A simple way to identify the degree of collusion under proportional reduction*

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Abstract

Proportional reduction is a common cartel practice, in which cartel members reduce their output by the same percentage. We develop a simple method to quantify this reduction relative to a benchmark market equilibrium scenario. Our measure is continuous, has a simple interpretation as the "degree of collusion" and nests the earlier models in the existing literature. More importantly, by exploiting firms ex post heterogeneity and optimality conditions, Corts (1999) critique can be addressed by estimating time-varying degree of industry monopolization from a short panel of firm-level observations. We illustrate the method in Monte-Carlo simulations and in application to the data from the Joint Executive Committee railroad cartel.

Keywords: Cartel, Proportional Reduction, Degree of collusion.

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1 Introduction

Measuring market power and assessing industry conduct remain among the major challenges in empirical Industrial Organization. These questions have important implications for welfare analysis and antitrust regulation. A wide variety of empirical models have been developed to measure the degree of competition in markets where reliable cost data are not available. The problem frequently boils down to estimating a "conduct parameter," which summarizes the level of competition in an industry. Typically, an econometrician specifies a supply relation where the conduct parameter takes on distinct values nesting Cournot, perfect competition (Bertrand), and perfect cartel (Monopoly) models. Estimated parameter values are then interpreted as the degree of collusiveness. In reality, however, the estimated parameter values are often significantly different from the values describing either of the conduct regimes, making it harder to interpret.[1] A problem that is perhaps more serious than the internal inconsistency between a theoretical model and its empirical implications is raised by Corts (1999), who shows that the estimated parameter values may fail to measure market power due to dynamic considerations of the firms. When firms are efficiently colluding, changes in the economic environment may affect the degree of collusion (for example, cartel sustainability as described in Rotemberg and Saloner, 1986), suggesting that the conduct parameter would change over time and would be an endogenous variable. Thus, across-time variation in the demand and supply conditions may fail to identify the industry conduct.

In this study, we propose an alternative way to evaluate the industry conduct, which overcomes the aforementioned problems in the literature. The key to our method is an assumption on the way collusion is implemented. Instead of assuming that the objective function of a cartel is known, e.g., joint profit maximization, we assume that firms employ Proportional Reduction (PR) collusive technology (as discussed in Schmalensee, 1987). Under the Proportional Reduction assumption, cartel members reduce their outputs proportionately relative to a benchmark market equilibrium output. We argue that Proportional Reduction is both theoretically and empirically plausible. For example, it

[1] In such cases, the industry competitiveness is evaluated in terms of the number of firms playing a particular equilibrium. This interpretation of the conjectural variation parameter is sometimes referenced as the "*as-if*" interpretation. For example, an industry with N firms is as competitive as if it were Cournot equilibrium with K players.

is hard to see why symmetric players who maximize joint industry profit would reduce their output non-proportionally.[2] From an empirical point of view, although it is difficult to show hard evidence that Cartel members are proportionately reducing their outputs, Proportional Reduction is indirectly supported. This is because Proportional Reduction can be seen as one particular type of market share allocation, which is frequently observed in practice and recognized by judicial systems (courts), as documented in Marshall and Marx (2008).[3]

Our method is simple and has several advantages over the traditional conduct parameter approach. First, our parameter takes values on a continuous interval, having a simple interpretation as the percentage reduction in the output relative to a well-defined benchmark competitive equilibrium outcome. Second, we show that firms' heterogeneity provides useful variation, which can be used to estimate time-varying degree of industry monopolization from a relatively short panel of firm level observations. This source of identification is present even when firms are symmetric ex ante, i.e., before realizations of iid innovations to their costs. The ability to estimate a time-varying degree of collusion is important to address Corts' critique regarding endogenously chosen levels of monopolization. Finally, while illustration of the method in this paper is provided using a very simple static framework, the method is extendable to more complex settings with dynamically optimizing agents and more flexible forms of the demand and cost functions. Therefore while strong in itself, our assumption about collusive technology can help to accommodate a wide variety of complex strategic interactions and can be used when a researcher prefers to stay agnostic about the objective function of a cartel.

Our work is closely related to studies by Bresnahan (1982), Lau (1982), and Porter (1983). Ellison (1994) provides a comprehensive empirical comparison of competing theories of collusion by Green and Porter (1984) versus Rotemberg and Saloner (1986). In these (and many other) articles, in order to derive an empirical specification for estimation a researcher has to assume that the objective function of a cartel is known, e.g., joint profit maximization. In reality, the objective function of a cartel is rarely known. It may be quite

[2]Symmetry and joint profit maximization are standard assumptions in the literature, in which case existing methods of identifying the parameter of interest (e.g., Bresnahan, 1982, Lau, 1982) are directly applicable within our framework.

[3]Keeping their market shares constant, cartel members must reduce their output proportionately from a benchmark equilibrium outcome.

complex and depend not only on the current and future states of the demand and supply conditions, but also on the probability of disclosure (which, in turn, may be a function of the level of collusion itself) and expected punishment by the antitrust authorities. Given feasible (potentially implementable in the real world) equilibrium supporting strategies not all levels of collusion can be sustained, as described by Rotemberg and Saloner (1986). Analysis of more complex settings, when the collusion occurs only along one of the dimensions, e.g., price fixing with competition in quality or capacity, is provided in Fershtman and Gandal (1994). Availability of reliable cost data facilitates estimation of industry conduct considerably. For example, Genesove and Mullin (1998) conduct a comprehensive comparison of various ways to estimate industry conduct and marginal costs in the sugar industry. Wolfram (1999) also considers a model with time-varying conduct parameters when direct measures of marginal costs are available. However, her identification still relies on the time-series variation in the data because in a duopolistic market the variation across firms is limited. A more structural way to address the Corts' critique can be found in Puller (2009).

The rest of the paper is organized as follows. Section 2 describes a general framework and provides a simple example with a linear demand function and constant marginal cost functions. We discuss extensions to the method using alternative demand and marginal cost specifications and describe how the method can be used in estimation of dynamic games. In Section 3 we discuss identification of our conduct parameter and evaluate its finite-sample properties using Monte Carlo simulations. We illustrate an application of our method using the well-known data of the Joint Executive Committee railroad cartel in 1880-1886 in Section 4. Section 5 concludes.

2 Model

In this section, we outline our framework by presenting a simple model with linear demand and constant marginal cost functions. Potential extensions of the model are discussed in Section 2.3. We begin by describing alternative ways of implementing collusion.

Schmalensee (1987) defines four distinct collusive technologies. The most profitable one is full collusion with side payments, where only the most efficient firms produce. This type of collusion is, perhaps, the least realistic for obvious reasons. The remaining three ways of colluding do not require side payments.[4] *Market Sharing* collusive technology involve assigning production quotas. For example, the quotas may be chosen to equate the critical discount factor among the cartel members, which would maximize sustainability of the cartel. Such arrangements generally would require solving a non-trivial bargaining problem, particularly when the firms are imperfectly informed about their rivals' costs. Collusion implemented through *Market Division* occurs when each firm is assigned to a part of the market and charges its optimal monopoly price in this segment. The possibility of arbitrage makes such a technology difficult to implement in practice. Finally, the last type of reward distribution is *Proportional Reduction*, when firms fix their market shares at some non-collusive (e.g. Cournot) values and each firm reduces the output by the same proportion. Even though PR technology may generate lower profits than some (or even all) of the alternatives, simplicity of its implementation may play a role. Another benefit of the proportional reduction is that frequently used concentration measures (e.g. HHI or C_n) would be observationally equivalent to a competitive outcome as the distribution of market shares does not change between competitive and collusive regimes. The latter is an important observation as it shows that PR technology is potentially empirically testable. For example, when the degree of collusion changes, the distribution of market shares stays the same, while aggregate output changes substantially.[5]

2.1 Basic setup

Consider a homogeneous product market with N firms competing in quantity over time, $t = 1, 2, \ldots, \infty$. Suppose each firm is characterized by a cost function denoted by $C_i(q_{it}, z_{it})$, where q_{it} is output and z_{it} is a vector of cost shifters. Let the inverse demand function be given by $P_t = P(Q_t, Y_t)$, where $Q_t = \sum_{i=1}^{N} q_{it}$ denotes total industry output and Y_t is a

[4] Without side payments, collusion would imply positive production levels even for the least efficient firms, making joint industry profit maximization infeasible.

[5] That being said, to construct an empirical test one would need to know exactly when the degree of collusion has changed.

vector of demand shifters. The per-period reward function is given by

$$\pi_{it} = P(Q_t, Y_t)q_{it} - C_i(q_{it}, z_{it}). \tag{1}$$

The firms in the industry interact repeatedly and can be engaged in tacit collusion agreements. Instead of making an assumption on the objective function of the cartel which is typically unknown to econometricians, we make the following assumption on the way the collusion is implemented.

Assumption 1: *In any collusive period firms reduce their individual output proportionally to the baseline Cournot quantities, i.e.,*

$$q_{it}^{C} = \theta_t q_{it}^{PR}, \ \forall i, t$$

where q_{it}^{C} and q_{it}^{PR} denote one period Cournot and collusive output levels under PR respectively, and $\theta_t \geq 1$ is the inverse of the percentage reduction in output.

Assumption 1 implies that knowing θ_t allows us to compute the counterfactual Cournot quantity by "inflating" observed output q_{it}^{PR} by a factor of θ_t. For example, suppose that in the collusive period each firm reduces its output by 10% relative to the Cournot quantity. Then, $\theta_t = 1/(1-0.1) = 1.11$. Under Assumption 1, the degree of collusion can be summarized by the parameter θ_t. Hence, our ultimate objective is to estimate θ_t from the observed data.

Before proceeding with how to recover θ_t, it is worth noting that we intentionally abstain from developing a particular structural model of collusion, i.e., our model avoids specifying the objective function of the cartel or the bargaining process, which we cannot learn from the data. However, one can think of simple collusion supported by grim trigger strategies with Cournot-Nash as the punishment phase. Lemma 1 in Appendix B shows that proportional reduction technology is profitable for all firms in the neighborhood of the Cournot equilibrium quantity. Therefore, it is straightforward to prove that there exists a common discount factor $\beta = \min\{\beta_1, \ldots, \beta_N\}, \ \beta_i \in (0,1) \ \forall i$, such that the collusion is sustainable. Section 2.3 addresses some of the potential caveats of Assumption 1 and our framework.

2.2 Simple model

Assume a linear inverse demand function $P_t = \alpha_0 + \alpha_1 Q_t + \alpha_2 Y_t + \nu_t^d$ and suppose in the data we observe $(q_{it}^{PR}, z_{it}, P_t^{PR}, Y_t),\ i = 1, \ldots, N;\ t = 1, \ldots, T$. Under Assumption 1, the following relationship must hold:

$$\begin{aligned} P(Q_t^C, Y_t) &= P(Q_t^C, Y_t) - P(Q_t^{PR}, Y_t) + P(Q_t^{PR}, Y_t) \\ &= \alpha_1 \left(\theta_t - 1\right) Q_t^{PR} + P_t^{PR}, \end{aligned}$$

where Q^{PR} and P^{PR} are collusive total output and equilibrium price, and $P(Q^C, Y)$ is an unobserved (counterfactual) Cournot equilibrium price.

