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Woodwind Instrument Bore Measurement

GAINING insight into the practices of a school of instrument making which has not survived to the present day is largely dependent on the degree to which its surviving instruments can be evaluated. This almost invariably includes consideration of the physical dimensions of the objects being studied, both in purely stylistic terms and in the form of numerical data.[1] Information of the latter sort obtained from the bores of woodwinds is of major concern in the study of this type of instrument and absolutely vital to all work involving reconstruction (modern 'copying' and certain aspects of restoration). Typological research aimed at determining the essential characteristics of specific instrumental types remains on a rudimentary level with many historical woodwinds. This includes, for example, defining often used terms such as the 'classical oboe' or the 'baroque bassoon', determining the differences between the styles of different makers of a given instrument, establishing the relationship of an individual maker to a broader regional or national school, etc. In its ultimate, perhaps utopian form this will entail analytical comparison of all extant representatives of the instrument and/or school in question, which in turn will require the availability of directly comparable numerical data for each individual object.

From the above it becomes obvious that the methods of woodwind bore measurement and evaluation require careful consideration from anyone involved in woodwind research. It also follows that many workers have developed and used such techniques.[2] The basic problems underlying the subject have not, however, been treated in the literature in more than a cursory fashion, and then only in regard to the problems of reconstruction. It is the purpose of this paper to discuss some of these problems and to encourage the development of uniform procedures for obtaining and recording the measurements of woodwind instrument bores.

The traditional method of boring a woodwind instrument involves the use of a number of reamers, each with a relatively simple shape.[3] When taken compositely these often give the finished bore a highly complex profile. For a conically-bored instrument on the most detailed level a series of spoon bits (or similar tools) is used to make a stepped cylindrical pilot hole which is then reamed out with a conical reamer. The conical bore is subsequently modified at various points with other conical reamers and the tips of spoon bits, leaving the finished bore with very few 'straight' surfaces. The final series of adjustments is (or perhaps was) traditionally a part of the tuning and voicing procedure (Appendix I) and the bore measurements of two externally and musically identical instruments from the same maker can often be astonishingly different. (This does not imply that very similar, perhaps 'mass-produced' instruments are not to be found.) The profiles of each individual tool used in finishing the bore are, however, usually apparent.

Ideally an examination of a large enough sample of instruments from one maker should reveal the dimensions of his entire battery of tools. However, since reamers change their dimensions from usage and sharpening this ideal may be difficult to realize. Our knowledge in this area can, despite this, be substantially improved.

An immediate goal of bore measuring thus becomes the determination of the dimensions of the individual reamers, with an ultimate goal being the use of information gathered from a large number of instruments to determine the logic behind the use of these tools (deducing the equivalent of Appendix I for other makers). A major difficulty in attaining the first of these goals lies in the fact that the dimensions of a wooden object are affected by its moisture content. Subsequently, the dimensions of the bore of a woodwind instrument which has been played and/or stored under climatic conditions different from those under which it was made, will not be identical to those of the corresponding reamers. The most common manifestation of this is shrinkage, which due to the anisotropic properties of wood[4] causes an elliptical instead of circular bore cross section. The degree of ellipticity can, in turn, be of use in extrapolating the pre-shrinkage (i.e. original) diameter. The task at hand reduces itself first to determining the bore profile at the time of measurement and then to determining how the original dimensions of the bore may have differed from this.

It will be useful to describe a bore in terms of an x-y coordinate system where the length of the instrument is the abscissa and the diameter of the bore is the ordinate.[5] From the preceding paragraph one would expect to find two y-values (the major and minor axes of the elliptical cross section; D_M and D_m) for every x (point along the length of the instrument, or joint under consideration; L). Depending on the measuring device used, either D_M and D_m will be measured for each L, or two values for L will be found for any given diameter (D). The net effects of both approaches are identical, although the former more readily reveals the degree of anisotropic shrinkage and the latter tends with simpler tools to give a greater degree of accuracy. No general rule can be given for determining the minimum number of points necessary for adequate bore description, but 0.1 mm increments in D and 1.0 mm increments in L should prove a totally adequate upper limit. Relatively even spacing of both L and D values is desirable. The tolerances in the measurements should be indicated (Appendix 2). Values obtained near fingerholes and other local irregularities should be specially recorded.

The data points are plotted on a graph with the D-scale several times that of the L-scale. (It is helpful to adjust the L and D scales to fill the long and short sides of a rectangular sheet respectively, with the bore on the diagonal.) On examining the graph it will be seen that the data points can be divided into several intervals, each of which can be approximated by a single curve or straight line (both will be referred to as curves). With the exception of mechanically damaged areas each interval should correspond to one of the reamers used in finishing the bore. Comparison of the graph with the tool marks on the bore surface can be used to confirm this and, if necessary, rectify the divisions on the graph.

