



ISTIC - IT & Electronics Department
Master EIT Digital in Data Science

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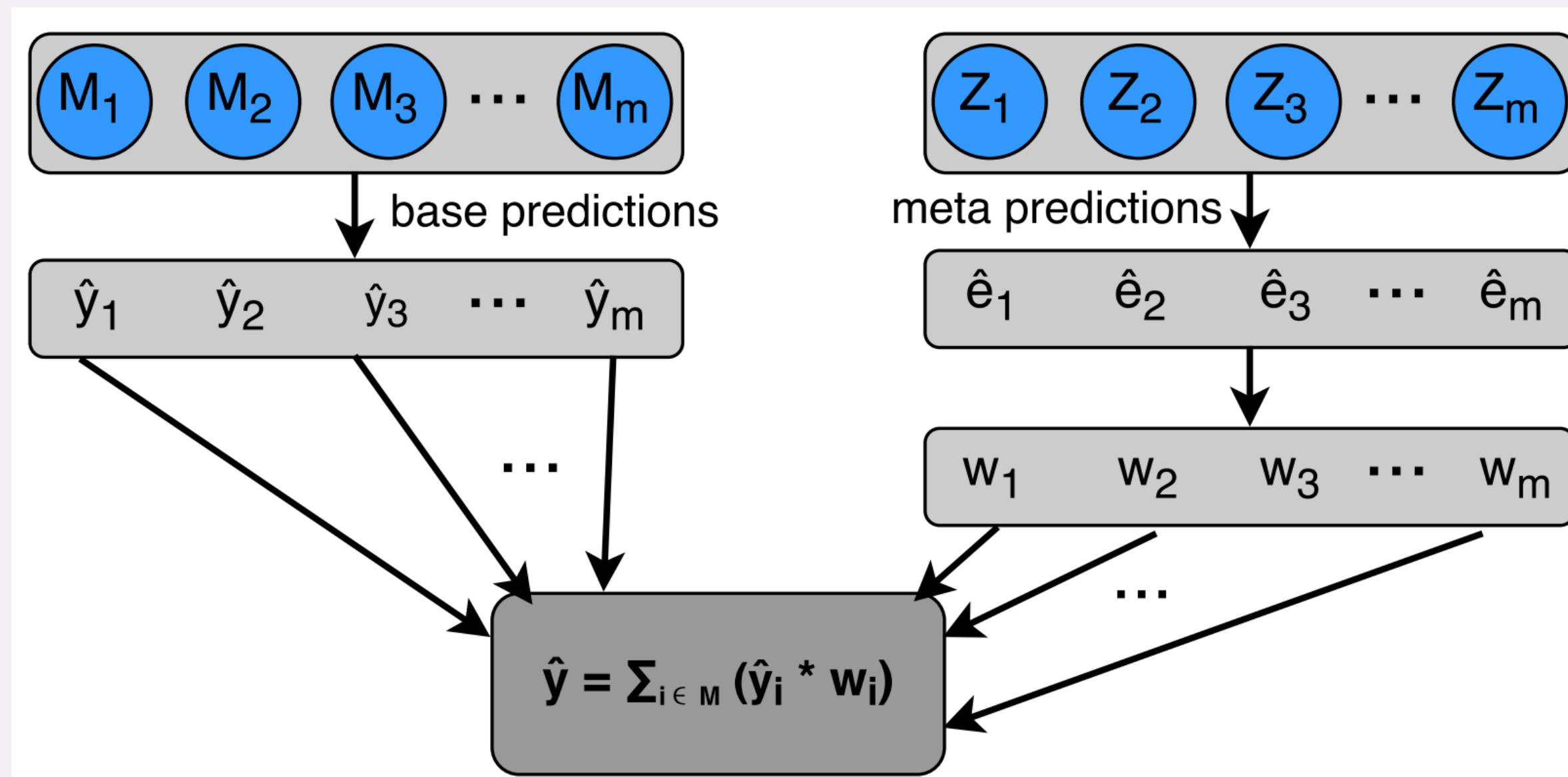
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ARBITRATED ENSEMBLING

SCOPE

- WHAT IS ARBITRATED DYNAMIC ENSEMBLE?
- REGRESSION:
 - WEIGHTING: EXPONENTIAL VS INVERSE
- CLASSIFICATION:
 - AVERAGING

ARCHITECTURE OF ADE



HOW WE SPLIT THE DATASET IN CHUNKS?