On the supply side, we assume a constant marginal cost function, i.e., $\partial C_i(q_{it}, z_{it}) / \partial q_{it} = \beta_{0i} + z_{it}\beta + \nu_{it}^s$, where z_{it} is a vector of observed cost shifters in the data and ν_{it}^s is unobserved cost component. In a Cournot NE, first-order conditions for firm i are given by

$$\alpha_1 q_{it}^C + P_t^C - \beta_{0i} - z_{it}\beta - \nu_{it}^s = 0. \tag{2}$$

Note that equation (2) would not hold with equality when evaluated at $\left(q_i^{PR}, Q^{PR}\right)$, as there would be incentives to deviate from the collusive quantity by expanding the output. However, we know the relationship between the collusive and competitive regimes and, therefore, can "restore" individual first-order conditions in terms of collusive values and the parameter θ_t as follows

$$\alpha_1 \theta_t q_{it}^{PR} + \alpha_1 \left(\theta_t - 1\right) Q_t^{PR} + P_t^{PR} - \beta_{0i} - z_{it}\beta - \nu_{it}^s = 0. \tag{3}$$

Even though one may attempt to identify both θ_t and α_1 using just the supply relation (3), we focus on identification of the conduct and cost function parameters and assume, throughout the rest of the paper, that α_1 can be consistently estimated using conventional instrumental variable techniques.[6]

From equation (3) one can already see that variation in q_{it}^{PR} across firms, holding Q_t^{PR} and P_t^{PR} fixed within a cross-section, provides additional identification power. More

[6]It is rather the degree of collusion that may change in response to changing economic environment.

formally, identification of our parameter of interest relies on the availability of firm-level exogenous cost shifters, which is summarized in the following Assumption 2.

Assumption 2: *Data contains information on exogenous demand and firm-level cost shifters, (Y_t and $z_{1t}, \ldots, z_{Nt}$ respectively), such that demand-side and cost-side innovations satisfy*

$$\mathrm{E}\left[\nu_{it}^{s} | z_{it}, z_{-it}, Y_t\right] = \mathrm{E}\left[\nu_{t}^{d} | Z_t, Y_t\right] = 0,$$

where $Z_t = \sum_i z_{it}$ and z_{-it} are cost shifters for firms other than the firm i.

For example, if $\beta_{0i} = \beta_0, \forall i$ and we observe just one cost shifter satisfying Assumption 2 in the data, we can identify all parameters in the model given that $T > 1$.[7] To see this, rewrite equation (3) as

$$\nu_{it}^{s} = P_t^{PR} - \beta_0 - z_{it}\beta - \alpha_1 Q_t^{PR} + \theta_t \alpha_1 (q_{it}^{PR} + Q_t^{PR}),$$

and define $X = (\mathbf{1}_{NT}, \mathbf{z}, \mathbf{q}, \mathbf{I}_T \otimes \mathbf{x}_t)$ and $Z = (\mathbf{1}_{NT}, \mathbf{z}, \mathbf{I}_T \otimes \mathbf{z}_{-it})$ where

- $\mathbf{1}_{NT}$ and $\mathbf{1}_T$ are vectors of ones of sizes $N \times T$ and T respectively;
- $\mathbf{z} = (z_{11}, \cdots, z_{N1}, \cdots, z_{1,T}, \cdots, z_{NT})'$;
- $\mathbf{q} = \left(\alpha_1 Q_1^{PR}, \cdots, \alpha_1 Q_1^{PR}, \cdots, \alpha_1 Q_T^{PR}, \cdots, \alpha_1 Q_T^{PR}\right)'$;
- $\mathbf{I}_T$ is identity matrix of size T;
- $\mathbf{x}_t = \Big(\alpha_1 \left(q_{1t}^{PR} + Q_t^{PR}\right), \cdots, \alpha_1 \left(q_{Nt}^{PR} + Q_t^{PR}\right)\Big)'$;
- $\mathbf{z}_{-it} = \Big(\sum_{j \neq 1} z_{jt}, \cdots, \sum_{j \neq N} z_{jt}\Big)'$;
- $\otimes$ denote Kronecker product.

The standard identification conditions for IV methods requires $Z'X$ to have a full column rank. With just one period of data, it is clear that X does not have a full column rank, due to the first and third columns. Hence, separate identification of the constant term in the marginal cost specification and the time-varying parameter θ_t requires data for at least two time periods.[8]

More generally, there are two sources of variation that help to identify the degree of output reduction. The first one is variation across asymmetric firms. The second one

[7]The system would be over-identified if in addition we observe demand shifters.
[8]Of course, to estimate demand parameter α_1 one would need longer time-series.

is variation over time in the demand and supply conditions. Under the proportional reduction, asymptotics is in terms of $N \times T^*$, where T^* is the number of time periods with constant conduct parameter θ_t. Of course, to fully utilize this property a researcher should specify observable cost shifters at the firm-level.

At this stage, it might be useful to compare our measure of market power to the conduct parameter from the earlier literature (e.g., Bresnahan, 1982). Typically, the existing literature identifies market power as a conduct parameter, λ, nesting three types of first order conditions within one equation,

$$P_t + q_{it}\frac{\partial P}{\partial Q}\lambda - mc_i = 0,$$

where λ can take 3 distinct values, depending on the underlying scenario of industry conduct. Table 1 compares values of our parameter θ for each theoretically admissible value of λ as a function of the number of firms, N.

[*** Table 1 appears about here ***]

The key difference is that our measure of market power, θ, is defined on a continuous interval, while the game theoretic approach dictates only discrete values for λ, outside of which interpretation of the parameter becomes vague. Of course, the number of firms puts bounds on the values of θ that can be rationalized by a static model of profit maximization.[9] Under ex ante symmetric firms such restrictions can be imposed to increase accuracy of the estimates. However, with asymmetric firms and/or dynamically optimizing agents it may be hard to make such sharp predictions. For example, in case of strong learning-by-doing effects, in the early periods firms may rationally price below marginal costs. That being said, dynamic optimization would require optimality conditions that are different from equation (3), e.g., include proper derivatives of the continuation value. Such extensions are beyond the scope of this paper.

[9] For example, with 9 symmetric competitors θ must be within $[0.9, 1.8]$.

2.3 Extensions

As we discussed in the previous section, in order to "restore" the individual first order conditions in terms of (supposedly collusive) values of the observed variables one needs to assume some functional form for the demand and cost functions.

Consider firm i's maximization problem in a competitive regime where the per-period profit function is given by equation (1). Assuming away any dynamic effects of the quantity choice, e.g., there is neither the motive of "learning-by-doing" nor of investment into the customer base in the case of state dependent demand schedules, first-order conditions for quantity choice are given by

$$\text{FOC}[q_{it}] : \frac{\partial P(Q_t, Y_t)}{\partial Q_t} q_{it} + P_t - \frac{\partial C_i(q_{it}, z_{it})}{\partial q_{it}} = 0. \tag{4}$$

When firms are in a collusive regime, under Assumption 1, equation (4) can be written in terms of observables $(P_t^{PR}, Q_t^{PR}, q_{it}^{PR})$ and the parameter θ_t as follows

$$P_t^{PR} + \Big[P(\theta_t Q_t^{PR}, Y_t) - P(Q_t^{PR}, Y_t)\Big] + \frac{\partial P(\theta_t Q_t, Y_t)}{\partial Q_t} \theta_t q_{it} - \frac{\partial C_i(\theta_t q_{it}, z_{it})}{\partial q_{it}} = 0, \tag{5}$$

where $\Big[P(\theta_t Q_t^{PR}, Y_t) - P(Q_t^{PR}, Y_t)\Big]$ represents a "collusive markup" over the Cournot price level, i.e., the difference between the observed outcomes and hypothetical competitive outcomes. This term measures price differences in the case of movement along the demand curve from the observed output levels to competitive output levels.

First, consider a particular case when the marginal cost function contains a linear-in-q_{it} term, i.e., when

$$\frac{\partial C_i(q_{it}, z_{it})}{\partial q_{it}} = \beta_{0i} + \beta_q q_{it} + z_{it}\beta + \nu_{it}^d.$$

This case substantially complicates estimation of the conduct parameter in the earlier literature (the problem description and potential solutions are discussed in Bresnahan, 1982, Lau, 1982). One of the frequently employed solutions would be to find exogenous variables affecting elasticity of the demand, i.e., in addition to the demand shifters one would need to find some demand "rotators". Interestingly, when assumption 1 holds, we can estimate parameters of the model without the demand rotators. To see this consider equation (3), which now becomes

$$(\alpha_1 - \beta_q)\theta_t q_{it}^{PR} + \alpha_1 (\theta_t - 1) Q_t^{PR} + P_t^{PR} - \beta_{0i} - z_{it}\beta - \varepsilon_{it} = 0. \tag{6}$$

As before, we assume that the slope of the demand function α_1 is estimated using the demand relationship (or that appropriate moment conditions are included into the GMM criterion function). Therefore, we can identify θ_t from the coefficient on Q_t^{PR}, while β_q is identified by the coefficient on q_{it}^{PR}. Since identification of θ_t now relies on variation in the aggregate output, it cannot be different for all t, i.e., one would need to assume that the conduct parameter is constant for several time periods in the data.[10] On the other hand, presence of non-linear in q_{it} terms in the cost function or non-linearity of the demand function facilitates identification of the conduct parameter. These results are not new and have been known since the early empirical literature on collusion. Therefore, we don't discuss them here.

One of the benefits of our method is that one does not have to assume that all firms are colluding. The framework is easily applicable to an industry with a few dominant players and a competitive fringe. As long as the researcher is willing to make assumptions regarding the identities of colluding and free-riding firms, the method can be directly applied (e.g., cheating firms would choose their output levels with $\theta_{it} = 1$ if the baseline NE is Cournot).

So far we have considered homogeneous product markets. Potentially, the method can be applied to differentiated product markets with the assumption that output levels (prices) are reduced (increased) by the same proportion. We do not consider this case here. However, the topic is very interesting and is left for further research.

Another potential extension would be to use a more structural approach and to model firms' maximization problem as a dynamic game. For example, assumption 1 can be used within the framework of Fershtman and Pakes (2000). However, this would require explicit assumptions on the objective function of the colluding firms as well as specifying punishment strategies, which is exactly what we want to avoid in this study. As long as the baseline scenario (relative to which firms reduce their outputs) is given by a static

[10]Simulation results for a model with linear-in-q_{it} cost function are available upon request.

NE, it remains a NE of a dynamic game. Hence, our parameter estimates can still be interpreted relative to a well-defined alternative conduct regime. An example of a more structural approach, addressing the critique by Corts (1999) when the firms are engaged in efficient collusion, is given in Puller (2009).

Finally, we briefly discuss testable implications of proportional reduction collusive technology. If firms are colluding with $\theta_t = \theta$ fixed within all periods in the observed data, the distribution of market shares would be identical to the one under the baseline scenario (Cournot in our case). However, if the regime of collusion changes at some point in time (e.g., as a result of price war) one can search for such evidence by inspecting variation in the aggregate or individual outputs and the distribution of market shares. One possible example would be to check if a concentration measure (say, HHI or C_N) is statistically different in periods before and after the time of the potential change in the conduct regime with a similar test (e.g., difference in means) used for the aggregate or individual levels of output. If the test rejects that the distribution of market shares are different while the difference in the output levels is significant, that would be consistent with a change in the degree of collusion.

