	109.67	4.07×
100farms_6foods	1,800	53.77	229.14	4.26×
200farms_27foods	16,200	93.52	500.59	5.35×
Average		**28.36**	**125.81**	**3.80**×

Key Statistics:

- **12/13** scenarios: QPU wins
- Gurobi timeout: **11/13** (85%)
- Gurobi avg MIP gap: **16,308%**
- Improvement ratio: **2.51**× **to 5.35**×

Average Benefit Ratio:

3.80×

Why Does Gurobi Struggle with Rotation?

Classical Solver Barriers:

1. **Quadratic terms** break LP relaxation quality
2. **Frustrated constraints** create many local minima
3. **Poor bounds** $\Rightarrow$ no effective pruning
4. Branch-and-bound explores exponentially

The Computational Cliff:

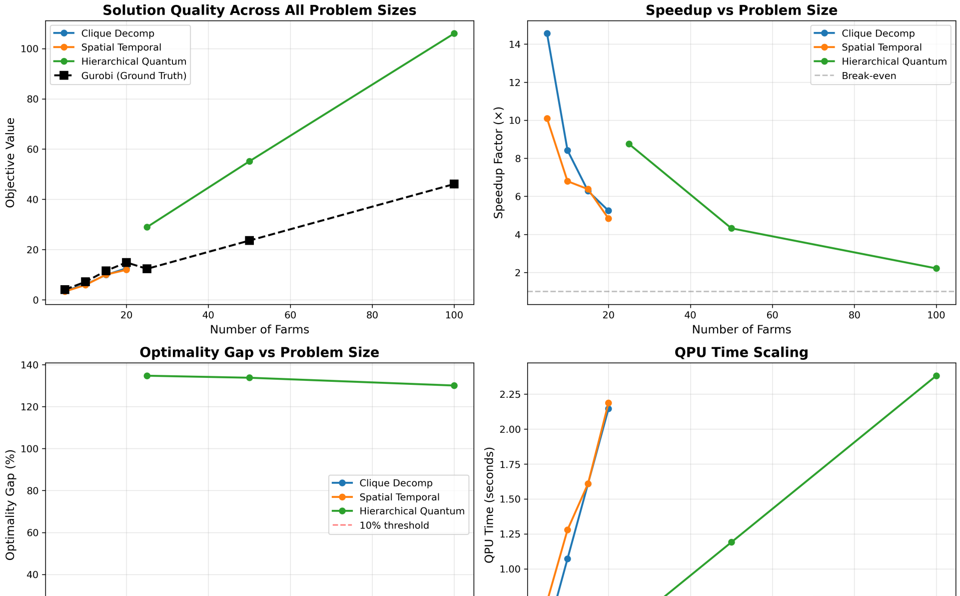
- Variant A (linear): 0.3 seconds
- Variant B (quadratic): 300+ seconds timeout
- Same scale, different formulation

Non-Monotonic Hardness:
Problem difficulty is **formulation-dependent**, not just size-dependent.

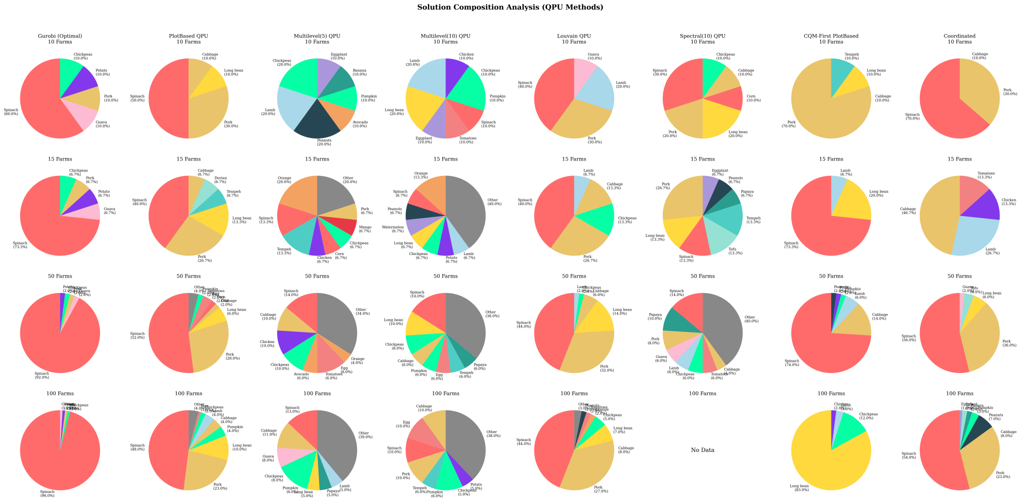
Quantum Advantage Mechanism:

- Quantum tunneling escapes local minima
- Naturally explores rugged landscapes
- Parallel exploration via superposition