BETA = 3

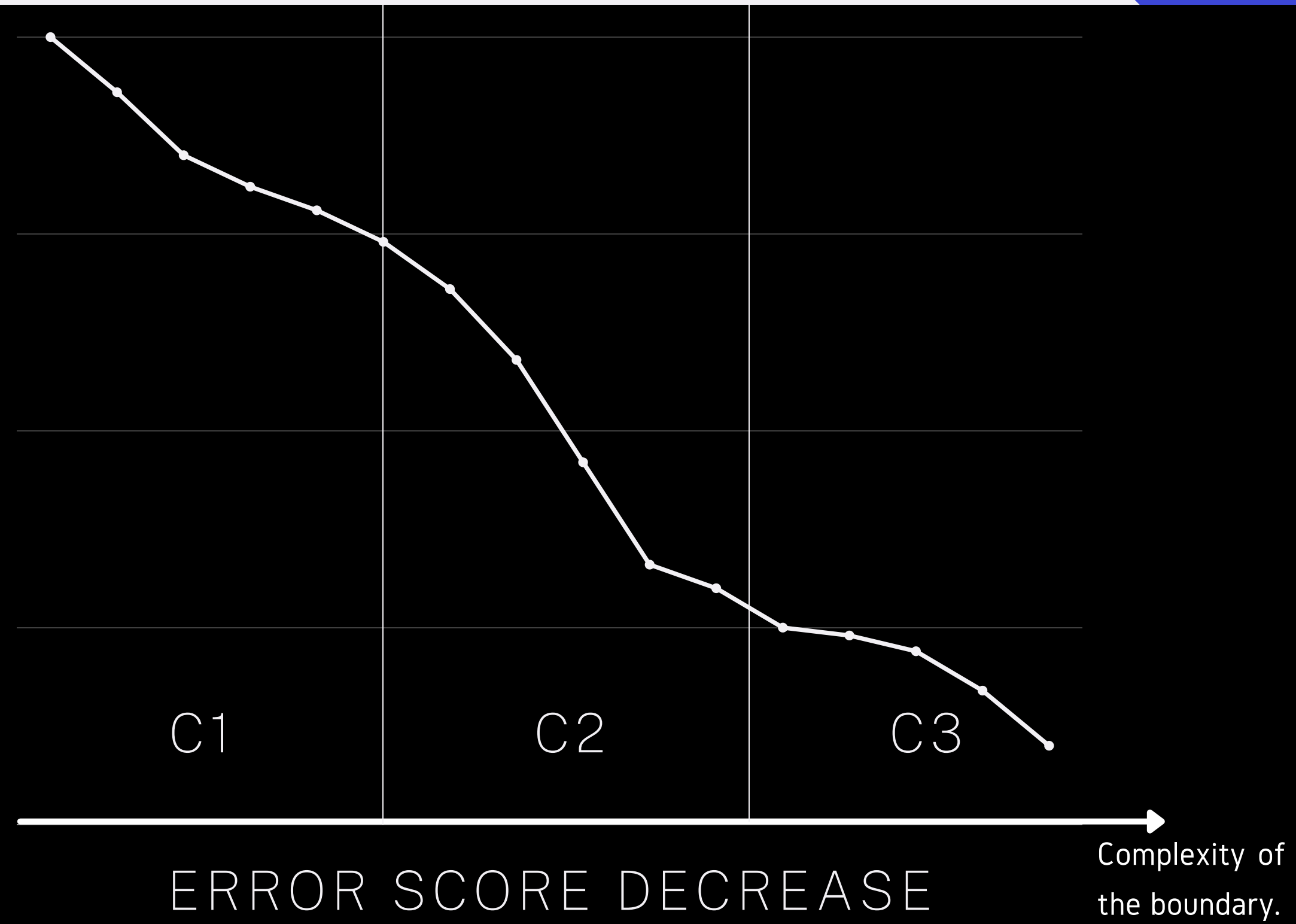
		#2		#1					
TRAIN	{	TRAIN	C1	{	1	X11	X12	X13	y1
					2	X21	X22	X23	y2
	{	TEST	C2	{	3	X31	X32	X33	y3
					4	X41	X42	X43	y4
TEST	{		C3	{	5	X51	X52	X53	y5
					6	X61	X62	X63	y6