The array of data points on each interval must be fitted with a curve. The most obvious means for doing this is freehand drawing. This has, however, the disadvantage of being dependent on individual judgement: different workers are therefore not likely to fit identical curves to the same data. It is, however, often satisfactory for workshop use. An alternative method is to assume a functional relationship between L and D and determine a mathematical equation by which this relationship can be described. This procedure is far less subjective than the freehand technique and permits quantification of the 'goodness' of the fit of the curve. Its weaknesses are primarily those inherent in the selected mathematical procedure (Appendix 3), and that accurate functional representations of tools with extreme cutting lengths may be no more convenient to deal with than the initial tabular data. A computing machine with at least the power of a scientific pocket calculator is required if the method is to be of practical use (preferrably programmable for extensive work), but no inordinate mathematical skill other than the ability to use such a device is necessary. The primary value of this approach is its usefulness in reducing tabularly recorded data to a standardized numerical form prior to further analysis. It thus provides both a well-defined curve for workshop purposes and numerical data useful for large-scale comparative analysis of substantial numbers of instruments. As a documentational procedure it is to be preferred. (Determining the equations of freehand curves, although possible, will not be considered, as it is far less precise and no more convenient.)

Whatever method is selected, the curve must be fitted to all the data points. Merely connecting either all D_M or all D_m points will not describe the bore profile. The D_M and D_m curves do, however, form an envelope through the middle of which the desired curve passes. (For any L, this 'middle' is theoretically the geometric mean of D_M and D_m.) Having satisfactorily fitted a curve to the data points of each interval, the bore profile on that interval at the time of measurement has been established. The original profile of any interval can be described by multiplying this curve by a correction factor after compensation has been made for mechanical deformation such as cracking, compression shrinkage of tenons,[6] etc.

This compensation is best made before any curve is fitted. Individual points which clearly deviate from reasonable positions can either be excluded from consideration or adjusted to 'smooth' the data. If the source of such local deviation cannot be deduced from direct examination of the bore surface, however, extreme caution should be exercised when doing this. If the cause is local damage, fingerholes, uneven junctions between reamers, or similar factors, this is usually obvious and no great difficulty ensues. Adjustments made to the bore by various scraping tools (as opposed to reamers) result in uneven data, but this usually occurs on clearly defined intervals which if treated individually cause no particular trouble. Compression-shrunk tenons cause smooth but implausible data which cannot necessarily be satisfactorily adjusted. If it can be assumed that the tenon bore was originally finished with a reamer that extended past the distorted area, the curve representing that reamer can be extended from the reliable area into the tenon and the distorted data ignored. If this results in an unacceptable fit between the extrapolated tenon bore and the bore in the adjacent joint, or would prevent the tenon from fitting its socket, a satisfactory value must be assumed for the end of the tenon and the curve adjusted accordingly. In other cases no general procedure can be recommended. Cracking is a source of serious difficulty. Closed cracks can usually be ignored, but open cracks, even if closed on the bore surface, can distort the data in a manner which is not necessarily obvious. As with all outright physical damage this results in a situation which resists systematic treatment. The subject is complex enough to warrant individual study and will not be dealt with further here.

The entire procedure to this point has been a fairly straightforward exercise in curve fitting. The remaining task is the adjustment of the set of curves to compensate for the effects of shrinkage. (It is assumed that expansion will not be encountered in the present situation.) For L this can be ignored, but for D it is both important and extremely difficult to treat in a practically useful fashion. A theoretical discussion of the problem will be necessary for a fuller understanding of its difficulties.

There are two factors which can contribute to the ellipticity of a bore cross section. The first is response to mechanical forces which cause deformation without appreciable loss of cross-sectional area. Such forces occur as tensions in a billet are released during turning and boring. To avoid the worst effects of this these operations are generally conducted in steps over a period of time. Thus the deformation which occurs after the removal of a small amount of wood is itself removed in a subsequent stage of production. This factor therefore is rarely of practical concern. In such a case the circular diameter is the geometric mean of the major and minor axes of the ellipse and can easily be determined:

$$D_1 = \sqrt{D_M D_m} \qquad (1)$$

The second factor is anisotropic shrinkage, which clearly results in reduced cross-sectional area. If the ratio between radial and tangential shrinkage is known, the circular diameter can be determined by adding to the major axis of the ellipse an amount which is the product of a function R of the shrinkage ratio and the difference between the major and minor axes of the ellipse:

$$D_2 = D_M + R(D_M - D_m) \qquad (2)$$

where

$$R = \frac{\sin\theta}{\cos\theta - \sin\theta}$$

$\theta = \arctan r$

r = ratio of radial shrinkage to tangential shrinkage.

(See Appendix 4 for the derivation of Formula 2. See Table 1 for the r and R of certain timbers.)

An important consequence of this is that the closer r is to unity, the less obvious shrinkage will become in terms of ellipticity. For boxwood (*Buxus sempervirens*) r is usually given as 11:15, and an instrument made of this material can shrink considerably before its cross section becomes noticably elliptical. (This is probably one of the reasons for boxwood's popularity amongst turners.) Published values of r are, however, based on average figures for the radial and tangential shrinkage of the specified timber when brought from green to a pre-selected moisture content (usually 12%) and the actual r of the specimen under examination can deviate from this value.

The following must also be considered. If the growth rings of the tree from which it was made appear as nearly parallel lines on an instrument's radial cross section, the radial and tangential directions are clearly defined and r is a valid concept. If the rings appear more nearly as concentric circles (taken from the centre of the tree) r is undetermined and not usable. In the latter case moisture loss will not result in anisotropic shrinkage, but in cracking. (The angular magnitude of such a crack can be used instead of r in extrapolating pre-shrinkage dimensions but the practical use of any mathematical description of this process is too tenuous to be worth pursuing.) There is a gradual change between these extremes, but intermediate r-values cannot easily be interpolated. If r is defined, little more can be done than to assume the suitability of published data and modify it if necessary. If r is undefined, any further procedure dependent on its use cannot be undertaken.