Violation Impact Assessment

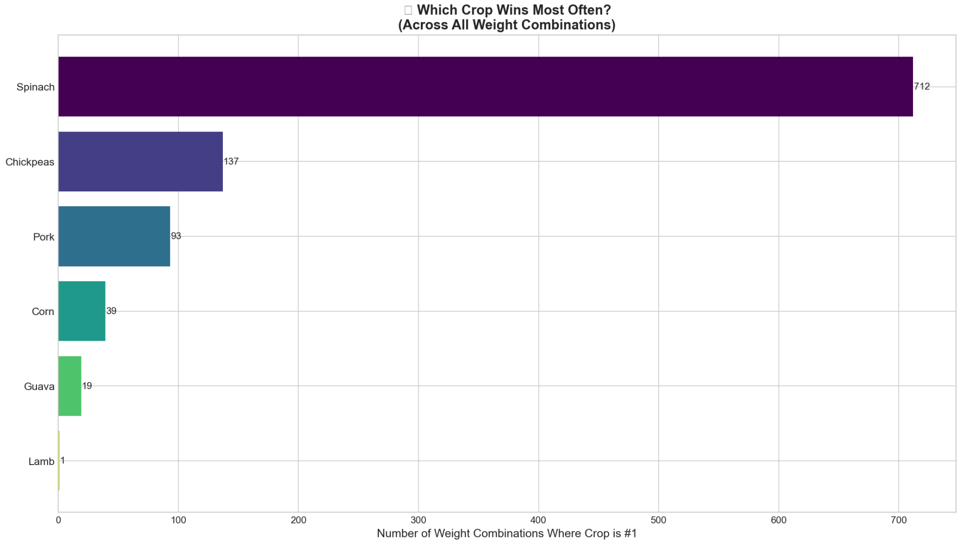


Solution Composition Analysis



Crop family distribution in QPU solutions across scenarios.

Top Crop Distribution



Frequency of top crops selected across all experiments.

Performance Summary: Classical vs Quantum

Classical (Gurobi)

Best for:

- Linear objectives (MILP)
- Well-structured constraints
- Tight LP relaxations

Performance:

- Variant A: **$<$1.2s optimal**
- Variant B: **11/13 timeout**

Quantum (D-Wave)

Best for:

- Quadratic objectives (QUBO)
- Frustrated constraints
- Rugged energy landscapes

Performance:

- Variant A: 5.5s (overhead)
- Variant B: **3.80$\times$ better**

Key Insight: Neither approach universally dominates. Optimal solver depends on **problem structure**, not just size.

Key Numerical Results at a Glance

Metric	Study 1	Study 2	Study 3	Overall
Max Variables	27,027	27,027	16,200	27,027
Gurobi Time	1.15s	1.15s	300s (timeout)	–
QPU Time (pure)	70ms	21.8s	1.1% of total	Linear $O(f)$
Solution Gap	0%	31.8–57.1%	N/A	–
QPU/Gurobi Ratio	$\sim$1.0$\times$	$<$1.0$\times$	**3.80**$\times$	–
Chain Break Rate	–	$<$2%	$<$2%	$<$2%
Embedding Overhead	–	95–99%	–	Dominant

3.80$\times$
Avg QPU Improvement

$<$**2%**
Chain Break Rate

$O(f)$
QPU Time Scaling

Hardware Effects Analysis

Chain Breaking:

- Rate: Consistently $<2\%$
- Auto-scaled chain strength effective
- Minimal impact on solution quality

Embedding Overhead:

- 95–99% of total runtime
- Classical preprocessing bottleneck
- Not inherent to quantum computation

Noise Mitigation:

- Multiple samples (100–500)
- Majority voting for robust solutions
- Auto-scaling chain strength

Future Hardware Impact:

- Higher connectivity $\rightarrow$ less embedding
- Longer coherence $\rightarrow$ better quality
- Native constraints $\rightarrow$ no penalty terms

The embedding bottleneck is a **classical preprocessing issue**, not a fundamental quantum limitation. Next-generation hardware (Advantage2) promises significantly reduced overhead.

Constraint Violation Analysis

Violation Statistics (Study 3):

- Average violation rate: 24.2%
- Most violations: Diversity constraints
- Severity: Generally minor (<5% of bound)

Violation Sources:

1. Penalty weight imbalance
2. Decomposition boundary effects
3. Insufficient samples at boundaries

Mitigation Strategies:

- Adaptive penalty scaling
- Post-processing repair
- Coordinated decomposition
- Constraint-aware partitioning

Violations represent a trade-off: higher objective vs constraint satisfaction.

Grid Refinement: Continuous to Binary Conversion

Convergence Analysis:

Grid n	**Gap (%)**	**Binary (s)**	**Cont. (s)**
5	12.63	0.08	0.12
10	6.42	0.15	0.31
25	2.58	0.42	1.24
50	1.29	1.21	4.12
100	0.00	3.87	14.32

Convergence Rate: $O(n^{-1})$

Key Findings:

- Binary is **3.7× faster** than continuous at $n = 100$
- Gap decreases monotonically
- $n = 50$ sufficient for most applications

Binary formulation preferred for QPU: faster embedding, comparable accuracy.