M trained on C1 and predict C2

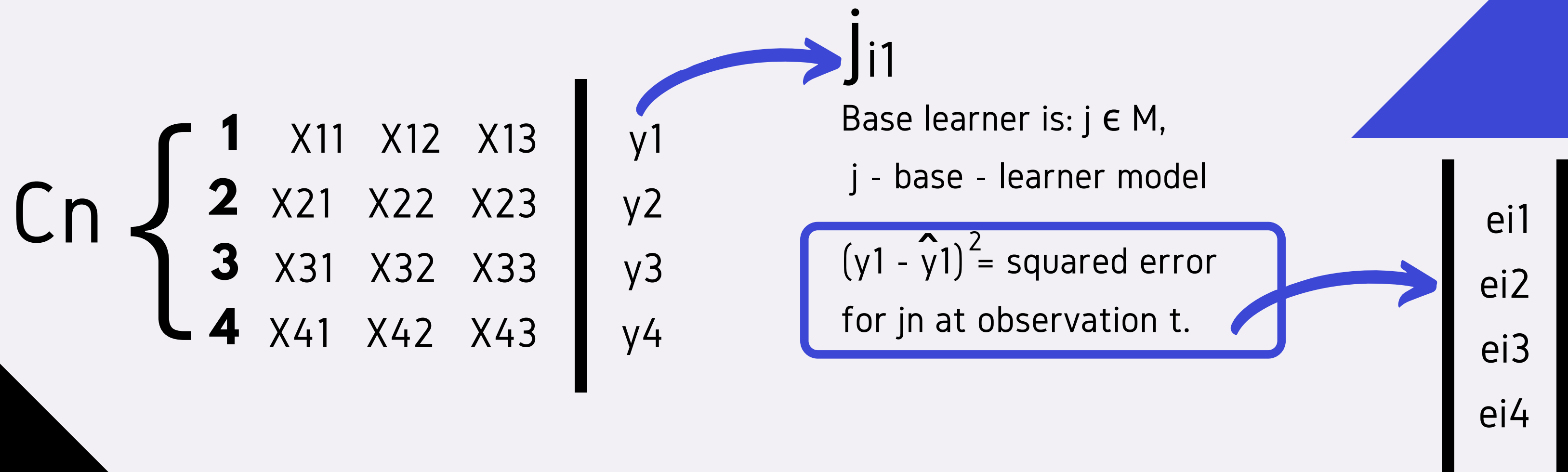
M trained on C1 + C2 and predict C3

M trained on C1 + ... + C_{n-1} and predict C_n

**WHAT
DOES IT
GIVE
US?**



REGRESSION



REGRESSION

$$C_n \left\{ \begin{array}{cccc|c|c} \mathbf{1} & X_{11} & X_{12} & X_{13} & y_1 & \hat{y}_1 \\ \mathbf{2} & X_{21} & X_{22} & X_{23} & y_2 & \hat{y}_2 \\ \mathbf{3} & X_{31} & X_{32} & X_{33} & y_3 & \hat{y}_3 \\ \mathbf{4} & X_{41} & X_{42} & X_{43} & y_4 & \hat{y}_4 \end{array} \right.$$

$$(y_1 - \hat{y}_1)^2 = e$$

**What do we need for
metalearner?**

e1 e2 e4 ... en

0.5 0.3 0.2 0.4

This is fitted to our
metalearner.

RFR

WHY WE CHOSE RANDOM FOREST REGRESOR (RFR)?

- GOOD PREDICTIVE PERFORMANCE OVER NOISY DATA.
- GOOD GENERALIZATION POTENTIAL.
- CONSISTENT FOR AVOIDING OVERFITTING.
- SUITABLE FOR DATASETS WITH HIGH NUMBER OF FEATURES.

REGRESSION PREDICTION

Meta-learner uses observation
to predict an error for each
base learner.