The above discussion assumes that the cross section of an instrument behaves as if it consisted of solid wood. In reality the bore accounts for a specific part of the total cross-sectional area, and two conditions occur as this fraction increases. Initially the resistance of the mass of the wood to the constrictive force generated by moisture loss decreases. After a certain point, however, the amount of constrictive force starts to diminish. (There is less wood to generate it.) The first factor increases and the second decreases ellipticity. Therefore the shrinkage characteristics at a given cross section become those of solid wood only when the ratio between the bore and outside diameters closely approaches either zero (infinitely narrow bore) or unity (infinitely thin walls).

There are three basic practical approaches to the solution of the problems described in the preceding paragraphs. The first is to ignore the shrinkage properties of the wood altogether and adjust the bore diameter according to external factors. The most common of these factors is metal fittings, the diameters of which can be compared to the diameter of the instrument. The effects of compression shrinkage often make it difficult to use this approach, but if correctly applied it is highly accurate.

The second method is to determine how much the instrument has shrunk on the basis of the difference between its moisture contents at the time of fabrication (estimated) and the time of measurement. For a given timber the extent of shrinkage can be calculated on the basis of these figures. Moisture content can, however, only be determined accurately either by destructive testing (measuring weight loss during total oven drying) or if the instrument has been stored for a long enough period of time at a known constant relative humidity (e.g. ideal museum conditions) for its moisture content to be in equilibrium with the atmosphere. This approach involves too great an amount of guesswork to be reliable when used alone, but can help confirm results calculated by the other two methods.

The third approach is a graphic extrapolation of the instrument's shrinkage behaviour as it would be if it were made of solid wood. A series of radial cross sections with varying wall thicknesses is selected. Each is plotted as a single point with x being the ratio between the bore diameters as calculated by Formulas 2 and 1 respectively (D_2 and D_1), and y being the ratio between the bore and exterior diameters as calculated by Formula 1. These points will lie on two curves which intersect at the point representing the most elliptical cross section. One of these curves turns towards, and if extended will intercept the x-axis. The other curve turns away from the x-axis and is asymtotic to a line parallel to the y-axis with the same x-intercept as the first curve. This x-value is the correction factor by which the previously determined diameter 'at the time of measurement' must be multiplied to give the original D for any L. This method is probably the most generally useful of the three and is quite accurate when the dimensions of the instrument are such as to permit its application. (The main requirement being the availability of a 'wide variety' of cross sections which are not immediately adjacent to cross sections with substantially different dimensions.) If the necessary curves cannot conveniently be found the correction factor can usually be approximated by locating the cross section for which the ratio between D_2 and D_1 is a minimum (least elliptical cross section), and reducing the fractional part of this value by 10%.

All these methods assume that r will be the same along the entire length of the instrument or joint under consideration. This will generally be true if the joint is not of extreme length and the wood is of clear and even growth, but irregular grain, knots, damage, etc. can all have a disturbing effect. It must also be remembered that wood is, despite everything, a living material which does not always behave as conveniently as the above treatment requires. Thus, each of the three procedures will at best produce slightly different results. At worst, one or another of them will not 'work' at all. Weighted averages of the results (adjusting r if necessary to avoid 'impossible' data), allow, however, many instruments to be treated satisfactorily. Hopefully, the examination of large numbers of 'well-behaved' instruments will reveal information of a generally applicable nature which can then be used in the analysis of uncertain material.

A demonstration of the practical application of the methods described in this paper is not possible without reference to graphic material in a format larger than that of this JOURNAL. Rather than requiring that the reader individually plot tabular data the following discussion will therefore be restricted to a verbal description of some of the problems encountered in the analysis of a Haka oboe (Musikhistoriska Museet, cat. no. MM 155).[7]

The upper joint of this instrument was measured with 0.10 mm. increments in both D_M and D_m resulting in 90 data points. A graph of these points revealed six individual curves which corresponded exactly to the tool marks on the bore surface. Of these, the two on either side of the narrowest part of the bore appeared to have been made with the same reamer, and one short segment revealed what could have been the initial conical reamer used for the entire joint. Each individual curve could adequately be described by a third degree polynomial.

Thus five reamers were used for this joint. The wood itself had irregular grain but this affected the distance between the major and minor axes only on two limited areas and caused no difficulty. The tenon was compression shrunk but the reamer with which it was bored penetrated half the length of the joint, and the extension into the tenon of the reliable portion of its curve resulted in entirely plausible data. The upper end of the joint was turned in the late 17th century Dutch 'pseudo-pirouette' style and capped with a loose fitting brass rosette. A circular mark made on the wood by this rosette when it was first mounted had shrunk 1.4% in relation to the rosette itself. A similar rosette on the bell left a mark which had shrunk 1.3%. With reference to the previously mentioned fact that the shrinkage characteristics of the radial cross-section of a bored instrument become those of solid wood when the ratio between the bore and external diameters approaches either zero (as at the upper end) or unity (as at the bell), the 1.3%–1.4% values should lie near the effective shrinkage factor for the whole instrument (assuming that one single factor is adequate to describe this behaviour). The graphic determination of this shrinkage factor as previously described gave 1.6%.