It is possible to show that exogenous changes in the demand and cost conditions in case of asymmetric firms must take very specific forms to generate proportional reduction in the individual output levels. In other words, it is very unlikely that in a Cournot NE firms would respond to the exogenous variation in a proportional way. For example, consider a change in the demand conditions Y_t. Proportional change in output of each firm implies that

$$\frac{\partial q_{it}/\partial Y_t}{q_{it}} = \frac{\partial q_{jt}/\partial Y_t}{q_{jt}} \Rightarrow \frac{\partial q_{it}/\partial Y_t}{\partial q_{jt}/\partial Y_t} = \frac{q_{it}}{q_{jt}}, \forall i, j.$$

By the implicit function theorem and Cournot first order conditions it is easy to show that to replicate proportional reduction as a result of aggregate demand shocks one must impose very strong restrictions on the underlying demand and cost functions. In particular, it is not possible in the case of linear demand and constant marginal costs (unless the firms are identical), while if the marginal costs are linear in quantity, the following would be required,

$$\frac{2\alpha_1 - \beta_q^{(j)}}{2\alpha_1 - \beta_q^{(i)}} = \frac{q_{it}}{q_{jt}}, \forall i, j$$

where $\beta^{(i)}$ and $\beta^{(j)}$ are cost parameters for firms i and j. Similar conclusions can be made regarding the cost shocks. While it might be possible to reverse-engineer a model (and/or competitive equilibrium concept) where firms do respond to some exogenous variation by replicating the proportional reduction collusive technology, we believe that for a very wide class of parametric empirical specifications used for estimation this is not true.

3 Monte Carlo Simulations

In order to demonstrate performance of our method and evaluate the properties of our estimator, Monte Carlo simulations are conducted. The details of the simulation design are as follows. Inverse demand and marginal cost functions are given by

$$P_t = \alpha_0 + \alpha_1 Q_t + \alpha_2 Y_t + \nu_t^d,$$
$$mc_i(q_{it}, z_{it}) = \beta_0 + \beta_1 z_{it} + \nu_{it}^s.$$

To make our simulations realistic, the following parameter values are chosen: demand side parameters are given by $\alpha_0 = 500$, $\alpha_1 = -1.0$, and $\alpha_2 = 1.0$, and supply side parameters are given by $\beta_{0i} = 10.0\ \forall i$ and $\beta_1 = 1.0$. The observable demand shifter, Y_t, the unobservable demand innovation, ν_t^d, the observed cost shifter, z_{it}, and the unobserved cost shock ν_{it}^s are randomly drawn from normal distributions, $Y_t \overset{iid}{\sim} N(0, 100)$, $\nu_t^d \overset{iid}{\sim} N(0, 1)$, $z_{it} \overset{iid}{\sim} N(1, 4)$ and $\nu_{it}^s \overset{iid}{\sim} N(0, 0.04)$, respectively. In every period, firms operated in one of three randomly chosen regimes with $\theta_t \in \{1.0, 1.2, 1.4\}$, where $\theta = 1.0$ implies Cournot NE. To see the effects of the number of firms, N, and time periods, T, a set of pairs of (N, T) is chosen from $\{10, 20, 30\} \times \{10, 20, 30\}$.

We simulate a data set 10,000 times and each time estimate parameters of the model using 2-step optimal GMM. The GMM criterion function is constructed using two sets of moment restrictions implied by Assumption 2. In particular, demand-side moment conditions are constructed by interacting ν_t^d with a constant, demand shifters and a sum of firm-level cost shifters. Supply-side moment conditions are obtained using products of ν_{it}^s with (z_{it}, z_{-it}, Y_t) and dummy variables for each regime. The weighting matrix is assumed to have a block-diagonal structure.

[*** Table 2 appears about here ***]

As our interest lies only in the estimates of the conduct parameter, Table 2 conveniently summarizes average estimates of θ_t, denoted by $\bar{\theta}$, standard deviation and average values of the estimated standard errors, denoted by Std. Dev. and ASE, respectively, for $(10, 30) \times (10, 30)$ sample sizes. The full set of estimation results can be found in Appendix C. In all cases, parameter estimates are precise and the standard deviations of the estimated coefficients are consistent with the mean values of the standard errors. As expected, the estimates become more accurate as the number of firms and/or the number of time-series observations increases. Monte Carlo simulations suggest that a longer panel (larger T) improves precision of the parameter estimates slightly better than a wider panel (larger N). We believe that this is because an increased number of time periods contributes to both the demand- and supply-side moment conditions, whereas an increased number of firms affects only the supply-side set of moment conditions.

4 Application: the Joint Executive Committee

In order to illustrate how our method works with real data, we apply our methodology to the Joint Executive Committee (JEC) railroad cartel data from Porter (1983) and Ellison (1994). The JEC was a legal cartel that controlled freight shipments from Chicago to the Atlantic seaboard in the 1880's. The cartel was created in 1879 – that is prior to the Sherman Act of 1880. The data contains firm-level information on prices, shipment volumes for grain and flour, and information about the availability of alternative transportation routes through the Great Lakes. A detailed description of the data can be found in Porter (1983) and Ellison (1994). It is worth noting that we provide this application primarily for illustrative purposes and the estimation results could be improved, if more detailed information were available, in particular on the individual firms' cost shifters.

4.1 Empirical Specification and Estimation

Let θ, α and β denote vectors of PR parameters, demand and cost function parameters respectively, and assume that the members of the JEC use proportional reduction collusive technology with parameter θ_t. Assume that the per-period profits of the firms within the JEC are given by

$$\pi(q_{it}, z_{it}, \nu_t^d, \nu_{it}^s; \theta, \alpha, \beta) = P(\theta_t Q_t, Y_t, \nu_t^d; \alpha)\theta_t q_{it} - C_i(\theta_t q_{it}, z_{it}, \nu_{it}^s; \beta),$$

where $Q_t = \sum_{i=1}^{N_t} q_{it}$, N_t is the number of firms in period t, Y_t is a vector of observed demand shifters, z_{it} is a vector of individual cost shifters, and (ν_t^d, ν_{it}^s) is a pair of demand and supply-side shocks, respectively. We assume the following functional forms

$$P(Q_t, Y_t, \nu_t^d; \alpha) = \alpha_0 + \alpha_1 Q_t + \alpha_2 Y_t + \nu_t^d,$$
$$C_i(q_{it}, z_{it}, \nu_{it}^s; \beta) = F_i + (\beta_{0i} + \beta_1 z_{it} + \nu_{it}^s)\, q_{it}.$$

When reporting estimation results the case where $\beta_{0i} \neq \beta_{0j}$ is referenced as "fixed effect" (FE), and the restriction of $\beta_{0i} = \beta_0,\ \forall i$ is denoted as "levels" (LE). In the data, we observe shipment volumes for both grain and flour. Because of the potential (dis-)economies of scope we define flour shipments to be an observable cost shifter z_{it} when evaluating collusion in the market for grain.[11]

Under our assumption of PR collusive technology, static Cournot first order conditions are given by (3). In order to estimate parameters of the model we estimate the demand and supply relations jointly. In particular, for any given vector of parameters, we isolate demand and supply shocks using the following system of equations,

$$\begin{cases} \nu_t^d = P_t - \alpha_0 - \alpha_1 Q_t - \alpha_2 Y_t, \\ \nu_{it}^s = \beta_{0i} + \beta_1 z_{it} - (\alpha_1 \theta_t q_{it}^{PR} + \alpha_1 (\theta_t - 1)\, Q_t^{PR} + P_t^{PR}). \end{cases}$$

Our estimation is based on the orthogonality restrictions following from the conditional

[11] We admit potential caveats related to the assumption of exogenous flour shipment volumes, however, available data do not provide us with better instrumental variables.

independence assumptions,

$$\mathrm{E}[\nu_t^d|Y_t, Z_t] = \mathrm{E}[\nu_{it}^s|Y_t, z_{it}, z_{-it}] = 0,$$

where $Z_t = \sum_{i=1}^{N_t} z_{it}$, $z_{-it} = \sum_{j\neq i} z_{jt}$. In practice, we interact z_{-it} with a set of dummy variables, one for each of the collusive regimes. We construct sample analogs of the population moment conditions, $G_d^N(Y_t, Z_t; \alpha)$ and $G_s^N(Y_t, z_{it}, z_{-it}; \alpha, \beta, \theta)$

$$G^N(Y_t, Z_t, z_{it}, z_{-it}; \alpha, \beta, \theta) = \begin{bmatrix} G_d^N(Y_t, Z_t; \alpha) \\ G_s^N(Y_t, z_{it}, z_{-it}; \alpha, \beta, \theta) \end{bmatrix},$$

and estimate parameters using the following GMM criterion function

$$(\alpha^*, \beta^*, \theta^*) = \underset{(\alpha,\beta,\theta)}{\arg\min} \left\{ G^N\left(Y_t, Z_t, z_{it}, z_{-it}; \alpha, \beta, \theta\right)' \cdot W \cdot G^N\left(Y_t, Z_t, z_{it}, z_{-it}; \alpha, \beta, \theta\right) \right\},$$

with a block-diagonal weighting matrix W.[12]

4.2 Estimation Results

4.2.1 Overall Results

As our main focus is again on the degree of collusion, Table 3 lists the inverse of the estimated degree of collusion ($1/\hat{\theta}$) in the FE specification. The full set of estimation results are documented in Appendix A. Parameter estimates obtained from the LE specification are similar and can be also found in Tables 6, 8, 10, and 12 in Appendix A.

[*** Table 3 appears about here ***]

Model (i) assumes that θ is constant for the entire sample period, regardless of the number of firms or other observables (see the results in the first column in Table 3). The detailed estimation results for this specification are documented in Table 7. According to

[12]In the first stage, the weighting matrix is obtained as inner product of the instrumental variables matrices, which would be optimal for linear model. In the second (and consecutive) stage(s), we compute optimal weighting matrix using empirical variance of the moment conditions.

the results, on average firms produced 31% more output than they would produce under the Cournot scenario.[13] Similarly, Model (ii) (in the second and third columns) assumes that the cartel is maintained at the same level of θ_1 during all collusive periods and that the firms produce ($1/\theta_0$-1)% more in competitive periods than they would do in Cournot. The estimates imply that, in the collusive period, the output was reduced to about 71% of hypothetical Cournot quantity. During price wars, on the other hand, firms produced 51% more than they would do in Cournot.

Estimation results become more plausible when the conduct regimes are defined as unique combinations of the number of firms and the indicator of collusion, because it is possible that these firms would target a different level of reductions, depending on the number of member firms. The results from Model (iii) under the fourth and fifth columns in Table 7 indicate that whenever the cartel indicator is equal to one, these firms produced 40 to 81% of the Cournot quantity. In the meantime, when the cartel broke down and the firms were involved in price wars, firms produce more than they would do in the Cournot equilibrium, except for the case of 7 firms. Interestingly, the estimated degree of monopolization monotonically declines in the number of firms, which is consistent with the presumption that larger cartels are less sustainable.

It is natural to believe that the firms collude on different levels depending on the existence of a competitor to the cartel, the Great Lakes, and thus we further use finer categorization in Model (iv). This specification assumes that the degree of monopolization depends on the number of firms, collusive indicator and the state of demand, i.e., whether the Great Lakes were open for navigation. The estimation results for this case are summarized in the last four columns of Table 3. Our estimates suggest that the degree of monopolization declines in the number of firms and is generally lower at lower states of demand. The latter speaks against the counter-cyclical cartel pricing patterns as in the model by Rotemberg and Saloner (1986). According to their predictions, a cartel would reduce the degree of monopolization at high states of demand to reduce incentives for cheating. Instead, we find that when facing competition from the Great Lakes transportation routes JEC members reduce their level of collusion.[14]

[13]Interestingly, this result is consistent with the findings in Porter (1983) where for constant θ the firms' behavior in collusive periods was roughly consistent with Cournot equilibrium (pp. 309-310).