$C_n + 1$ {
1 X11 X12 X13
2 X21 X22 X23
3 X31 X32 X33
4 X41 X42 X43

$\hat{e}1$ $\hat{e}2$ $\hat{e}3$ $\hat{e}4$
0.1 0.2 0.5 0.7
0.4 0.1 0.6 0.5
0.3 0.2 0.5 0.7
0.2 0.1 0.6 0.5

**Meta-
learner**

$C_n + 1$ {
1 X11 X12 X13
2 X21 X22 X23
3 X31 X32 X33
4 X41 X42 X43

j1 j2 j3 j4
2 5 7 8
9 10 5 4
8 1 2 4
5 4 6 7

**Base-
learner**

REGRESSION PREDICTION

Meta-learner uses observation
to predict an error for each
base learner.

Cn + 1 {	1	X11	X12	X13	$\hat{e}1$	$\hat{e}2$	$\hat{e}3$	$\hat{e}4$	Meta- learner
	2	X21	X22	X23	0.1	0.2	0.5	0.7	
	3	X31	X32	X33	0.4	0.1	0.6	0.5	
	4	X41	X42	X43	0.3	0.2	0.5	0.7	
Cn + 1 {	1	X11	X12	X13	0.2	0.1	0.6	0.5	Base- learner
	2	X21	X22	X23	j1	j2	j3	j4	
	3	X31	X32	X33	2	5	7	8	
	4	X41	X42	X43	9	10	5	4	
					8	1	2	4	
					5	4	6	7	

REGRESSION PREDICTION: EXPONENTIAL WEIGHTING

$$w_t^j = \frac{\exp(-\hat{e}_t^j)}{\sum_{j \in M} \exp(-\hat{e}_t^j)}$$

$$W11 = \frac{\exp(-0.1)}{\exp(-0.1) + \exp(-0.2) + \exp(-0.5) + \exp(-0.7)}$$

$$W11 = 0.32$$

REGRESSION PREDICTION: EXPONENTIAL WEIGHTING

REGRESSION PREDICTION

Meta-learner uses observation to predict an error for each base learner.

$C_n + 1$ { $\begin{matrix} \mathbf{1} & X_{11} & X_{12} & X_{13} \\ \mathbf{2} & X_{21} & X_{22} & X_{23} \\ \mathbf{3} & X_{31} & X_{32} & X_{33} \\ \mathbf{4} & X_{41} & X_{42} & X_{43} \end{matrix}$

$\hat{e}1 \quad \hat{e}2 \quad \hat{e}3 \quad \hat{e}4$
0.1 0.2 0.5 0.7
0.4 0.1 0.6 0.5
0.3 0.2 0.5 0.7
0.2 0.1 0.6 0.5

Meta-learner

$C_n + 1$ { $\begin{matrix} \mathbf{1} & X_{11} & X_{12} & X_{13} \\ \mathbf{2} & X_{21} & X_{22} & X_{23} \\ \mathbf{3} & X_{31} & X_{32} & X_{33} \\ \mathbf{4} & X_{41} & X_{42} & X_{43} \end{matrix}$

$j1 \quad j2 \quad j3 \quad j4$
2 5 7 8
9 10 5 4
8 1 2 4
5 4 6 7

Base-learner

8

$$\hat{y}_t^{ADE} = \sum_{j \in M} \hat{y}_t^j w_t^j$$

$$W_{jt} = (0.32, 0.24, 0.27, 0.28)$$

$$\hat{y}_1^{ADE} = 2 \cdot 0.32 + 5 \cdot 0.24 + 7 \cdot 0.27 + 8 \cdot 0.28 = 6.09$$

$$\hat{y}_{1...4}^{ADE} = (6.09, 7.86, 4.5, 6.24)$$

REGRESSION PREDICTION

Meta-learner uses observation to predict an error for each base learner.

Cn + 1 {					Meta-learner
				ê1 ê2 ê3 ê4	
				0.1 0.2 0.5 0.7	
				0.4 0.1 0.6 0.5	
				0.3 0.2 0.5 0.7	
Cn + 1 {					Base-learner
				j1 j2 j3 j4	
				2 5 7 8	
				9 10 5 4	
				8 1 2 4	
				5 4 6 7	

REGRESSION PREDICTION: INVERSE WEIGHTING

PROPORTIONING

$$p_{jt} = \frac{\hat{e}_t}{\sum_{j \in M} \hat{e}_t^j}$$

$$p_{1n} = \frac{0.1}{0.1 + 0.2 + 0.5 + 0.7} = 0.066$$

$$p_{1...4n} = (0.066, 0.25, 0.17, 0.14)$$

REGRESSION PREDICTION

Meta-learner uses observation to predict an error for each base learner.