At the time of measurement the instrument has been stored for at least four years in a constant 50% relative humidity atmosphere. Assuming that the moisture content of the wood was in equilibrium with the atmospheric RH, 1.5% shrinkage would indicate that the equilibrium atmospheric RH at the time of manufacture was *circa* 70%, which is an entirely plausible value.[8] Adopting 1.015 as the shrinkage correction factor thus appears justified, although it must be noted that the accuracy of this figure is considerably less than that of the initial measurements. The bore profile as measured is therefore better defined than the calculated original profile. To avoid inaccuracy as a result of this each reamer was represented in a standardized form by five numbers: the four coefficients of the cubic curve plus the correction factor.

The lower joint was measured in the same manner and provided 104 data points. Three distinct curves corresponding to tool marks on the bore surface were manifest in the data and these presumably represent three reamers. A portion of the bore had, however, been chambered with some type of scraping tool and it is possible that this obliterated a point of junction between two reamers. In addition, the minor axis of the bore was severely perturbed by a deeply set slot for the C key and lever causing loss of bore volume[9] which was apparently compensated for by the scraped chamber and, as is often the case, the tenon was compression shrunk. Thus the data was well-behaved only as far down as the key slot. (The initial dimensions of the bore were probably not musically satisfactory, but as the instrument now possesses excellent playing qualities the subsequent adjustment seems to have been successful.) The erratic portion of the minor axis could justifiably be replaced by a suitable extension of the remainder of that axis at an appropriate distance from the major axis. (The wood of this joint is clear-grained and both axes are largely 'parallel'.) The scraped portion of the bore presented both a problem in itself and prevented sufficiently objective treatment of the tenon bore. For purposes of comparative analysis this area of the bore should be given less statistical importance than more clearly defined areas. For purposes of reconstruction a reamer should probably be devised which both approximates the data as well as possible and provides the volume of the scraped chamber.

The bell bore consisted of one short reamed section and a free-turned curve, and was easily treated. The standardized numerical representation used for the upper joint was, with the above-mentioned qualifications, suitable for the other joints as well.

NOTES

1 See virtually any *GSJ* article dealing with the physical examination of instruments.

2 See, for example, Hailperin, *GSJ* 1975 and Marvin, *GSJ* 1972.

3 Such tools are illustrated in Bergeron's *Manuel du Tourneur*, see *GSJ* 1976, Pl. 1, and in Diderot's *Encyclopédie*, see *GSJ* 1977, Pl. XIX *a*.

4 If an end-grain section of wood is considered, shrinkage due to moisture loss will be markedly greater parallel to the growth rings than it will be perpendicular to them.

5 Marvin and Hailperin as per note 2.

6 If a piece of wood is wetted and mechanically constrained from swelling it will upon drying have smaller dimensions than it initially had. This process can be repeated and thread-lapped tenons on a much used woodwind instrument will often display considerable compression shrinkage.

7 This instrument is of interest both due to its high quality and the fact that it plays well at a pitch of $a'=440$ Hz. (It is presumably a D instrument pitched 'one tone low'.) The reader interested in following more closely the analysis of this instrument can obtain additional documentary material including a technical drawing and detailed measurements from the Musikhistoriska Museet in Stockholm.

8 This is based on very approximate figures. For a discussion of this and other basic problems of wood technology see, Brommelle and Werner, 'Deterioration and Treatment of Wood' in *Problems of Conservation in Museums*, ICOM 1969.

9 Much of what is said in this article implies that the bore volume of an older woodwind instrument is likely to be less than it was when the instrument was new. Inasmuch as the pitch of an instrument is dependent on the volume of its bore, shrinkage is of potentially profound musical consequence. There are, however, other 'pitch-altering' effects of aging and these often cancel each other out. The original pitch of a 'shrunken' woodwind can therefore not be extrapolated solely on the basis of bore-measurement analysis. I suspect despite this that generally accepted historical pitch levels as established on the basis of characteristics of surviving woodwinds may in light of future research need revision 'downwards'.

The subject of tolerance in measurement and mathematical curve fitting are briefly discussed in Appendices 2 and 3. The following literature may be of help to the reader interested in more detailed study of these matters:

M. R. Spiegel, *Theory and Problems of Statistics*, SI Edition, McGraw-Hill, New York 1972.

N. C. Barford, *Experimental Measurements: Precision, Error and Truth*, Addison-Wesley, London 1967.

J. Mandel, *The Statistical Analysis of Experimental Data*, John Wiley, New York 1964.

R. W. Hammig, *Numerical Methods for Scientists and Engineers*, Second Edition, McGraw-Hill, New York 1973.

APPENDIX 1 ON OBOE MAKING

The following is a translation of a text written by Karl F. Golde (d. 1873), a Dresden instrument maker specializing in oboes. It is a series of somewhat random thoughts and was apparently written at an important client's insistence. Golde's style is awkward and often obscure in meaning and the translation has therefore not been kept absolutely literal throughout. The text is part of a manuscript which was in the possession of the Gewerbemuseum in Markneukirchen until the 2nd world war and is now lost. The source used for this translation is a transcript of the original in an article in the *Zeitschrift für Instrumentenbau* 52 (1932), pp. 258–9, by F. Drechsel entitled 'Über den Bau der Oboe'. Much of Golde's material is not immediately relevant to the present article but is included for the sake of general interest. Basic for the purpose at hand are Golde's references both to a specific series of reamers used for boring an oboe and to an unspecified number of other reamers to be used for chambering the bore at various points. It should be noted that the criteria used in judging the successfulness of the bore are solely musical and that the absolute measurements of the reamers and the subsequent finished bore profile do not appear to be of any direct concern whatsoever.

