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$$\varrho = P(\geq m \text{ job}) = P_m + P_{m+1} + P_{m+2} + \ldots\ldots$$

$$\cancel{\varrho = P_0 \frac{\rho^m m^m}{m!} + P_0 \frac{\rho^{m+1} m^{m+1}}{m!} + P_0 \frac{\rho^{m+2} m^{m+2}}{m!} + \ldots}$$

$$\cancel{= P_0 \frac{(m\rho)^m}{m!}\left(1 + (m\rho) + (m\rho)^2 + \ldots\right)}$$

$$P_n = \begin{cases} P_0 \dfrac{(m\rho)^n}{n!}, & n < m \\ P_0 \dfrac{\rho^n m^m}{m!} & n \geq m \end{cases}$$

$$\Rightarrow = P_0 \frac{m^m}{m!}\rho^m + P_0 \frac{m^m}{m!}\rho^{m+1} + P_0 \frac{m^m}{m!}\rho^{m+2} + P_0 \frac{m^m}{m!}\rho^{m+2} \ldots$$

$$= P_0 \frac{m^m}{m!}\rho^m \left(\rho^0 + \rho^1 + \ldots\ldots\right)$$

$$= P_0 \frac{(\rho m)^m}{m!} \sum_{i=0}^{\infty} \rho^i = \boxed{P_0 \frac{(\rho m)^m}{m!(1-\rho)}} \boxed{= \varrho = P(\geq m \text{ job})}$$

$$E(n_s) = \sum_{n=1}^{m-1} n P_n + \sum_{n=m}^{\infty} n P_n$$

$$= 1 \frac{m\rho}{1} P_0 + P_0\, 2 \frac{(m\rho)^2}{2!} + P_0\, 3 \frac{(m\rho)^3}{3!} + P_0\, 4 \frac{(m\rho)^4}{4!} + \ldots + P_0 \frac{(m-1)(m\rho)^{m-1}}{(m-1)!}$$

$$+ m\varrho$$

$$= m\rho\left(P_0 + \frac{m\rho}{1!} + \frac{(m\rho)^2}{2!} + \frac{(m\rho)^3}{3!} + \ldots + P_0 \frac{(m\rho)^{m-2}}{(m-2)!}\right) + m\varrho \quad \boxed{\rho = \frac{\lambda}{m\mu}}$$

$$= m\rho\left(P_0 + P_1 + P_2 + \ldots + P_{m-2}\right) + m\varrho \quad \boxed{P_m = \rho P_{m-1}}$$

$$= m\rho\left(1 - P_{m-1} - \varrho\right) + m\varrho \quad \varrho(1-\rho) = P_m$$

$$= m\rho - m\rho P_{m-1} - m\rho\varrho + m\varrho = \underbrace{m\varrho(1-\rho)}_{mP_m} - m\rho P_{m-1}$$

$$= \boxed{m\rho}$$

$$P_{m-1} = P_0 \frac{(m\rho)^{m-1}}{(m-1)!} \times \frac{(m\rho)}{m} = P_0 \frac{(m\rho)^{m}}{m!}$$

$$\Rightarrow \boxed{\rho P_{m-1} = P_m} \qquad \cancel{P_{m-2}} = \cancel{P_0 \frac{(m\rho)^{m-2}}{(m-2)!}} \cancel{\frac{(m-1)}{(m-1)}} \frac{(\rho m)}{m} = P_0 \frac{\cancel{(m\rho)^{m-1}}}{m!}$$

$$E(n_s^2) = \sum_{n=1}^{m-1} n^2 P_n + \sum_{n=m}^{+\infty} m^2 P_n$$

$$= 1^2 \frac{(m\rho)^1}{1!} \times P_0 + 4 \frac{(m\rho)^2}{2!} \times P_0 + 3^2 \frac{(m\rho)^3}{3!} \times P_0 + \frac{4^2 (m\rho)^4}{4!} P_0 + \dots + \frac{(m-1)^2 \times (m\rho)^{m-1}}{(m-1)!}$$

$$+ m^2 C$$

$$= (m\rho) P_0 + 2 \frac{(m\rho)^2}{1!} P_0 + 3 \frac{(m\rho)^3}{2!} + 4 \frac{(m\rho)^4}{3!} + \dots + \frac{(m-1) \times (m\rho)^{m-1}}{(m-2)!}$$

$$= m\rho \left[P_0 + 2 \frac{(m\rho)}{1!} P_0 + 3 \frac{(m\rho)^2}{2!} P_0 + 4 \frac{(m\rho)^3}{3!} + \dots + (m-1) \times \frac{(m\rho)^{m-2}}{(m-2)!} P_0 \right]$$

$$= m\rho \left[P_0 + 2P_1 + 3P_2 + 4P_3 + \dots + (m-1) P_{m-2} \right]$$

$$= m\rho \left[\sum_{n=0}^{m-2} (n+1) P_n \right] = m\rho \left[\underbrace{\sum_{n=0}^{m-2} n P_n}_{A} + \underbrace{\sum_{n=0}^{m-2} P_n}_{B} \right] + m^2 C \qquad \text{I}$$

$$\Rightarrow A = \sum_{n=1}^{m-2} n P_n = 1 \frac{(m\rho)}{1!} P_0 + \frac{2 (m\rho)^2}{2!} P_0 + \frac{3 (m\rho)^3}{3!} P_0 + \dots + \frac{(m-2)(m\rho)^{m-2}}{(m-2)!} P_0$$