Cn + 1 {				1	X11	X12	X13	ê1	ê2	ê3	ê4	Meta-learner
				2	X21	X22	X23	0.1	0.2	0.5	0.7	
				3	X31	X32	X33	0.4	0.1	0.6	0.5	
				4	X41	X42	X43	0.3	0.2	0.5	0.7	
Cn + 1 {				1	X11	X12	X13	j1	j2	j3	j4	Base-learner
				2	X21	X22	X23	2	5	7	8	
				3	X31	X32	X33	9	10	5	4	
				4	X41	X42	X43	8	1	2	4	
								5	4	6	7	

REGRESSION PREDICTION: INVERSE WEIGHTING

$$p_{1j}^{-1}x_t + p_{2j}^{-1}x_t + p_{3j}^{-1}x_t + \dots + p_{nj}^{-1}x_t = 1$$

$$p_{1...4n} = (0.066, 0.25, 0.17, 0.14)$$

$$p_{1...4n}^{-1} = (15, 4, 5.66, 7)$$

BALANCING

$$w_{jt} = p_{jt}^{-1}x_t$$

$$15 * X + 4 * X + 5.66 * X + 7 * X = 1$$

$$X = 0.031$$

$$0.47 + 0.13 + 0.18 + 0.22 = 1$$

REGRESSION PREDICTION

Meta-learner uses observation to predict an error for each base learner.

Cn + 1 {					1	X11	X12	X13	ê1	ê2	ê3	ê4	Meta-learner
					2	X21	X22	X23	0.1	0.2	0.5	0.7	
					3	X31	X32	X33	0.4	0.1	0.6	0.5	
					4	X41	X42	X43	0.3	0.2	0.5	0.7	
					4	X41	X42	X43	0.2	0.1	0.6	0.5	

Cn + 1 {					1	X11	X12	X13	j1	j2	j3	j4	Base-learner
					2	X21	X22	X23	2	5	7	8	
					3	X31	X32	X33	9	10	5	4	
					4	X41	X42	X43	8	1	2	4	
					4	X41	X42	X43	5	4	6	7	

8

REGRESSION PREDICTION: INVERSE WEIGHTING

$$\hat{y}_t^{ADE} = \sum_{j \in M} \hat{y}_t^j w_t^j$$

$$0.47 + 0.13 + 0.18 + 0.22 = 1$$

$$\hat{y}_1^{ADE} = 0.47 * 2 + 0.13 * 2 + 0.18 * 7 + 0.22 * 8 = 4.6$$

$$\hat{y}_{1...4}^{ADE} = (4.6, 7.3, 5.15, 5.49)$$

DATASETS

Regression Datasets		
Name	Attributes	# instances
Cancer	14	3047
Bike Sharing	15	17379
Insurance (Charge)	6	1338
Real Estate	6	415

Time series Datasets	
Name	# instances
Bike Sharing	17379
IBM stock price	1008
Ozon	518
Temperature	1461

EVALUATION

PRIORIZATION

As we evaluate the mean of the predicted error the lower value achieved the better.

Regression: Base-learners								
Dataset	SVR	RFR	DTR	LR	LAS	ELAS	BAY	ARD
Cancer	817.30	477.36	586.67	520.72	519.38	719.79	528.71	509.76
Bike Sharing	23306	13118	13881	22150	22159	26643	22152	22154
Insurance	1.64	1.33	1.35	1.2713	1.2713	1.30	1.2765	1.28
Real Estate	60.10	58.51	60.02	71.88	71.74	74.40	71.26	105.82

Regression: ADE								
Dataset	Ens3	Ens3P	ExEns3	ExEns3P	Ens5	Ens5P	ExEns5	ExEns5P
Cancer	582.04	519.29	604.34	503.87	626.60	708.31	584.01	708.31
Bike Sharing	22344	23719	26655	23719	21292	23508	26654	23508
Insurance	1.32	1.63	3.24	1.63	1.29	1.6484	3.24	1.6484
Real Estate	63.65	70.45	64.36	70.45	60.71	72.25	63.37	72.25