[14]Again, this finding is similar to the one in Ellison (1994), where no evidence of the countercyclical

Since estimated parameter values imply a relatively high degree of collusion compared to a hypothetical Cournot equilibrium, we conducted the following experiment. Given our estimates of the cost function parameters, we calculated optimal monopoly and perfectly competitive quantity levels for each firm. The smallest optimal monopoly output among the colluding firms defines a lower bound on the total quantity of the cartel, while the largest (Bertrand) competitive quantity among the participating firms would impose an upper bound consistent with rational behavior. Figure 1 summarizes the results for the firm fixed-effect specification. Figure 3 in Appendix A presents same statistics for the specification in levels. As is apparent from the top panel of the figure, in most cases observed quantities stay in-between the upper and lower bounds. In particular, for the FE specification in 202 out of 328 weeks (62%) JEC produces more than the standalone monopoly quantity for the least efficient firm in a given week, and for the specification in levels this occurs 205 out of 328 times. The same observation can be made when output levels are averaged for each of the potential collusive regimes (bottom panel).

[*** Figure 1 appears about here ***]

To further confirm our estimation results, own price elasticity of the demand is calculated and presented in Table 4. As expected, the degree of monopolization is positively related to the absolute value of price elasticity, i.e., the higher the degree of monopolization the larger price elasticity of demand with correlation coefficient of 0.77. On average, during collusive regimes price elasticity of demand is -5.11, which is almost twice as big as the elasticity during non-collusive regimes of -2.75.

[*** Table 4 appears about here ***]

Lastly, we conducted several robustness checks of our specifications. First, we excluded observations with 6 and 8 firms when the cartel indicator is zero and the Great Lakes are open for navigation (see the note in Table 3). Estimation results do not change qualitatively as can be seen from Tables 14 and 15 in Appendix A. Second, we estimated the model using two alternative specifications for the cost function. Namely, in Table 16

pricing was found.

we report estimation results where the marginal cost function is given by either

$$mc_i(q_{it}, z_{it}; \beta) = \beta_{0i} + \beta_1 z_{it} + \beta_2 q_{it} + \nu_{it}^s,$$

or

$$mc_i(q_{it}, z_{it}; \beta) = \beta_{0i} + \beta_1 z_{it} + (\beta_2 + 1) q_{it}^{\beta_2} + \nu_{it}^s.$$

Columns 2 and 4 of Table 16 summarize the results. It turns out that including a linear or non-linear term in quantity does not effect our estimates of the conduct parameter substantially. Besides, the coefficients on the own quantity variable in the cost functions are statistically not different from zero at any reasonable significance level. Unfortunately, we do not have other instrumental variables to explore much richer specifications.

4.2.2 Absence of the Cartel Indicator

So far we use the cartel indicator, reported in the data, to tabulate regimes with a constant level of collusion. In practice, however, econometricians or competition authorities do not know whether or not firms collude. Thus, we must be able to define regimes relying only on observed variation in the output levels and market shares, not the cartel indicator. Therefore, without using the cartel indicator, we conduct two final empirical exercises: (i) we create our own index describing potential regimes of JEC operations and estimate the model with the new index, and (ii) we estimate the model at the monthly-level assuming that θ_t is constant within a month.

Our New Indicator In order to create our own index of collusion, we inspect the data for candidate collusive periods. Our criteria require a stable distribution of market shares and reduction in output relative to the adjacent time intervals. To test the stability of market shares, we use a t-test for difference in means, which accounts for serial correlation. In particular, the test compares sub-intervals within a given interval.[15] We find 9 such intervals with 662 observations in total. Table 5 reports parameter estimates for each of

[15] We assumed AR(1) process for serial correlation and computed equivalent sample size using approximation $\hat{n}^e = n \, \frac{1-\hat{\rho}_1}{1+\hat{\rho}_1}$. Then statistic for $H_0 : \mathrm{E}[HHI_1] = \mathrm{E}[HHI_2]$ was computed using $t = \frac{\mu_1 - \mu_2}{\left(\sigma_1^2/n_1^e + \sigma_2^2/n_2^e\right)^2}$ with significance level 0.05.

the collusive regimes with full estimation results listed in Table 17 in Appendix A.

[*** Table 5 appears about here ***]

For all regimes our estimates suggests at least some degree of collusion with the output levels below static Cournot NE (in 8-firms period weeks 191-196 the output level was very close to Cournot), as the percent reduction is almost always below one for both LE and FE specifications. To check the validity of our method, we create a Cartel Index, the average value of the reported cartel indicator during each period, expecting that the percent reduction and the Cartel Index are negatively correlated. If the Cartel Index is zero, for example, we must expect that the firms compete severely, yielding close to the Cournot output. The correlation coefficient between the percent reduction and the Cartel Index for the FE specification is -0.56, which indicates that the estimates are likely to be able to detect the existence of the cartel.

Monthly-Level As a last step, we estimate monthly-level θ_t to examine whether our methodology can detect the cartel for each month.[16] Figure 2 plots the estimated monthly-level θ_t. The black solid line shows the estimated value, whereas the gray solid lines indicate the confidence interval. To examine the performance of our methodology, the gray dashed line records the Cartel Index, which is an average value of the cartel indicator within a month. Whenever our estimated θ's go below one, the firms indeed failed to collude, indicated by the Cartel Index falling below one. Therefore, this observation validates our methodology.

[*** Figure 2 appears about here ***]

[16] Although our methodology allows us to estimate θ_t for weekly-level in principle, the JEC had a small number of firms, between 5 and 8 firms depending on the time period. Estimating one parameter (weekly-level θ_t), relying on only five to eight observations, might not yield statistically significant results. Therefore, we estimate the model with monthly-level θ_t for stacking at least 20 observations for estimating θ_t for each period.

5 Conclusions

In this paper we develop a method to estimate the time-varying degree of industry monopolization. The methodology does not impose any restrictions on the objective function of colluding firms. Instead, we impose an assumption on how collusion is implemented. We believe that our method has several advantages over the traditional empirical literature on collusion. First of all, proportional reduction would be a natural way to implement collusion with symmetric firms. Therefore, most of the earlier literature on estimating conduct parameters can be viewed as a special case of our model. Asymmetricity in the firms' cost functions provides useful variation that can be utilized to identify the degree of industry monopolization conditional on observing firm-level cost shifters. Second, the parameter measuring the degree of industry monopolization is a continuous measure relating observed levels of output to the hypothetical stage game Nash equilibrium. As a result, it has a simple interpretation as the percentage of output reduction relative to a well defined competitive equilibrium. Third, the fact that we do not require explicit assumptions about the objective function of the cartel allows us to accommodate a wide range of fairly complex models of collusion as long as the proportional reduction assumption is satisfied. The latter fact can be empirically tested. Fourth, we show that the variation in output levels across asymmetric firms allows time-varying estimates of the degree of monopolization. This way one can address the critique by Corts (1999) of the conjectural variation literature when the industry conduct is endogenous to the changing demand and supply conditions. Finally, we believe that simplicity of the method is appealing to industry practitioners because estimation can be done using standard statistical software. Perhaps, the best application of our framework would be at the stage of pre-screening procedures conducted by an antitrust authority when deciding about taking the case for a thorough investigation or dismissing it.

Monte Carlo simulations illustrate finite-sample properties of the parameter estimates and show that our method performs well even with medium sample sizes consisting of 100 to 300 data points. Thus, the parameter of interest can be estimated from relatively short panels of firm-level observations. To further investigate the practicality of our method, we use the Joint Executive Committee railroad cartel from the 19^{th} century. Our analysis using the available cartel indicator demonstrates that it strongly correlates with the estimated degree of collusion. Finally, we estimate the time-varying degree of monopolization at a monthly level. Estimation results imply substantial variability in the degree of collusion over time, with the output levels during price wars sometimes exceeding quantities predicted by the Cournot equilibrium.

Table 1: Traditional measure of industry conduct vs θ.

scenario	existing literature, λ	our measure, θ
Bertrand	0	N/(N+1)
Cournot	1	1
Monopoly	N	2N/(N+1)

Table 2: Monte Carlo Simulation for $N = 10, 30$ and $T = 10, 30$

	T=10			T=30		
Regime 1: True parameter value = 1.000						
	$\bar{\theta}$	Std. Dev.	ASE	$\bar{\theta}$	Std. Dev.	ASE
N = 10	1.000	0.005	0.004	1.000	0.002	0.002
N = 30	1.000	0.002	0.002	1.000	0.001	0.001
Regime 2: True parameter value = 1.200						
	$\bar{\theta}$	Std. Dev.	ASE	$\bar{\theta}$	Std. Dev.	ASE
N = 10	1.200	0.012	0.010	1.200	0.006	0.006
N = 30	1.200	0.008	0.007	1.200	0.005	0.004
Regime 3: True parameter value = 1.400						
	$\bar{\theta}$	Std. Dev.	ASE	$\bar{\theta}$	Std. Dev.	ASE
N = 10	1.400	0.019	0.016	1.400	0.010	0.010
N = 30	1.400	0.015	0.012	1.400	0.008	0.008

Note: $\bar{\theta} = \sum_{s=1}^{ns} \hat{\theta}_s$ and "Std. Dev." is defined as $\sqrt{\frac{1}{ns-1}\sum_{s=1}^{ns}(\hat{\theta}_s - \bar{\theta})}$. ASE is the average of standard errors for each simulation.

Table 3: Summary of the monopolization parameter estimates using FE specifications

	Model (i) Table 8	Model (ii) Table 10		Model (iii) Table 12		Model (iv) Table 14			
		C = 0	C = 1	C=0	C=1	C=0	C=0	C=1	C=1
N						L=0	L=1	L=0	L=1
5	1.31	1.51	0.71	–	0.40	–	–	0.44	0.54
6				1.46	0.55	0.91	–	0.65	0.66
7				0.68	0.64	–	0.93	0.75	0.82
8				1.36	0.81	1.42	–	0.92	1.17

Note: N, C, and L denote the number of firms, the cartel indicator, and the Great Lake operation dummy, respectively. Parameter estimates for the cases (N=6,C=0,L=1) and (N=8,C=0,L=1) are not statistically significant at any reasonable significance levels and therefore are not reported.

Table 4: Estimated parameters versus price elasticity across regimes

regime	θ	% of Cournot	$p-$elasticity
N=5, C=1, L=0	2.29	0.44	-6.00
N=5, C=1, L=1	1.85	0.54	-6.34
N=6, C=0, L=0	1.10	0.91	-3.17
N=6, C=1, L=0	1.54	0.65	-4.31
N=6, C=1, L=1	1.51	0.66	-6.75
N=7, C=0, L=1	1.07	0.93	-4.52
N=7, C=1, L=0	1.33	0.75	-4.98
N=7, C=1, L=1	1.21	0.82	-5.23
N=8, C=0, L=0	0.70	1.42	-2.72
N=8, C=1, L=0	1.09	0.92	-3.28
N=8, C=1, L=1	0.85	1.17	-3.88
avg.	1.32	0.84	-4.65

Table 5: Estimation results for 9 selected periods satisfying PR assumption, 662 obs.