Six Key Takeaways

1. **Formulation-Dependent Performance**
   Classical excels on MILP; quantum excels on frustrated QUBO problems
2. **Linear QPU Scaling**
   Pure quantum time scales $O(f)$ — embedding is the bottleneck
3. **3.80$\times$ Quantum Improvement**
   On rotation problems, QPU achieves significantly higher benefit
4. **Computational Cliffs Exist**
   Problem hardness is non-monotonic; rotation creates classical barriers
5. **Diversity Paradox**
   Quantum naturally produces more diverse, agriculturally robust solutions
6. **Hardware Constraints Remain**
   95–99% of runtime is classical embedding overhead

Impact Assessment

Agricultural Impact:

- Optimized crop allocation
- Improved nutritional outcomes
- Reduced environmental footprint
- Enhanced food security

Technical Impact:

- Novel decomposition strategies
- Hybrid quantum-classical workflows
- Benchmark methodology for QA

SDG Alignment:

- **SDG 2:** Maximize nutrition, food security
- **SDG 3:** Diverse diets, health outcomes
- **SDG 12:** Reduce waste, sustainable production
- **SDG 13:** Lower emissions, climate resilience

Quantum-optimized allocation addresses food security, sustainability, and economics simultaneously.

Future Directions

Hardware Improvements:

- Higher qubit connectivity
  $\Rightarrow$ Reduce embedding overhead
- Longer coherence times
  $\Rightarrow$ Better solution quality
- Native constraint support
  $\Rightarrow$ Avoid penalty terms

Algorithm Development:

- Improved decomposition strategies
- Adaptive penalty scaling
- Problem-specific embeddings

Application Extensions:

- Larger farm networks (10,000+)
- Multi-year planning horizons
- Climate adaptation scenarios
- Supply chain integration

Near-Term Target:
With Advantage2 hardware, embedding overhead could drop below 50%, making quantum competitive for simpler problems.

Thank You

Questions & Discussion

Edoardo Spigarolo

edoardo.spigarolo@epfl.ch

EPFL | Open Quantum Initiative

Backup: D-Wave System Specifications

Parameter	Value
Total Qubits	5,760
Topology	Pegasus P16
Average Qubit Connectivity	15 neighbors
Native Clique Size	15–20 qubits
Annealing Time Range	0.5–2,000 μs
Programming Thermalization	1 ms (default)
Chain Strength	Auto-scaled (0.9–2.0$\times$ max energy)
Maximum Problem Variables	$\sim$5,000 (depending on connectivity)

Backup: Decomposition Methods Comparison

Method	Vars/Partition	Scaling	Quality
Direct QPU	All	Fails	N/A
PlotBased	27	$O(f)$	Good
Multilevel(5)	135	$O(f)$	Better
Multilevel(10)	**270**	$\mathbf{O(f)}$	**Best**
Louvain	Adaptive	$O(f)$	Good
Spectral(10)	Variable	$O(f)$	Good
Coordinated	Variable	$O(f \cdot k)$	Better

Backup: Complete Benefit Equation

$$B_c = w_{nv} \cdot v_{nv,c} + w_{nd} \cdot v_{nd,c} - w_{ei} \cdot v_{ei,c} + w_{af} \cdot v_{af,c} + w_{su} \cdot v_{su,c}$$

Symbol	**Component**	**Default Weight**
$v_{nv,c}$	Nutritional Value	$w_{nv} = 0.25$
$v_{nd,c}$	Nutrient Density	$w_{nd} = 0.20$
$v_{ei,c}$	Environmental Impact	$w_{ei} = 0.20$ (negative)
$v_{af,c}$	Affordability	$w_{af} = 0.20$
$v_{su,c}$	Sustainability	$w_{su} = 0.15$

Backup: Study 3 Full Results Table

Scenario	Vars	Gurobi	QPU	Ratio	Gurobi Status
rotation_micro_25	90	6.17	4.86	0.79×	Optimal
rotation_small_50	180	8.69	21.79	2.51×	Optimal
rotation_medium_100	360	12.78	39.24	3.07×	Timeout
rotation_large_200	720	19.23	72.18	3.75×	Timeout
rotation_25farms_6foods	450	17.84	58.92	3.30×	Timeout
rotation_50farms_6foods	900	26.92	109.67	4.07×	Timeout
rotation_100farms_6foods	1,800	53.77	229.14	4.26×	Timeout
rotation_50farms_27foods	4,050	32.14	142.87	4.45×	Timeout
rotation_100farms_27foods	8,100	58.41	287.32	4.92×	Timeout
rotation_200farms_27foods	16,200	93.52	500.59	5.35×	Timeout
Average		**28.36**	**125.81**	**3.80**×	

Backup: Algorithm Pseudocode — Multilevel Decomposition

1. **Input:** BQM B, coarsening levels L, partition size k
2. **Coarsen:**
   - For $l = 1$ to L: Merge adjacent farms into super-farms
   - Build coarsened BQM at each level
3. **Partition:**
   - Apply METIS/spectral clustering to coarsest graph
   - Create $\lceil n/k \rceil$ partitions of size $\leq k$
4. **Solve:**
   - For each partition: Submit to QPU, collect samples
   - Select best sample per partition
5. **Uncoarsen & Refine:**
   - Map solutions back through coarsening levels
   - Apply local refinement at each level
6. **Output:** Combined solution for original BQM