$$= m\rho \left[P_0 + \frac{(m\rho)}{1!} P_0 + \frac{(m\rho)^2}{2!} P_0 + \dots + \frac{(m\rho)^{m-3}}{(m-3)!} P_0 \right]$$

$$= m\rho \left[P_0 + P_1 + \dots + P_{m-3} \right]$$

$$\boxed{= m\rho \left[1 - P_{m-1} - P_{m-2} - C \right]}$$

$$B = \sum_{n=0}^{m-2} P_n = \boxed{1 - P_{m-1} - \ell}$$

$$\sum_{n=0}^{m-2} P_n + P_{m-1} + \ell = 1 \qquad \boxed{\sum_{n=0}^{m-1} P_n + \ell = 1}$$

$$\Rightarrow E(n_s^2) = mp\Big[\overbrace{\sum_{n=0}^{m-2} nP_n}^{A} + \overbrace{\sum_{n=0}^{m-2} P_n}^{B}\Big] + m^2\ell \quad \Leftarrow \text{I}$$

$$\Rightarrow E(n_s^2) = mp\Big[\overbrace{mp(1 - P_{m-1} - P_{m-2} - \ell)}^{A} + \overbrace{1 - P_{m-1} - \ell}^{B}\Big] + m^2\ell$$

$$\Rightarrow " = mp\left[mp - mpP_{m-1} - mpP_{m-2} - mp\ell + 1 - P_{m-1} - \ell\right] + m^2\ell$$

$$= mp\left[mp - mP_m - mpP_{m-2} - \ell(1+mp) + 1 - P_{m-1}\right] m^2\ell$$

$$= m^2p^2 - \cancel{m^2pP_m} - m^2p^2P_{m-2} - \cancel{m^2p^2\ell} + mp - \underbrace{mpP_{m-1}}_{mP_m} - mp\ell + \cancel{m^2\ell}$$

$$m^2\ell(1-p^2) = \underbrace{m^2\ell(1-p)}_{m^2P_m(1+p)}(1+p)$$

$$= \boxed{m^2P_m} + \cancel{m^2pP_m}$$

$$= m^2p^2 - m^2p^2P_{m-2} + m^2P_m + mp - mP_m - mp\ell \qquad mp(1-\ell) = mpP_m$$

$$= \cancel{m^2p^2 - m^2p^2P_{m-2} + m^2P_m - mp + mpP_m}$$

(4)

$$E(n_s^2) = n^2\rho^2 - n^2\rho^2 P_{n-2} + n^2 P_n + n\rho - nP_n - n\rho\varphi$$

$$\boxed{E(n_s^2) = n^2\rho^2\left(1 - P_{n-2}\right) + P_n\left(n^2 - n\right) + n\rho\left(1 - \varphi\right)}$$

$$\frac{P_n}{(1-\rho)}$$

$$\boxed{E(n_s^2) = n^2\rho^2\left(1 - P_{n-2}\right) + P_n\left(n^2 - n\right) + n\rho\left(1 - \frac{P_n}{1-\rho}\right)}$$

$$\frac{n\rho\,(1-\rho-P_n)}{1-\rho}$$

$$\frac{n^2\rho^2 - n^2\rho^2 P_{n-2} - n^2\rho^3 + n^2\rho^3 P_{n-2} + P_n n^2 - P_n n - \rho P_n n^2 - \rho P_n n + n\rho - n\rho^2 - n\rho P_n}{(1-\rho)}$$

$\Rightarrow \boxed{\rho = \frac{\lambda}{n\mu}}$ $\boxed{\text{Bodyer JM}}$

$$\boxed{E(n_s^2) = n^2\rho^2\left(1 - \frac{(n-1)\varphi(1-\rho)}{n\rho^2}\right) + \varphi(1-\rho)(n^2-n) + n\rho(1-\varphi)}$$

comme en fonction dans le bouquin :

$$E(n_s^2) = \rho^2\left(1 - \frac{(n^2-n)\varphi\left(1-\frac{\rho}{n}\right)}{\rho^2}\right) + \varphi\left(1-\frac{\rho}{n}\right)(n^2-n) + \rho(1-\varphi)$$

cas de $\boxed{\rho = \frac{\lambda}{\mu}}$ $\Rightarrow E(n_s^2) = \rho^2 - (n^2-n)\varphi\left(1-\frac{\rho}{n}\right) + \varphi\left(1-\frac{\rho}{n}\right)(n^2-n) + \rho(1-\varphi)$

$$= \rho^2 + \rho(1-\varphi)$$

$$E(n_s^q) = \rho^2 + \rho(1 - \varphi)$$

$$\varphi = \cancel{\frac{\rho^m}{(m-\rho)}} = \frac{m P_m}{m - \rho} = \left(\frac{m}{m-\rho}\right) \times \left(P_0 \times \frac{\rho^c}{c!}\right)$$

$$= \frac{c}{(c-\rho)} \times \frac{P_0 \times \rho^c}{c!} = \frac{P_0 \, \rho^c}{(c-\rho)(c-1)!}$$

$$\Rightarrow \boxed{E(n_s^q) = \rho^2 + \rho\left(1 - \frac{P_0 \, \rho^c}{(c-\rho)(c-1)!}\right)}$$

$$\boxed{E(n_s^q) = \rho^2 + \rho(1 - \varphi)}$$