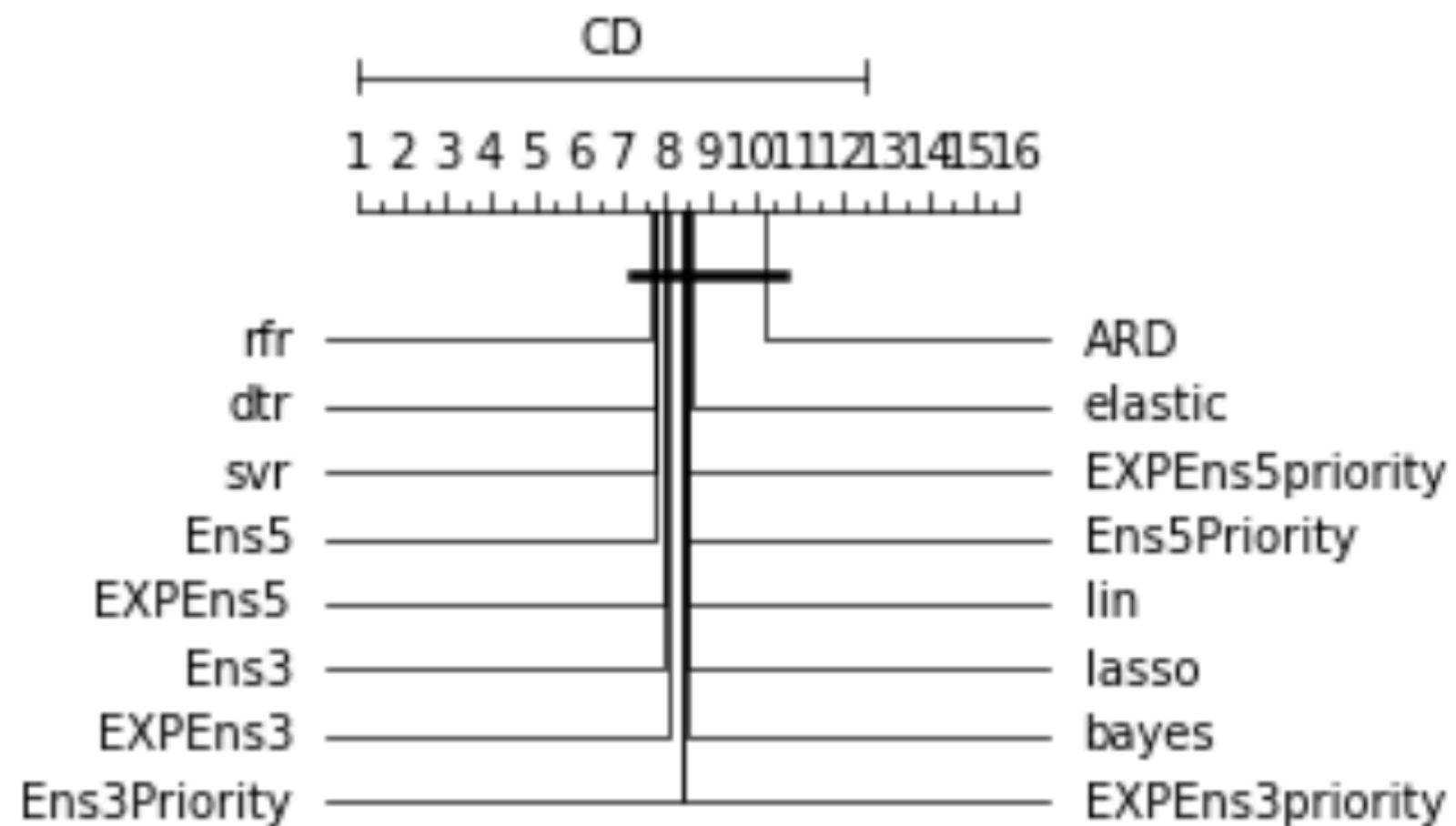
RESULTS

Results of the datasets used in the report are shown for regression and for time series.