Regimes.	LE Specification			FE Specification			Cartel Index
	1^{st} Est.	2^{nd} Est.	% Reduction	1^{st} Est.	2^{nd} Est.	% Reduction	
θ_1 (N=6, 68-75)	1.548 (0.150)	1.531 (0.144)	0.65	1.534 (0.148)	1.522 (0.145)	0.66	0.71
θ_2 (N=6, 116-131)	1.447 (0.138)	1.418 (0.131)	0.71	1.423 (0.138)	1.394 (0.132)	0.72	1.00
θ_3 (N=6, 131-166)	1.650 (0.165)	1.616 (0.156)	0.62	1.630 (0.167)	1.600 (0.160)	0.63	0.97
θ_4 (N=7, 171-181, 324)	1.583 (0.168)	1.545 (0.159)	0.65	1.572 (0.171)	1.536 (0.163)	0.65	0.83
θ_5 (N=8, 184-189)	1.694 (0.185)	1.651 (0.174)	0.61	1.672 (0.187)	1.632 (0.178)	0.61	1.00
θ_6 (N=8, 191-196)	1.025 (0.058)	1.013 (0.056)	0.99	1.016 (0.058)	1.004 (0.056)	1.00	0.67
θ_7 (N=8, 254-259)	1.200 (0.070)	1.184 (0.067)	0.84	1.186 (0.072)	1.170 (0.069)	0.85	0.83
θ_8 (N=8, 258-263)	1.109 (0.063)	1.074 (0.058)	0.93	1.091 (0.065)	1.051 (0.061)	0.95	0.83
θ_9 (N=8, 313-318)	1.212 (0.127)	1.231 (0.128)	0.81	1.192 (0.128)	1.220 (0.130)	0.82	0.67

Note: 1^{st} and 2^{nd} Est. report 1st and 2nd stage GMM esimates. % Reduction demonstrates the how much firms reduce their output compared to Cournot outcomes. Cartel Index is calculated the average value of the cartel indicator during the sample periods.

Figure 1: Upper and low bounds on the total output by week (top panel) and by regime (bottom panel) for estimates with firm FE's (Table 13)

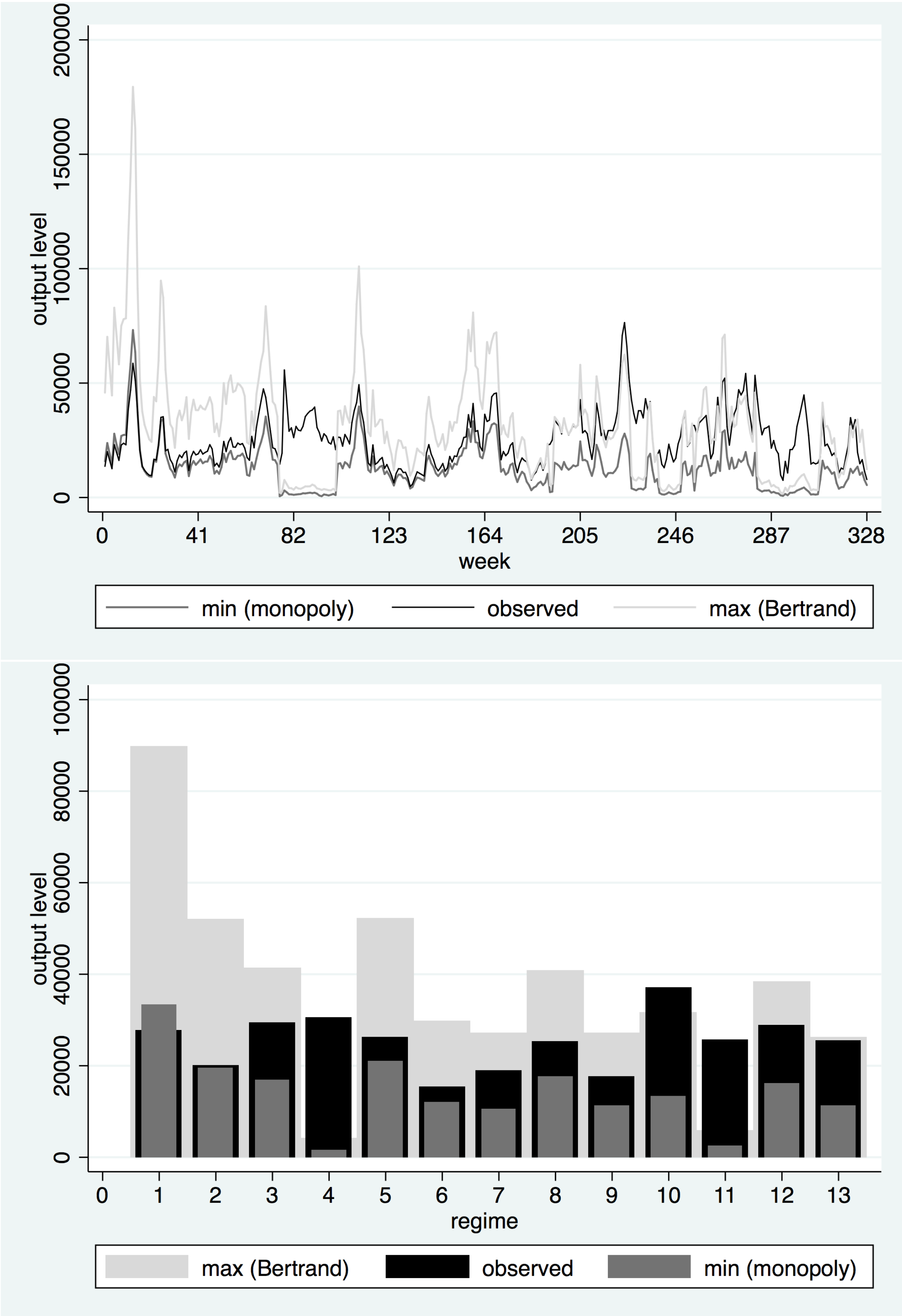
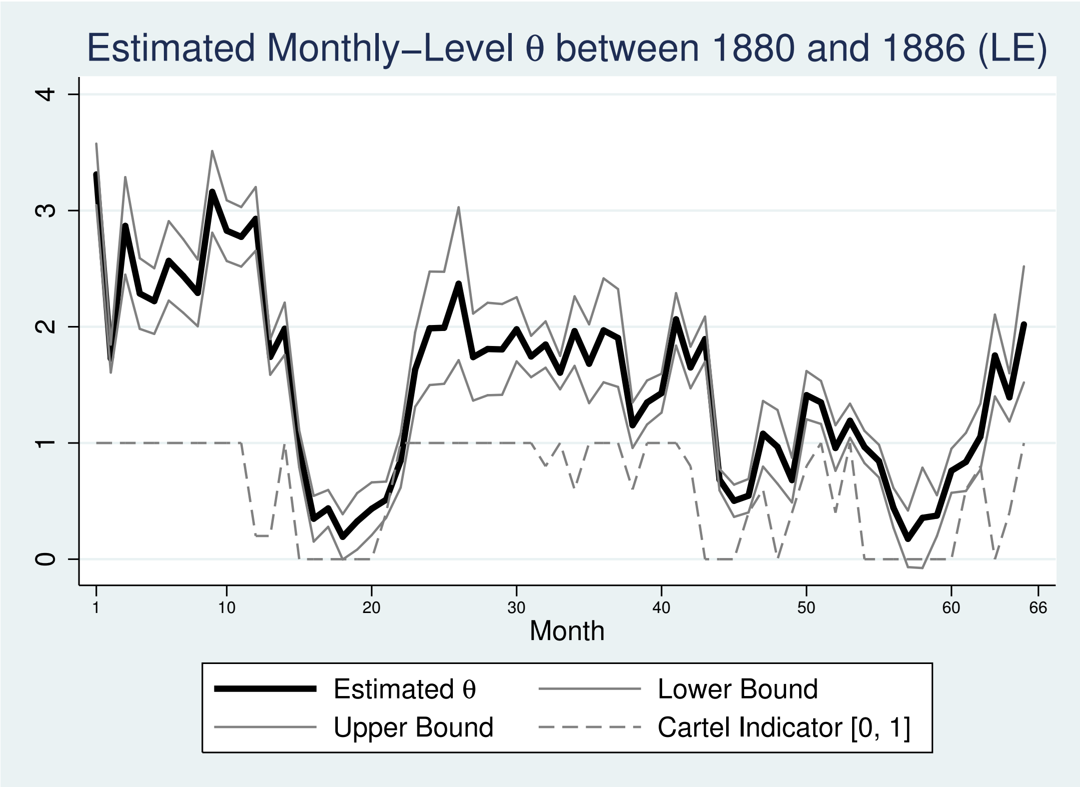


Figure 2: Monthly Level θ



Appendix A Estimation results

Table 6: Constant conduct parameter, $mc_i = \beta_0 + \beta_1 z_{it} + \nu_{it}$ (levels)

param.	1^{st} stage	%Cournot	2^{nd} stage.	%Cournot	cont.-update	%Cournot
α_0	35847.846		36107.822		36100.903	
	(1846.092)		(1851.162)		(1851.018)	
α_1	-0.294		-0.302		-0.302	
	(0.052)		(0.052)		(0.052)	
α_2	-6510.624		-6604.251		-6601.853	
	(893.286)		(895.631)		(895.564)	
θ	0.677	1.48	0.763	1.31	0.761	1.31
	(0.078)		(0.069)		(0.069)	
β_0	25472.798		24388.318		24333.122	
	(452.696)		(461.001)		(461.278)	
β_1	0.381		0.371		0.367	
	(0.160)		(0.162)		(0.162)	
$f-val$	2794.6981		277.2121		268.8319	

Table 7: Constant conduct parameter, $mc_i = \beta_{0i} + \beta_1 z_{it} + \nu_{it}$ (FE)

param.	1^{st} stage	%Cournot	2^{nd} stage.	%Cournot	cont.-update	%Cournot
α_0	35856.839		36429.472		36413.757	
	(1846.005)		(1857.807)		(1857.448)	
α_1	-0.294		-0.311		-0.311	
	(0.052)		(.052)		(0.052)	
α_2	-6512.883		-6719.050		-6713.762	
	(893.302)		(898.722)		(898.556)	
θ	0.666	1.50	0.749	1.34	0.722	1.39
	(0.083)		(0.074)		(0.077)	
β_1	0.496		0.587		0.815	
	(0.216)		(0.224)		(0.238)	
$f-val$	2735.5657		281.4734		271.1445	

Table 8: Regimes defined by the cartel indicator only, $mc_i = \beta_0 + \beta_1 z_{it} + \nu_{it}$ (levels)

param.	1^{st} stage	%Cournot	2^{nd} stage.	%Cournot	cont.-update	%Cournot
α_0	35852.778		36396.104		36374.390	
	(1845.908)		(1857.047)		(1856.558)	
α_1	-0.294		-0.310		-0.310	
	(0.052)		(0.052)		(0.052)	
α_2	-6511.863		-6707.138		-6699.672	
	(893.259)		(898.378)		(898.154)	
θ_0 (C=0)	0.637	1.57	0.663	1.51	0.659	1.52
	(0.076)		(0.070)		(0.071)	
θ_1 (C=1)	1.398	0.72	1.409	0.71	1.424	0.70
	(0.099)		(0.096)		(0.098)	
β_0	22669.676		21900.779		21762.925	
	(413.988)		(420.671)		(423.670)	
β_1	0.051		-0.016		-0.054	
	(0.146)		(0.147)		(0.147)	
$f-val$	1870.773		238.657		227.135	

Table 9: Regimes defined by the cartel indicator only, $mc_i = \beta_{0i} + \beta_1 z_{it} + \nu_{it}$ (FE)

param.	1^{st} stage	%Cournot	2^{nd} stage.	%Cournot	cont.-update	%Cournot
α_0	35855.274		36602.543		36597.780	
	(1845.726)		(1861.554)		(1861.425)	
α_1	-0.294		-0.316		-0.316	
	(0.052)		(0.052)		(0.052)	
α_2	-6512.490		-6780.817		-6779.655	
	(893.222)		(900.470)		(900.409)	
θ_0 (C=0)	0.625	1.60	0.670	1.49	0.659	1.52
	(0.078)		(0.069)		(0.070)	
θ_1 (C=1)	1.371	0.73	1.372	0.73	1.363	0.73
	(0.108)		(0.101)		(0.100)	
β_1	0.156		0.196		0.281	
	(0.193)		(0.197)		(0.202)	
$f-val$	1844.615		270.430		264.910	

