

# DATA.ML.200 Pattern Recognition and Machine Learning

## Homework 1: Estimation theory

This homework prepares you for the next week exercises.

1. **pen&paper** *LS estimator.*

You have  $n = 0, \dots, N - 1$  samples from a sensor that can be modeled using a sinusoid

$$x[n] = A \cos(2\pi f_0 n + \phi) . \quad (1)$$

The estimation problem is to estimate the model parameters, one, two or all three of them,  $\theta = (A, f_0, \phi)$ .

The LS estimator  $\theta_{LSE}$  estimates the model parameters by finding the values that minimise the sum of squared errors (SSE) “loss”  $\ell_{SSE}$ . Define how the loss is computed for  $N$  samples  $x[n]$ .

2. **pen&paper** *ML estimator.*

LS estimator does not explicitly model the noise in samples, but the noise  $w[n]$  is intrinsic property of real measurements. The model with the noise term is defined as

$$x[n] = A \cos(2\pi f_0 n + \phi) + w[n] , \quad (2)$$

where  $w[n] \sim \mathcal{N}(0, \sigma^2)$ .

The ML estimator  $\theta_{ML}$  estimates the model parameters by finding the values that maximise the probability of obtaining them. This is done by computing the product of likelihood of each sample  $p(x[n]; \theta)$ . Define the total likelihood of  $N$  samples.