Regression for Time series: Base-learners								
Dataset	SVR	RFR	DTR	LR	LAS	ELAS	BAY	ARD
Bike Sharing	26904	8302	9589	11711	11711	11792	11712	11710
IBM	44.38	27.46	58.88	16.30	16.31	16.56	16.35	16.35
Ozone	543.5	358.5	569.8	373.4	373.5	386.3	375.0	389.1
Temperature	22.74	17.86	21.06	16.81	16.85	16.97	16.85	16.87

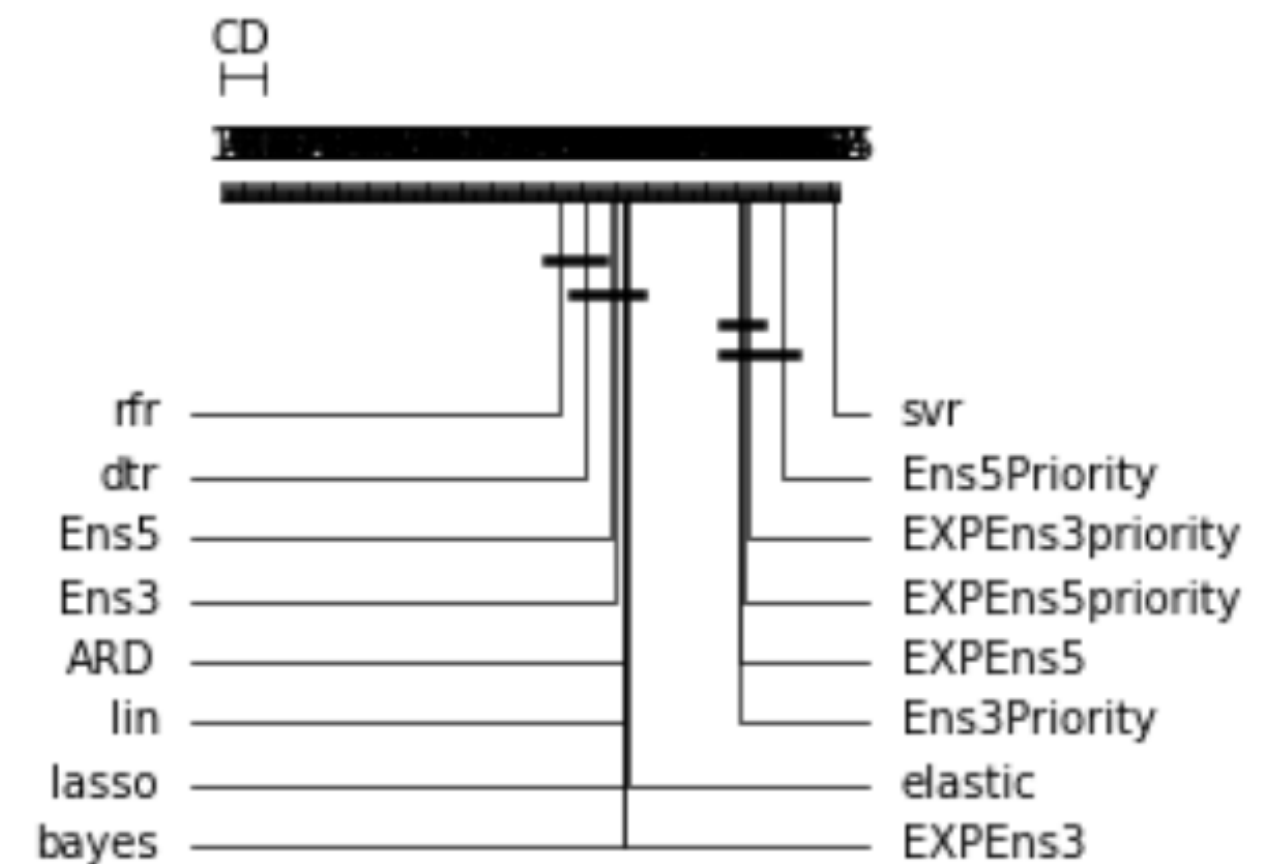
Regression for Time series: ADE								
Dataset	Ens3	Ens3P	ExEns3	ExEns3P	Ens5	Ens5P	ExEns5	ExEns5P
Bike Sharing	11250	19070	11734	19731	10869	22414	19145	19378
IBM	17.11	119.50	16.94	119.50	16.35	44.29	16.33	44.29
Ozone	413.0	703.4	396.3	703.4	562.8	610.2	608.1	610.2
Temperature	20.92	26.03	24.45	26.03	20.37	23.07	22.56	23.07

NEMENYI TEST



REGRESSION

Mean errors with Real Estate dataset.