'In oboe-making the bore, above all, must be precisely executed; the lower joint not being too narrow and the upper joint not too wide. The bore in both joints has a sword profile [*sackig oder gewölbt*—my use of the term 'sword profile' is a reluctant concession to common usage]. The ease of speech of the upper and lower registers and the beauty of the middle register depend on this. The upper and lower joints must be bored in the same proportion, the tone developing in the upper joint and radiating from the lower. To ease the speech of the lower register, the upper portion of the upper joint must not be too wide and the lower joint, between the C♯ hole and the middle F hole, must be adequately wide and chambered from below. A powerful low register and a full tone are thus attained. Instruments which do not have a sword profile (*nicht gewölbt gebohrt*) have a thin nasal tone, as that of the French and Viennese oboes.

'The choice of wood is very important. Clear knot-free boxwood, preferably soft than hard, is best suited. It gives a mild soft tone, whereas hard firm wood gives a hard tone. Hard wood can sooner be used for the upper joint than for the lower, as this is responsible for resonance and the tone becomes milder through the soft vibrations. With hard wood the vibrations are shorter and lighter and this is why many notes which naturally tend to be flat, as for example the middle

D, become more in tune when hard rather than soft wood is used. The middle D can, however, be sharpened by very slightly chambering (*Nachbohren mit gewölbtem Bohrer*) the upper joint between the C hole and a point just below the narrowest point in the bore. If this chamber becomes too large the middle D becomes unstable and tends to overblow. The lower D hole is then cautiously enlarged. The C hole on the bell can be of especially great help. The bore of the bell can also be left somewhat narrower from this hole onwards, but not too narrow as this will cause the low C and B to be too sharp. In general, it will be more beneficial for the middle register when the low B, C and D are sharp than when they are flat. In the latter case the E, F and G in both octaves will also be too flat, and the middle and high Ds and the A will be too sharp.

'The G in both octaves is usually flat and becomes more so when the low notes are too flat. If the low G is too flat its hole can be conically undercut, or the upper end of the lower joint bore can be gently widened from above, or the upper joint can be slightly chambered from its lower end up to just below the A hole.

'The ease of speech of the high C and D requires that the E and F holes on the lower joint are heavily conically undercut. The C hole [*sic* E?] primarily influences the ease of speech of the high C and the middle F hole that of the high D.

'The holes on the upper joint must be rather undercut. Care must be taken, however, to avoid undercutting the C hole too much as this will cause the middle C to be too sharp and sound poorly. The B hole can, in contrast, be more undercut.

'The double holes for A must be drilled and undercut so that their edges meet at the bore and almost form a single hole. This improves the speech of the A. If these holes are drilled diagonally towards the tenon they must be made larger. This gives the A the same strength as the G. These holes must be significantly smaller if they are drilled perpendicular to the bore as are the other holes, since the effectively lower-placed diagonal holes must be wide rather than narrow. It is preferable to leave the A holes somewhat small and to enlarge them when tuning, since both the As easily become sharp. If the A remains slightly flat a small chamber must be made between its hole and the B hole. Also, if the middle C and D are too flat the narrowest part in the bore can be enlarged through the reed socket with the long reamer. The clarity of the middle D depends, however, on the lower C and D holes being adequately enlarged. The open-standing key must also open very wide or the middle D will be muffled and flat even if the holes have been enlarged sufficiently to cause the low C and D to become sharp. Incidentally, for high pitch both these lower holes must be moved upwards.

'The third very short lower joint reamer must fit into the lower joint up to its tang (*bis ans erste an der Angel*—?). This gives body to the lower and middle registers and greatly improves the speech of the high C. A flat middle D is usually caused by the upper part of the bell being too wide, which also sharpens the lower notes. If, however, the bell is narrower both octaves are equally pure.

'If the low C and D of an oboe or English horn are too flat and too hardblowing one may chamber slightly only from the tenon to the C hole.'

APPENDIX 2 TOLERANCE

If a given dimension of an object is measured repeatedly under unaltered conditions, the values of the individual measurements will differ from each other by amounts which may or may not be of importance for the purpose at hand. If gross error such as incorrect reading or recording of values can be discounted, there are two primary sources of discrepancy. The first is systematic and mainly concerned with the accuracy with which the measuring instruments are calibrated and the conditions under which they are used (most measuring devices are calibrated for use at a specified temperature). This includes a personal factor caused by differences in the way individual workers use the same device, such as the pressure with which a caliper is closed, the way in which a scale is read, etc. Systematic error can be quantified to a high degree and can thus be largely compensated for.

