SWE 580 Midterm Question 1
SWE580 Complex Networks Spring 2021

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1 Clustering Coefficient for 1-Dimensional Lattice

In a one-dimensional lattice network having N vertex, the nodes are arranged in a circular configuration and each has $k = 2r$ links, which are linked to their r nearest neighbors.We calculate clustering coefficient (C), by getting average of the local clustering coefficients (C_i) for all node i, using the general formula:

$$C = \frac{1}{N}\sum_{i=1}^{N} C_i$$

Local clustering coefficients (C_i) for node i, is found by dividing number of edges, and all possible edges in the lattice network.

$$C_i = \frac{\text{Number of Edges}}{\text{Number of All Possible Edges}}$$

Since this is a ring lattice, every vertex is connected to the nearest r nodes on the left and r nodes on the right so, number of edges are degree of vertex, k :

$$k = 2r$$

For any vertex i, there is for sure a left and right connection. If we divide the circle into 2 equal parts referencing vertex i, we can use formula below in order to find the number of vertices at the left half of ring,

$$n_L = N \text{ div } 2$$

Number of all possible edges at left half of circle, m_L, can be calculated by combination of any 2 vertices in left half-of the circle:

$$m_L = C(n_L, 2)$$

Since the left and right halves would have same number of vertices, number of all possible edges at right half of circle, m_R, will be equal to m_L. Therefore, number of all possible edges, M, will be :

$$\begin{aligned} M &= m_L + m_R \\ &= 2m_L \\ &= 2C(n_L, 2) \\ &= 2\frac{n_L(n_L-1)}{2} \qquad = n_L(n_L-1) \end{aligned}$$

By consolidating the formula for local clustering coefficient for vertex i, (C_i), the formula will be :

$$C_i = \frac{\text{Number of Edges}}{\text{Number of All Possible Edges}} = \frac{k}{M}$$

$$C_i = \frac{2r}{(N \text{ div } 2)((N \text{ div } 2) - 1)}$$

For lattice networks, the local clustering coefficient, C_i, is the same for each vertex i. In this case, formula for clustering coefficient (C), can be consolidated as follows:

$$C = \frac{1}{N}\sum_{i=1}^{N} C_i$$
$$= \frac{1}{N} N C_i = C_i$$

$$C = \frac{2r}{(N \text{ div } 2)((N \text{ div } 2) - 1)}$$

1.1 Clustering coefficients for various combinations of N and r

Table 1 shows some sample results for clustering coefficient in different combinations of N and r.

No	N	r	k	C
1	10	2	4	0.20000
2	10	4	8	0.40000
3	25	2	4	0.02782
4	25	5	10	0.06956
5	25	8	16	0.11130
6	50	2	4	0.00666
7	50	5	10	0.01666
8	50	10	20	0.03333
9	50	15	30	0.05000
10	100	15	30	0.01224
11	100	20	40	0.01632
12	100	35	70	0.02857

Table 1: Example: Sample results for various combinations of N and r