Table 10: Regimes defined by N and cartel indicator, $mc_i = \beta_0 + \beta_1 z_{it} + \nu_{it}$ (levels)

param.	1^{st} stage	%Cournot	2^{nd} stage.	%Cournot	cont.-update	%Cournot
α_0	35834.452		35546.398		35545.746	
	(1845.755)		(1840.689)		(1840.681)	
α_1	-0.294		-0.285		-0.285	
	(0.052)		(0.052)		(0.052)	
α_2	-6507.259		-6403.890		-6403.764	
	(893.140)		(890.764)		(890.759)	
θ_1 (N=5, C=1)	2.550	0.39	2.346	0.43	2.304	0.43
	(0.322)		(0.291)		(0.283)	
θ_2 (N=6, C=0)	0.907	1.10	0.629	1.59	0.401	2.49
	(0.086)		(0.106)		(0.135)	
θ_3 (N=6, C=1)	2.023	0.49	1.840	0.54	1.817	0.55
	(0.208)		(0.183)		(0.180)	
θ_4 (N=7, C=0)	1.648	0.61	1.565	0.64	1.516	0.66
	(0.148)		(0.138)		(0.131)	
θ_5 (N=7, C=1)	1.686	0.59	1.651	0.61	1.619	0.62
	(0.145		(0.142)		(0.137)	
θ_6 (N=8, C=0)	0.876	1.14	0.772	1.30	0.743	1.35
	(0.048)		(0.057)		(0.061)	
θ_7 (N=8, C=1)	1.364	0.73	1.297	0.77	1.262	0.79
	(0.092)		(0.085)		(0.080)	
β_0	20294.021		20954.043		21236.366	
	(412.154)		(405.708)		(411.161)	
β_1	-0.321		-0.297		-0.263	
	(0.152)		(0.149)		(0.151)	
$f-val$	1014.420		163.301		164.366	

Table 11: Regimes defined by N and cartel indicator, $mc_i = \beta_{0i} + \beta_1 z_{it} + \nu_{it}$ (FE)

param.	1^{st} stage	%Cournot	2^{nd} stage.	%Cournot	cont.-update	%Cournot
α_0	35837.177		35657.267		35645.881	
	(1845.907)		(1842.674)		(1842.474)	
α_1	-0.294		-0.288		-0.288	
	(0.052)		(0.052)		(0.052)	
α_2	-6507.943		-6443.455		-6439.437	
	(893.192)		(891.680)		(891.586)	
θ_1 (N=5, C=1)	2.471	0.40	2.488	0.40	2.511	0.40
	(0.311)		(0.319)		(0.324)	
θ_2 (N=6, C=0)	0.817	1.22	0.687	1.46	0.589	1.70
	(0.088)		(0.096)		(0.105)	
θ_3 (N=6, C=1)	1.904	0.53	1.802	0.55	1.778	0.56
	(0.201)		(0.187)		(0.184)	
θ_4 (N=7, C=0)	1.546	0.65	1.467	0.68	1.431	0.70
	(0.144)		(0.134)		(0.130)	
θ_5 (N=7, C=1)	1.595	0.63	1.572	0.64	1.551	0.64
	(0.139)		(0.138)		(0.134)	
θ_6 (N=8, C=0)	0.799	1.25	0.738	1.36	0.723	1.38
	(0.057)		(0.062)		(0.063)	
θ_7 (N=8, C=1)	1.274	0.78	1.240	0.81	1.221	0.82
	(0.089)		(0.086)		(0.084)	
β_1	0.011		-0.003		0.006	
	(0.189)		(0.186)		(0.186)	
$f-val$	1003.630		203.199		216.512	

Table 12: Regimes defined by N, cartel indicator and state of demand, $mc_i = \beta_0 + \beta_1 z_{it} + \nu_{it}$ (levels)

param.	1^{st} stage	%	2^{nd} stage.	%	cont.-update	%
α_0	35839.113		35739.791		35741.825	
	(1845.186)		(1843.389)		(1843.426)	
α_1	-0.294		-0.291		-0.291	
	(0.052)		(0.052)		(0.052)	
α_2	-6508.430		-6472.906		-6473.643	
	(893.011)		(892.168)		(892.186)	
θ_1 (N=5, C=1, L=0)	2.257	0.44	2.348	0.43	2.354	0.42
	(0.279)		(0.298)		(0.299)	
θ_2 (N=5, C=1, L=1)	1.859	0.54	1.923	0.52	1.924	0.52
	(0.185)		(0.198)		(0.198)	
θ_3 (N=6, C=0, L=0)	1.114	0.90	1.155	0.87	1.155	0.87
	(0.114)		(0.117)		(0.117)	
θ_4 (N=6, C=0, L=1)	0.137	7.30	0.165	6.06	0.166	6.02
	(0.152)		(0.149)		(0.149)	
θ_5 (N=6, C=1, L=0)	1.574	0.64	1.616	0.62	1.617	0.62
	(0.132)		(0.140)		(0.140)	
θ_6 (N=6, C=1, L=1)	1.535	0.65	1.604	0.62	1.606	0.62
	(0.136)		(0.147)		(0.147)	
θ_7 (N=7, C=0, L=1)	1.091	0.92	1.143	0.87	1.145	0.87
	(0.077)		(0.082)		(0.082)	
θ_8 (N=7, C=1, L=0)	1.356	0.74	1.393	0.72	1.394	0.72
	(0.088)		(0.094)		(0.094)	
θ_9 (N=7, C=1, L=1)	1.231	0.81	1.284	0.78	1.286	0.78
	(0.100)		(0.106)		(0.106)	
θ_{10} (N=8, C=0, L=0)	0.736	1.36	0.752	1.33	0.752	1.33
	(0.057)		(0.056)		(0.056)	
θ_{11} (N=8, C=0, L=1)	0.211	4.74	0.240	4.17	0.242	4.13
	(0.141)		(0.138)		(0.137)	
θ_{12} (N=8, C=1, L=0)	1.125	0.89	1.154	0.87	1.155	0.87
	(0.062)		(0.065)		(0.065)	
θ_{13} (N=8, C=1, L=1)	0.869	1.15	0.904	1.11	0.905	1.10
	(0.057)		(0.056)		(0.056)	
β_0	23810.582		23529.588		23519.281	
	(409.084)		(410.955)		(411.084)	
β_1	-0.294		-0.338		-0.339	
	(0.130)		(0.132)		(0.132)	
$f - val$	8.251		3.376		3.311	

Table 13: Regimes defined by N, cartel indicator and state of demand, $mc_i = \beta_{i0} + \beta_1 z_{it} + \nu_{it}$ (FE)

param.	1^{st} stage	%	2^{nd} stage.	%	cont.-update	%
α_0	35839.791		35831.157		35831.293	
	(1845.995)		(1845.835)		(1845.838)	
α_1	-0.294		-0.294		-0.294	
	(0.052)		(0.052)		(0.052)	
α_2	-6508.600		-6505.512		-6505.561	
	(893.227)		(893.152)		(893.153)	
θ_1 (N=5, C=1, L=0)	2.212	0.45	2.287	0.44	2.294	0.44
	(0.274)		(0.288)		(0.290)	
θ_2 (N=5, C=1, L=1)	1.800	0.56	1.847	0.54	1.849	0.54
	(0.184)		(0.192)		(0.193)	
θ_3 (N=6, C=0, L=0)	1.056	0.95	1.095	0.91	1.094	0.91
	(0.112)		(0.114)		(0.114)	
θ_4 (N=6, C=0, L=1)	0.089	11.24	0.118	8.47	0.119	8.40
	(0.155)		(0.151)		(0.150)	
θ_5 (N=6, C=1, L=0)	1.508	0.66	1.540	0.65	1.541	0.65
	(0.135)		(0.140)		(0.140)	
θ_6 (N=6, C=1, L=1)	1.456	0.69	1.509	0.66	1.511	0.66
	(0.140)		(0.147)		(0.147)	
θ_7 (N=7, C=0, L=1)	1.033	0.97	1.074	0.93	1.075	0.93
	(0.082)		(0.085)		(0.085)	
θ_8 (N=7, C=1, L=0)	1.306	0.77	1.332	0.75	1.333	0.75
	(0.090)		(0.094)		(0.094)	
θ_9 (N=7, C=1, L=1)	1.173	0.85	1.213	0.82	1.215	0.82
	(0.103)		(0.107)		(0.107)	
θ_{10} (N=8, C=0, L=0)	0.690	1.45	0.704	1.42	0.705	1.42
	(0.062)		(0.061)		(0.061)	
θ_{11} (N=8, C=0, L=1)	0.166	6.02	0.195	5.13	0.197	5.08
	(0.143)		(0.139)		(0.139)	
θ_{12} (N=8, C=1, L=0)	1.068	0.94	1.090	0.92	1.091	0.92
	(0.067)		(0.069)		(0.069)	
θ_{13} (N=8, C=1, L=1)	0.823	1.22	0.852	1.17	0.853	1.17
	(0.060)		(0.060)		(0.060)	
c_1	-0.106		-0.143		-0.143	
	(0.163)		(0.164)		(0.164)	
$f-val$	6.659		2.842		2.798	

Table 14: Reduced sample, $mc_i = \beta_0 + \beta_1 z_{it} + \nu_{it}$

param.	1^{st} stage	%	2^{nd} stage.	%	cont.-update	%
α_0	33652.344		33480.469		33485.943	
	(2085.344)		(2084.071)		(2084.114)	
α_1	-0.220		-0.215		-0.215	
	(0.061)		(0.061)		(0.061)	
α_2	-3198.509		-3125.633		-3128.026	
	(1050.390)		(1048.918)		(1048.963)	
θ_1 (N=5, C=1, L=0)	2.678	0.37	2.846	0.35	2.853	0.35
	(0.525)		(0.585)		(0.586)	
θ_2 (N=5, C=1, L=1)	2.122	0.47	2.265	0.44	2.265	0.44
	(0.349)		(0.398)		(0.397)	
θ_3 (N=6, C=0, L=0)	1.139	0.88	1.221	0.82	1.221	0.82
	(0.160)		(0.174)		(0.174)	
θ_4 (N=6, C=1, L=0)	1.747	0.57	1.842	0.54	1.843	0.54
	(0.243)		(0.274)		(0.274)	
θ_5 (N=6, C=1, L=1)	1.661	0.60	1.801	0.56	1.803	0.55
	(0.235)		(0.274)		(0.274)	
θ_6 (N=7, C=0, L=1)	1.080	0.93	1.191	0.84	1.193	0.84
	(0.110)		(0.127)		(0.127)	
θ_7 (N=7, C=1, L=0)	1.463	0.68	1.542	0.65	1.543	0.65
	(0.159)		(0.184)		(0.184)	
θ_8 (N=7, C=1, L=1)	1.265	0.79	1.378	0.73	1.379	0.73
	(0.151)		(0.176)		(0.176)	
θ_9 (N=8, C=0, L=0)	0.639	1.56	0.676	1.48	0.677	1.48
	(0.106)		(0.100)		(0.100)	
θ_{10} (N=8, C=1, L=0)	1.145	0.87	1.212	0.83	1.213	0.82
	(0.093)		(0.108)		(0.108)	
θ_{11} (N=8, C=1, L=1)	0.804	1.24	0.874	1.14	0.875	1.14
	(0.091)		(0.087)		(0.086)	
β_0	24175.641		23713.288		23705.834	
	(455.280)		(461.587)		(461.803)	
β_1	-0.249		-0.291		-0.291	
	(0.139)		(0.142)		(0.142)	
$f-val$	9.710		3.550		3.439	