Despite the greatest care, however, there will be a residual error which is irregular in nature and the second main source of error in measurement. Irregular error has two important properties: it is more likely to be small than large, and it is as likely to be positive as negative. Thus, if a given dimension of an object is measured an infinite number of times and the only source of error is irregular, the true value of that dimension will be the arithmetic mean of the individual measurements. One can say with 100% confidence that the calculated value and the true value are identical. Given less than an infinite number of measurements the degree of confidence with which such a statement can be made is no longer 100%. In practice one generally deals with clearly limited numbers of individual measurements and it is not possible to be 100% confident that the true value has been determined. There is an amount of uncertainty in the measurement which is expressed as its tolerance.

The mean value of a series of measurements should be near the true value of the dimension under consideration. The tolerance is a statement of how far from the calculated mean the true value can be expected to lie on any desired confidence level. The expression, 9.85±.02 mm., on a 95% confidence level means that it is a 95% certainty that the true value, which 9.85 mm. approximates, lies between 9.83 and 9.87 mm. Tolerances are calculated on the basis of the selected confidence level (usually 95% or more rarely 99%), the number of measurements taken, and the dispersion of the individual measurements about their mean. If the measurements of several dimensions of an object are all to be given with tolerances on the same confidence level, the sizes of the individual tolerances are likely to vary.

The requirements of accuracy in measurement depend entirely on the purpose for which the measurements are being made. If, for example, the bore of an oboe is measured to enable a reconstruction with 0.1 mm. bore tolerances, there is no reason to demand 0.001 mm. accuracy from the measurements. Conversely, if the oboe is measured by a procedure which does not allow greater than 0.1 mm. accuracy, 0.01 mm. tolerance in the reconstruction will not be attainable. (In reality it is extremely difficult to establish musically relevant tolerances for reconstruction and most such expressions are little more than intuitively derived. It is, however, perfectly possible to determine the accuracy of the measuring procedure used when gathering the initial numerical data and it might be advisable to establish the tolerances of reconstruction in accordance with this.) If the ultimate use of the measurements is not known when they are being recorded it is especially important to indicate their inherent accuracy. With the commonly used measuring techniques this is not as great as might be assumed and it is unlikely that its full utilization can be regarded as excessive.

Tolerances are calculated as follows:
The individual measurements are added together and the sum divided by the number of measurements. This defines the mean ($\hat{x}$):

$$\hat{x} = \Sigma x/n,$$

where

Σx = The sum of all x values
n = The number of x values.

The dispersion of the individual measurements about the mean is the standard deviation (s):

$$s = \sqrt{(x-\hat{x})^2/(n-1)}$$

or in a form more suitable for use with a pocket calculator:

$$s=\sqrt{\Sigma x^2/(n-1)-\{\Sigma x/(n-1)\}^2}$$

The mean value expressed with a tolerance range is written as:

$$\hat{x}\pm t_c s$$

where t_c depends on the desired confidence level and n. A table of t_c values for 95%, 99% and 99.99% confidence levels is given in Table 2–1.

(The expression $\hat{x}\pm t_c s$ is usually encountered as $\hat{x}\pm t_p(s/\sqrt{n})$ where t_p is a percentage point of the Student's t-distribution and $s/\sqrt{n}$ is the standard error of the mean. The simplification is justified by the present context but the t_c values were calculated according to the t-distribution. Further information can be found in virtually any text on statistics.) It must be remembered that this computation can only deal with irregular error. All tolerances resulting from systematic error must be considered separately. In practice the systematic factors taken into account are the tolerances of the measuring tools themselves and corrections for their use at temperatures other than those prescribed. Thus, the tolerances computed above must be increased by the tolerances of the measuring devices used.

To illustrate the application of the above, the Haka oboe described in the text may be considered. The bore was measured using telescoping and ball gauges pre-set with a micrometer to desired dimensions. The gauges were then inserted into the instrument until they firmly contacted the sloping walls of the bore, making sure that the depth of insertion was either a maximum or a minimum, and that the depth was recorded. The greatest source of uncertainty with this procedure is varying pressure when bringing the tool into contact with the instrument. The maximum depth to which a telescoping gauge set to 8.60 mm. could be inserted was measured ten times and the mean value of the measurements found to be 107.3 mm. with a standard deviation of 0.42 mm. On the 95% confidence level, this gives a tolerance of 107.3±.3 mm. The scale on which the depth was measured was made according to the standard DIN 866/II which specifies a tolerance of ±.06 mm. The depth value thus becomes 107.3±.36. For D=8.60 mm., L=107.3±.36 mm. It is, however, desired that D be treated as a function of L. If L is fixed at 107.3 mm, its tolerance must therefore be transferred to the D value. This is done by multiplying the tolerance in L by the slope of the bore at L. (If the bore profile is described as a polynomial, the slope of the bore is the derivative of D with respect to L.

If, $D=a_0+a_1L+a_2L^2+a_3L^3 \;.\;.\;.\; +a_nL^n$,
its derivative, $D'=a_1+2a_2L+3a_3L^2 \;.\;.\;.\; +na_nL^{n-1}$.)

In the case at hand this results in D=8.60±.01 mm. at L=107.3 mm. The micrometer with which the telescoping gauge was set was made according to the standard DIN 863/II which specifies a maximum tolerance of ±.01 mm. This gives D=8.60±.02 mm. The tolerances determined for the other measurements of the Haka oboe bore were of similar magnitude. Thus, measurements of this bore are adequately expressed on a 0.1 mm. level but are uncertain on a 0.01 mm. level. (See Table 2–2 for tolerances of various measuring tools.)

Had the measuring devices been individually calibrated prior to use and a larger number of measurements made, 0.01 mm. accuracy probably could have been obtained. Using more complicated measuring apparatus under laboratory conditions, still finer tolerances are possible. In practice, however, it is rare that repeated measurements are made except at particularly uncertain or critical spots in the bore. Thus, realistic tolerances for general work are clearly uncertain below the 0.1 mm. level. The preceding quasi-statistical discussion and workshop practice are not entirely reconcilable but it may prove worthwhile for individual workers to determine the tolerances inherent in their own measuring procedures.

APPENDIX 3 MATHEMATICAL CURVE FITTING

The problem of curve fitting is of great concern to many fields which deal with the evaluation of numerical data and there is consequently much literature on the subject. The interested reader will find pertinent material in virtually any text on numerical analysis, engineering mathematics, or statistics under the headings of curve fitting, non-linear regression, and least squares. The following discussion is intended only to provide a general orientation.

It is obviously not possible for all data points to collocate on a single smooth curve. The individual points will deviate from any curve by differing amounts and the best fitting curve will be the one from which the total deviation is a minimum. The most widely-used measurement of this deviation is the sum of the squares of the individual point deviations. The best fitting curve when so defined is called the least square curve and the residual error determines its goodness of fit. The best straight line that can be fitted to a given data array will be the least square line, the best parabola will be the least square parabola, etc. Thus one must first determine what type of curve best approximates the data and then define the best curve of that type in the least squares sense.

There are usually several functions which can approximate a given relationship and the choice of which to use can often present a problem. To avoid this difficulty polynomials of suitable degree are commonly used as 'all-purpose' functions and it will be found that they adequately describe the relationships encountered in woodwind bores.

Considering D as a function of L:

A first degree polynomial, $D=a_0+a_1L$, is a straight line.

A second degree polynomial, $D=a_0+a_1L+a_2L^2$, is a parabola.

An nth degree polynomial, $D=a_0+a_1L+a_2L^2+\ \ldots\ +a_nL^n$, effectively resembles a 'modified parabola'.

The process of least square polynomial curve fitting envolves determining the values of a_0, a_1, a_2, . . . a_n.

Although each successive degree will theoretically provide a better fit to a non-linear relationship, a point is reached beyond which this gain is of no practical consequence. Using a polynomial of unnecessarily high degree can in fact introduce an element of inaccuracy into the calculation. For present purposes third degree polynomials (cubic curves) are usually satisfactory. If they are not, subdivision of the interval under consideration into several cubic curves will generally be preferable to chosing a higher degree polynomial.

The least square cubic curve has the equation,

$$D=a_0+a_1L+a_2L^2+a_3L^3,$$

where the constants a_0, a_1, a_2, and a_3 can be determined by solving the following system of simultaneous equations:

$$\begin{aligned}
\mathbf{\Sigma}D &= a_0N + a_1\mathbf{\Sigma}L + a_2\mathbf{\Sigma}L^2 + a_3\mathbf{\Sigma}L^3\\
\mathbf{\Sigma}LD &= a_0\mathbf{\Sigma}L + a_1\mathbf{\Sigma}L^2 + a_2\mathbf{\Sigma}L^3 + a_3\mathbf{\Sigma}L^4\\
\mathbf{\Sigma}L^2D &= a_0\mathbf{\Sigma}L^2 + a_1\mathbf{\Sigma}L^3 + a_2\mathbf{\Sigma}L^4 + a_3\mathbf{\Sigma}L^5\\
\mathbf{\Sigma}L^3D &= a_0\mathbf{\Sigma}L^3 + a_1\mathbf{\Sigma}L^4 + a_2\mathbf{\Sigma}L^5 + a_3\mathbf{\Sigma}L^6,
\end{aligned}$$

where N=number of data points and $\mathbf{\Sigma}D$= sum of all D values ($\mathbf{\Sigma}L^5$=the sum of the fifth powers of all L values, etc.).

(This system of equations can be altered for use with polynomials of degree n by writing n+1 lines with no terms with constants greater than a_n.)

Programmable pocket calculators capable of 4 x 4 matrix analysis can easily be used for both the necessary summing and the solution of the simultaneous equations. If such a device is available further reference can be made to its accompanying literature. Otherwise, the equations can be solved by Gaussian elimination, the technique of which is described in any algebra text either under that heading or under the solution of simultaneous equations.

APPENDIX 4 FORMULA DERIVATION

If the cross section of a bore is considered as its shrinks from circularity to ellipticity as a result of moisture loss, it will be seen that D_m shrinks r times more rapidly than does D_M. If this process were to be reversed and an elliptical cross section allowed to 'grow' to circularity it would be expected that D_m would expand r times more rapidly than D_M. At a certain point D_m would be equal to D_M and their magnitude would be identical to that of the diameter of the original circular cross section. This model will permit mathematical calculation of D from D_M, D_m, and r (although it must be remembered that in reality the shrinkage and expansion mechanisms are not identical) using Formula 2 in the text. This formula can be derived as follows:

A scale on which D_M and D_m can be plotted is taken as the abscissa of a rectangular coordinate system. An elliptical cross section is considered and from the point representing D_M a line is drawn at a 45° angle to the abscissa. From the point representing D_m a line is drawn at angle θ to the abscissa. The ordinate is scaled arbitrarily. Considering the D_M and D_m lines with reference to their points of origin a given increment in y will result in an x-increment along the D_m line which will be r times as great as the x-increment along the D_M line. The previously described 'shrinkage reversal' can thus be simulated by considering the x values along the D_M and D_m lines for successively higher values of y. Where the D_M and D_m lines intersect at point P their x values will be equal both to each other and to D. A line dropped from P perpendicular to the abscissa will thus intersect it at a point representing D. This gives the following diagram:

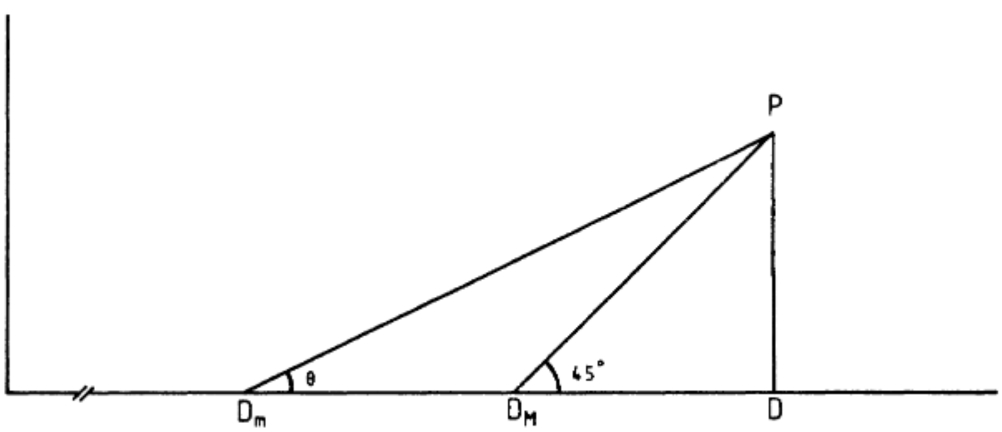

FIG. I

It will be seen trigonometrically that:

$$D_M D = D_m D_M \frac{\sin\theta \cos 45^\circ}{\sin(45^\circ - \theta)}$$

which, as $\sin 45^\circ = \cos 45^\circ$, reduces to

$$D_M D = D_m D_M \frac{\sin\theta}{\cos\theta - \sin\theta}$$

As θ is a function of r, the expression $\sin\theta/(\cos\theta - \sin\theta)$ is also a function of r and can conveniently be termed R. Making this substitution and considering only the x coordinates of all points gives:

$D = D_M + R(D_M - D_m)$.

Inasmuch as D as calculated by this formula is identical to D as calculated by Formula 1 in the text only when r=1:2 (in which case $D = D_M + (D_M - D_m)$), the D values as calculated by these formulas respectively have been termed D_2 and D_1.

TABLE 1

r and R for various timbers:

	r	R
Maple (*Acer saccharum*)	2.5 : 5.0	1.00
Boxwood (*Buxus sempervirens*)	11.0 : 15.0	2.75
Ebony (*Diospyros crassiflora*)	5.5 : 6.5	5.50
Pear (*Pirus communis*)	4.6 : 9.1	1.02
Cherry (*Prunus avium*)	3.5 : 6.5	1.17

The values for maple, ebony, and cherry were taken from *Handbook of Hardwoods*, Second Edition, Princes Risborough Laboratory, HMSO, London 1972. The values for boxwood and pear were taken from H. H. Bosshard, *Holzkunde* Band I, Birkhäuser, Basel 1974.

Published values of r will vary in different sources due both to differing test criteria and sample variation. The magnitude of these discrepancies varies for different species (boxwood having quite similar values in most reference works) but is small for most hardwoods.

TABLE 2–1

t_c Values

%/n	2	3	4	5	6	7	8
95	8.98	2.48	1.59	1.24	1.05	0.92	0.84
99	45.01	5.73	2.92	2.06	1.65	1.52	1.24
99.99	4501.58	57.73	14.0	6.95	4.56	3.43	2.79

%/n	9	10	11	12	13	14	15	30
95	0.77	0.72	0.67	0.64	0.60	0.58	0.55	0.37
99	1.12	1.03	0.96	0.90	0.85	0.80	0.77	0.50
99.99	2.37	2.09	1.87	1.71	1.58	1.47	1.38	0.82

TABLE 2–2

DIN norms for measuring tool tolerances:
T = maximum permitted deviation from scale value ± in μm
L = measured distance as shown on scale
l = maximum distance measurable by the device

Vernier calipers	DIN 862	$T = 50 + 0.1l$	
Micrometer	DIN 863/I	$T = 4 + 0.01l$	
	DIN 863/II	$T = 10 + 0.02l$	
Steel ruler	DIN 866/1	$T = 20 + 0.02L$	
	DIN 866/11	$T = 50 + 0.05L$	
Dial gauge	DIN 878/I	$T = 12$ if $l = 3$ mm	$T = 17$ if $l = 10$ mm
	DIN 878/II	$T = 18$ if $l = 3$ mm	$T = 28$ if $l = 10$ mm

Note that the mechanical bore gauges which are used with dial gauges have transmission tolerances of circa 2 μm.