Table 15: Reduced sample, $mc_i = \beta_{i0} + \beta_1 z_{it} + \nu_{it}$

param.	1^{st} stage	%	2^{nd} stage.	%	cont.-update	%
α_0	33653.329		33664.271		33663.994	
	(2087.356)		(2087.450)		(2087.447)	
α_1	-0.220		-0.220		-0.220	
	(0.061)		(0.061)		(0.061)	
α_2	-3198.860		-3203.499		-3203.382	
	(1051.114)		(1051.213)		(1051.210)	
θ_1 (N=5, C=1, L=0)	2.619	0.38	2.734	0.37	2.744	0.36
	(0.517)		(0.549)		(0.551)	
θ_2 (N=5, C=1, L=1)	2.044	0.49	2.129	0.47	2.130	0.47
	(0.345)		(0.368)		(0.368)	
θ_3 (N=6, C=0, L=0)	1.061	0.94	1.134	0.88	1.131	0.88
	(0.155)		(0.163)		(0.163)	
θ_4 (N=6, C=1, L=0)	1.658	0.60	1.716	0.58	1.717	0.58
	(0.241)		(0.255)		(0.256)	
θ_5 (N=6, C=1, L=1)	1.559	0.64	1.645	0.61	1.647	0.61
	(0.236)		(0.255)		(0.256)	
θ_6 (N=7, C=0, L=1)	0.995	1.01	1.071	0.93	1.072	0.93
	(0.119)		(0.126)		(0.127)	
θ_7 (N=7, C=1, L=0)	1.391	0.72	1.435	0.70	1.436	0.70
	(0.158)		(0.169)		(0.169)	
θ_8 (N=7, C=1, L=1)	1.180	0.85	1.253	0.80	1.255	0.80
	(0.155)		(0.166)		(0.167)	
θ_9 (N=8, C=0, L=0)	0.573	1.75	0.602	1.66	0.602	1.66
	(0.117)		(0.111)		(0.111)	
θ_{10} (N=8, C=1, L=0)	1.062	0.94	1.103	0.91	1.104	0.91
	(0.099)		(0.105)		(0.105)	
θ_{11} (N=8, C=1, L=1)	0.735	1.36	0.786	1.27	0.787	1.27
	(0.100)		(0.095)		(0.095)	
β_1	-0.056		-0.074		-0.075	
	(0.175)		(0.177)		(0.177)	
$f - val$	6.764		2.630		2.564	

Table 16: Alternative specifications for marginal cost function, GMM 2nd stage

param.	$mc_{it} = \beta_{i0} + \beta_1 z_{it} + \beta_2 q_{it}$		$mc_{it} = \beta_{i0} + \beta_1 z_{it} + (\beta_2 + 1) q_{it}^{\beta_2}$	
	coef/s.e.	%	coef/s.e.	%
α_0	35805.207		35419.501	
	(1845.260)		(1838.694)	
α_1	-0.293		-0.281	
	(0.052)		(0.052)	
α_2	-6509.235		-6370.224	
	(892.857)		(889.753)	
θ_1 (N=5, C=1, L=0)	2.609	0.38	2.117	0.47
	(1.430)		(1.118)	
θ_2 (N=5, C=1, L=1)	2.085	0.48	1.686	0.59
	(1.063)		(0.874)	
θ_3 (N=6, C=0, L=0)	1.193	0.84	0.981	1.02
	(0.470)		(0.427)	
θ_4 (N=6, C=0, L=1)	0.101	9.90	0.074	13.51
	(0.175)		(0.148)	
θ_5 (N=6, C=1, L=0)	1.689	0.59	1.411	0.71
	(0.689)		(0.620)	
θ_6 (N=6, C=1, L=1)	1.649	0.61	1.372	0.73
	(0.642)		(0.601)	
θ_7 (N=7, C=0, L=1)	1.152	0.87	0.975	1.03
	(0.362)		(0.366)	
θ_8 (N=7, C=1, L=0)	1.443	0.69	1.228	0.81
	(0.501)		(0.478)	
θ_9 (N=7, C=1, L=1)	1.307	0.77	1.107	0.90
	(0.429)		(0.427)	
θ_{10} (N=8, C=0, L=0)	0.741	1.35	0.632	1.58
	(0.194)		(0.204)	
θ_{11} (N=8, C=0, L=1)	0.187	5.35	0.148	6.76
	(0.153)		(0.138)	
θ_{12} (N=8, C=1, L=0)	1.158	0.86	1.002	1.00
	(0.322)		(0.333)	
θ_{13} (N=8, C=1, L=1)	0.906	1.10	0.772	1.30
	(0.253)		(0.263)	
β_1	-0.014		-0.115	
	(0.371)		(0.343)	
β_2	-0.230		0.773	
	(0.880)		(0.507)	
$f - val$	3.735		2.339	

Figure 3: Upper and low bounds on the total output by week (top panel) and by regime (bottom panel) for estimates in levels (Table 12)

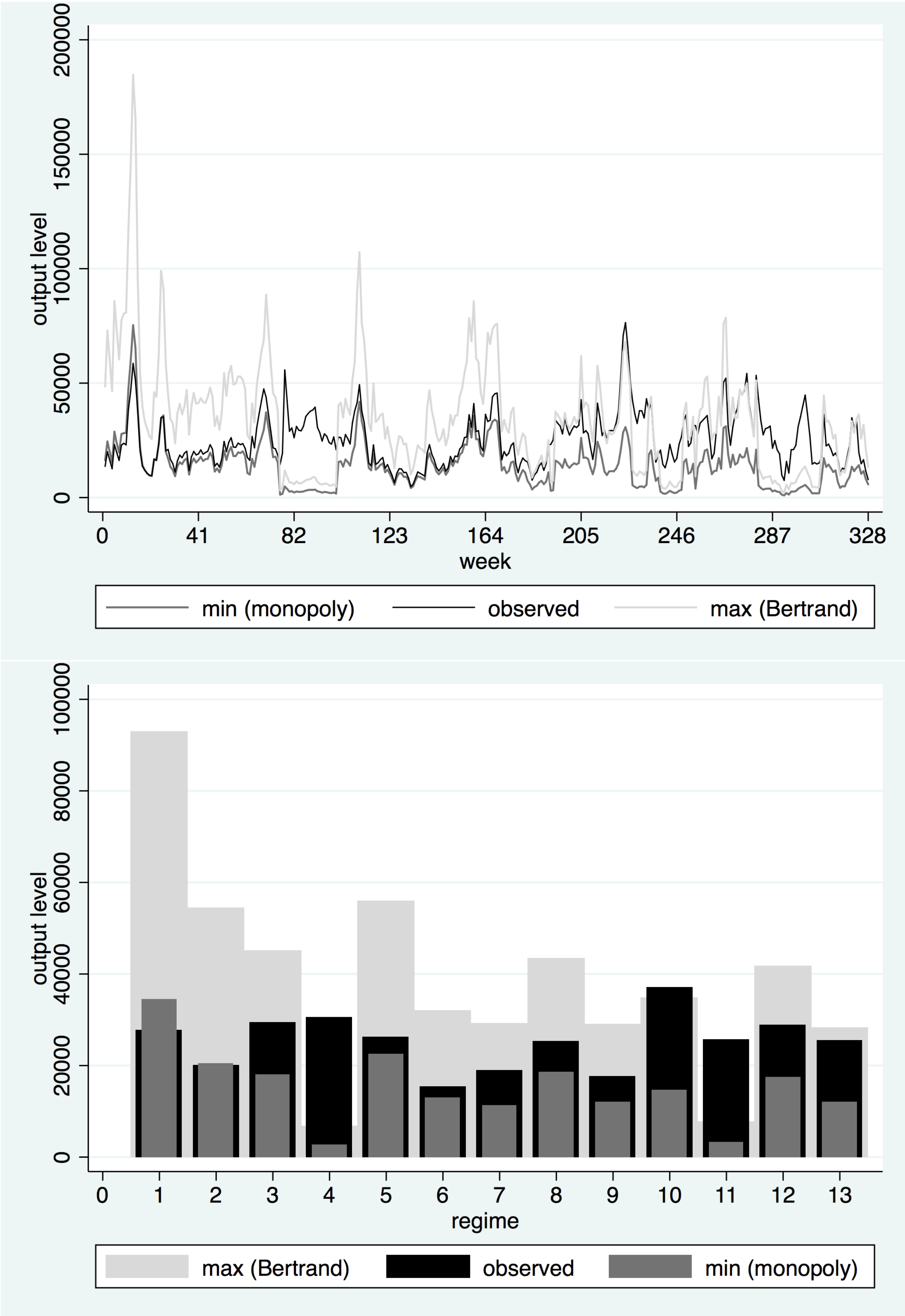


Table 17: Estimation results for 9 selected periods satisfying PR assumption, 662 obs.

param.	levels			firm fixed-effects		
	1^{st}	2^{nd}	%	1^{st}	2^{nd}	%
α_0	35329.948	35522.669		35326.973	35427.255	
	(2123.370)	(2130.429)		(2125.596)	(2129.231)	
α_1	-0.281	-0.286		-0.281	-0.284	
	(0.058)	(0.058)		(0.058)	(0.058)	
α_2	-6356.179	-6427.317		-6355.350	-6392.351	
	(1018.042)	(1020.952)		(1018.649)	(1020.149)	
θ_1 (N=6, 68-75)	1.548	1.531	0.65	1.534	1.522	0.66
	(0.150)	(0.144)		(0.148)	(0.145)	
θ_2 (N=6, 116-131)	1.447	1.418	0.71	1.423	1.394	0.72
	(0.138)	(0.131)		(0.138)	(0.132)	
θ_3 (N=6, 131-166)	1.650	1.616	0.62	1.630	1.600	0.63
	(0.165)	(0.156)		(0.167)	(0.160)	
θ_4 (N=7, 171-181, 324)	1.583	1.545	0.65	1.572	1.536	0.65
	(0.168)	(0.159)		(0.171)	(0.163)	
θ_5 (N=8, 184-189)	1.694	1.651	0.61	1.672	1.632	0.61
	(0.185)	(0.174)		(0.187)	(0.178)	
θ_6 (N=8, 191-196)	1.025	1.013	0.99	1.016	1.004	1.00
	(0.058)	(0.056)		(0.058)	(0.056)	
θ_7 (N=8, 254-259)	1.200	1.184	0.84	1.186	1.170	0.85
	(0.070)	(0.067)		(0.072)	(0.069)	
θ_8 (N=8, 258-263)	1.109	1.074	0.93	1.091	1.051	0.95
	(0.063)	(0.058)		(0.065)	(0.061)	
θ_9 (N=8, 313-318)	1.212	1.231	0.81	1.192	1.220	0.82
	(0.127)	(0.128)		(0.128)	(0.130)	
β_0	22220.813	22305.381		-		
	(319.017)	(317.656)				
β_1	-0.229	-0.214		-0.170	-0.150	
	(0.104)	(0.104)		(0.128)	(0.127)	
$f-val$	37.892	3.158		46.627	4.106	

Appendix B Profitability of PR collusive technology

Lemma 1 *Proportional reduction collusive technology is profitable for all firms in the neighborhood of Cournot equilibrium.*

Proof A Cournot competitor first-order conditions are given by

$$P'(Q_t)q_{it} + P(Q_t) - C_i'(q_{it}) = 0.$$

Consider a cartel, which sets overall industry output to $\bar{Q}_t = Q_t^{Cournot}$ and assigns market shares such that $\bar{Q}_t s_{it} = q_{it}^{Cournot}, \forall i = 1, \dots, n$, where s_{it} is market share of firm i in period t. Then, profit of a cartel member is given by $\pi^m(s_{it}, \bar{Q}_t) = P(\bar{Q}_t)\bar{Q}_t s_{it} - C_i(\bar{Q}_t s_{it})$ and, by construction, is identical to the non-cooperative Cournot outcome.

Consider a derivative of this profit function with respect to $\bar{Q}_t$,

$$\begin{aligned}
\frac{\partial \pi^m(s_{it}, \bar{Q}_t)}{\partial \bar{Q}_t} &= P'(\bar{Q}_t)\bar{Q}_t s_{it} + P(\bar{Q}_t)s_{it} - C_i'(\bar{Q}_t s_{it})s_{it} \\
&= C_i'(q_{it}) - P(\bar{Q}_t) + P(\bar{Q}_t)s_{it} - C_i'(\bar{Q}_t s_{it})s_{it} \\
&= (1 - s_{it})\left(C_i'(q_{it}) - P(\bar{Q}_t)\right) < 0,
\end{aligned}$$

where the second equality is obtained by replacing $P'(\bar{Q}_t)\bar{Q}_t s_{it}$ with $C_i'(q_{it}) - P(\bar{Q}_t)$ and the inequality follows from the fact that $C_i'(q_{it}) - P(\bar{Q}_t) = P'(Q_t)q_{it} < 0$. ∎

Appendix C MC simulations

The data generating process for our Monte-Carlo simulations is as follows. We assume the following inverse demand and cost functions,

$$P_t = \alpha_0 + \alpha_1 Q_t + \alpha_2 Y_t + \nu_t^d,$$
$$mc_i(q_{it}, z_{it}) = \beta_0 + \beta_1 z_{it} + \nu_{it}^s.$$

Table 18 summarizes parameter values and the distribution of the variables. We simulated data 10,000 times for each of the following combinations of (N, T): $(10,10)$, $(10,20)$, $(10,30)$, $(20,10)$, $(20,20)$, $(20,30)$, $(30,10)$, $(30,20)$, and $(30,30)$. Each time parameters were estimated using 2-step optimal GMM.

Table 18: Summary of parameter values for data-generating process in MC-simulations

parameter / variable	value / distribution
α_0	500
α_1	-1.0
α_2	1.0
β_0	10.0
β_1	1.0
Y_t	N(0,100)
ν_t^d	N(0,1)
z_{it}	N(1,4)
ν_{it}^s	N(0,0.04)
θ	$\{1.0, 1.2, 1.4\}$

We present summary statistic for a typical data set generated for $N = 30, T = 30$ in Table 19.

Table 19: Summary statistics for simulated data, N=30, T=30.

variable	mean	p50	min	max	sd
q_{it}	13.397	13.306	7.688	21.319	2.369
Q_t	401.918	396.420	331.500	488.790	51.116
P_t	100.003	106.385	26.574	166.800	49.782
Y	1.944	1.789	-16.740	26.532	9.000
z_{it}	1.102	1.107	-4.765	7.932	1.969
z_{-it}	31.967	29.714	12.246	58.177	10.174
regime 1	0.267	0.000	0.000	1.000	0.442
regime 2	0.467	0.000	0.000	1.000	0.499
regime 3	0.267	0.000	0.000	1.000	0.442
Y_t	1.944	1.789	-16.740	26.532	9.000

Table 20: Summary statistic for MC simulations.

var	N,T	coefficient mean	p50	sd	min	max	standard error mean	p50	sd	min	max
	10,10	500.049	499.810	13.858	358.310	668.440	11.501	9.266	11.525	0.453	507.380
	20,10	499.957	500.010	12.857	381.650	599.580	11.000	9.175	7.517	0.611	85.738
	30,10	500.167	500.080	12.979	356.070	591.330	10.973	9.143	7.527	0.654	73.862
	10,20	499.926	499.950	9.286	454.870	556.630	8.809	8.766	3.748	1.123	83.080
α_0	20,20	500.108	499.965	9.426	453.780	551.090	8.778	8.850	3.563	1.163	39.685
	30,20	500.098	500.050	9.148	453.710	553.730	8.714	8.806	3.528	1.150	26.701
	10,30	499.959	499.930	7.864	453.770	606.620	7.677	7.780	6.317	1.373	591.800
	20,30	499.968	499.950	7.716	458.510	537.970	7.536	7.702	2.344	1.395	16.097
	30,30	500.026	499.960	7.621	464.530	536.820	7.441	7.644	2.307	1.533	17.287
	10,10	-1.000	-1.000	0.037	-1.407	-0.625	0.031	0.025	0.030	0.001	1.318
	20,10	-1.000	-1.000	0.033	-1.253	-0.688	0.028	0.023	0.019	0.002	0.220
	30,10	-1.000	-1.000	0.032	-1.212	-0.615	0.027	0.023	0.019	0.002	0.181
	10,20	-1.000	-1.000	0.025	-1.155	-0.879	0.023	0.023	0.010	0.003	0.217
α_1	20,20	-1.000	-1.000	0.024	-1.125	-0.877	0.022	0.022	0.009	0.003	0.109
	30,20	-1.000	-1.000	0.023	-1.132	-0.884	0.022	0.022	0.009	0.003	0.066
	10,30	-1.000	-1.000	0.021	-1.272	-0.877	0.020	0.021	0.016	0.004	1.512
	20,30	-1.000	-1.000	0.020	-1.097	-0.898	0.019	0.020	0.006	0.004	0.040
	30,30	-1.000	-1.000	0.019	-1.094	-0.909	0.019	0.019	0.006	0.004	0.043
	10,10	1.001	1.001	0.087	-0.359	2.032	0.071	0.049	0.083	0.004	2.580
	20,10	1.000	0.999	0.078	0.054	1.929	0.067	0.048	0.057	0.005	0.642
	30,10	1.001	1.001	0.079	0.034	1.985	0.067	0.049	0.057	0.004	0.630
	10,20	1.000	1.000	0.042	0.650	1.316	0.041	0.034	0.023	0.006	0.262
α_2	20,20	1.000	1.000	0.041	0.699	1.314	0.041	0.035	0.023	0.007	0.278
	30,20	1.000	0.999	0.041	0.716	1.303	0.040	0.035	0.022	0.008	0.291
	10,30	1.000	1.000	0.031	0.783	1.201	0.031	0.028	0.015	0.008	0.236
	20,30	1.000	1.000	0.031	0.774	1.204	0.031	0.028	0.014	0.007	0.139
	30,30	1.000	1.000	0.031	0.782	1.306	0.031	0.028	0.014	0.008	0.154
	10,10	1.000	1.000	0.005	0.956	1.166	0.004	0.004	0.008	0.001	0.700
	20,10	1.000	1.000	0.003	0.975	1.023	0.002	0.002	0.001	0.001	0.017
	30,10	1.000	1.000	0.002	0.990	1.023	0.002	0.002	0.001	0.001	0.011
	10,20	1.000	1.000	0.003	0.988	1.016	0.003	0.003	0.001	0.001	0.016
θ_1	20,20	1.000	1.000	0.002	0.993	1.008	0.002	0.002	0.000	0.001	0.006
	30,20	1.000	1.000	0.001	0.993	1.007	0.001	0.001	0.000	0.001	0.003
	10,30	1.000	1.000	0.002	0.983	1.014	0.002	0.002	0.001	0.001	0.074
	20,30	1.000	1.000	0.001	0.994	1.007	0.001	0.001	0.000	0.001	0.003
	30,30	1.000	1.000	0.001	0.996	1.005	0.001	0.001	0.000	0.001	0.002
	10,10	1.200	1.200	0.012	1.097	1.447	0.010	0.008	0.014	0.002	1.023
	20,10	1.200	1.200	0.009	1.155	1.314	0.008	0.006	0.005	0.001	0.071
	30,10	1.200	1.200	0.008	1.159	1.349	0.007	0.006	0.004	0.001	0.072
	10,20	1.200	1.200	0.008	1.161	1.242	0.007	0.007	0.003	0.002	0.053
θ_2	20,20	1.200	1.200	0.006	1.172	1.234	0.006	0.006	0.002	0.001	0.024
	30,20	1.200	1.200	0.005	1.172	1.231	0.005	0.005	0.002	0.001	0.015
	10,30	1.200	1.200	0.006	1.140	1.243	0.006	0.006	0.003	0.002	0.259
	20,30	1.200	1.200	0.005	1.178	1.227	0.005	0.005	0.001	0.001	0.011
	30,30	1.200	1.200	0.005	1.178	1.225	0.004	0.005	0.001	0.001	0.011
	10,10	1.401	1.400	0.019	1.235	1.723	0.016	0.013	0.020	0.002	1.325
	20,10	1.400	1.400	0.015	1.320	1.605	0.013	0.011	0.009	0.002	0.129
	30,10	1.400	1.400	0.015	1.324	1.675	0.012	0.010	0.008	0.002	0.134
	10,20	1.400	1.400	0.012	1.334	1.468	0.012	0.012	0.005	0.002	0.087
θ_3	20,20	1.400	1.400	0.011	1.349	1.462	0.010	0.010	0.004	0.002	0.042
	30,20	1.400	1.400	0.010	1.349	1.457	0.010	0.010	0.004	0.002	0.027
	10,30	1.400	1.400	0.010	1.300	1.471	0.010	0.010	0.005	0.002	0.436
	20,30	1.400	1.400	0.009	1.361	1.449	0.009	0.009	0.003	0.002	0.019
	30,30	1.400	1.400	0.008	1.360	1.446	0.008	0.008	0.002	0.002	0.020
	10,10	9.941	9.990	2.006	-62.295	24.268	1.496	1.259	3.196	0.423	211.650
	20,10	10.024	10.015	1.066	2.599	22.927	0.973	0.889	0.385	0.371	7.414
	30,10	10.009	10.010	0.842	4.176	15.053	0.789	0.726	0.286	0.309	4.553
	10,20	9.994	9.996	0.843	5.929	15.393	0.810	0.781	0.182	0.396	2.352
β_0	20,20	9.996	10.002	0.581	7.405	12.843	0.567	0.550	0.116	0.288	1.467
	30,20	10.006	10.005	0.477	7.793	12.339	0.463	0.450	0.092	0.254	1.163
	10,30	10.005	10.007	0.646	7.419	12.924	0.624	0.610	0.139	0.361	9.947
	20,30	10.007	10.007	0.451	8.027	11.789	0.440	0.433	0.069	0.266	0.790
	30,30	10.003	10.004	0.365	8.433	11.529	0.359	0.354	0.054	0.219	0.691
	10,10	1.000	1.000	0.029	0.685	1.472	0.025	0.020	0.023	0.008	0.970
	20,10	1.000	1.000	0.027	0.766	1.194	0.024	0.019	0.014	0.006	0.184
	30,10	1.000	1.000	0.027	0.704	1.176	0.023	0.019	0.015	0.005	0.152
	10,20	1.000	1.000	0.019	0.912	1.111	0.018	0.018	0.007	0.006	0.168
β_1	20,20	1.000	1.000	0.019	0.902	1.095	0.018	0.018	0.007	0.005	0.081
	30,20	1.000	1.000	0.019	0.903	1.107	0.018	0.018	0.007	0.005	0.054
	10,30	1.000	1.000	0.016	0.908	1.216	0.016	0.016	0.013	0.006	1.215
	20,30	1.000	1.000	0.016	0.915	1.079	0.015	0.016	0.004	0.005	0.033
	30,30	1.000	1.000	0.016	0.928	1.080	0.015	0.015	0.004	0.005	0.035

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