TIME SERIES

Mean errors with Bike Sharing dataset.

CLASSIFICATION

				j1	j2	j3	j4	z1	z2	z3	z4				
C_{n+1}	1	x11	x12	x13	\hat{y}_1	\hat{y}_1	\hat{y}_1	\hat{y}_1	\hat{e}_{i1}	\hat{e}_{i1}	\hat{e}_{i1}	\hat{e}_{i1}			
	2	x21	x22	x23	\hat{y}_2	\hat{y}_2	\hat{y}_2	\hat{y}_2	\hat{e}_{i2}	\hat{e}_{i2}	\hat{e}_{i2}	\hat{e}_{i2}			
	3	x31	x32	x33	\hat{y}_3	\hat{y}_3	\hat{y}_3	\hat{y}_3	\hat{e}_{i3}	\hat{e}_{i3}	\hat{e}_{i3}	\hat{e}_{i3}			
	4	x41	x42	x43	\hat{y}_4	\hat{y}_4	\hat{y}_4	\hat{y}_4	\hat{e}_{i4}	\hat{e}_{i4}	\hat{e}_{i4}	\hat{e}_{i4}			
				Instances				base-learners predictions				meta-learners predictions			

**HOW TO
CALCULATE
THE ERROR?**

CLASSIFICATION: ERROR CALCUALTION

$$\hat{y}_t^{ADE} = c, \text{ where } \hat{e}_{tc} = \min \left(\frac{\sum_{j \in MC} \hat{e}_{jc}}{N_{MC}}, \text{ for each } c \text{ where } c \in C \right)$$

X11 X12 X13 | **y**
Dog

	Cat	Dog	Wolf
j1	0.2	0.5	0.3
j2	0.4	0.3	0.3
j3	0.1	0.8	0.1
j4	0.2	0.3	0.5

$(1 - y_{j_{\text{Dog}}}) = \text{error for } j_n$
at observation t.

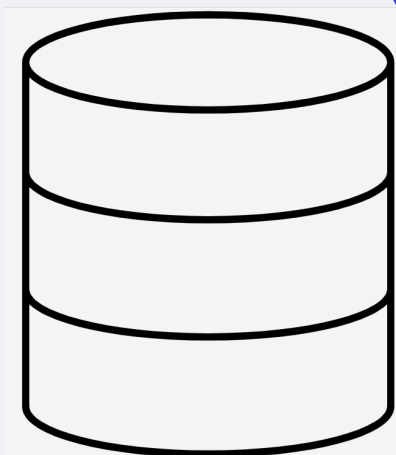
Errors

j1	0.5
j2	0.7
j3	0.2
j4	0.3

X11 X12 X13 | **y** | e1 e2 e3 e4
Dog | 0.5 0.7 0.2 0.3

CLASSIFICATION PREDICTION

X11 X12 X13 | ?



Dataset

Possible classes:

{cat, dog, wolf,
rabbit, fox, squirrel}

y

?

	\hat{y}	\hat{e}
j1	wolf	0.4
j2	dog	0.2
j3	cat	0.6
j4	dog	0.3
j5	wolf	0.4
j6	wolf	0.5

$$\text{cat} = [0.6] / N_{\text{cat}} = 0.6$$

$$\text{dog} = [0.2, 0.3] / N_{\text{dog}} = 0.25$$

$$\text{wolf} = [0.4, 0.4, 0.5] / N_{\text{wolf}} = 0.43$$

$$\text{rabbit, fox, squirrel} = \emptyset$$

$$\hat{y}_1^{\text{ADE}} = \text{dog}$$

EVALUATION

DATASETS

2 datasets for binary classification and 2 for multinomial classification.

Classification Datasets			
Name	Attributes	Classes	# Instances
Car	6	3	1728
Obesity	16	4	2211
Chess	36	2	3196
Tic-tac-toe	9	2	958

RESULTS

Results of the datasets used in the report are shown for classification. As we evaluate the accuracy the higher value achieved the better.

Classification: Base-learners								
Dataset	MLR	SVM	SGD	RFC	multNB	bernNB	KNN	ADA
Car	0.917	0.973	0.905	0.967	0.855	0.863	0.946	0.795
Obesity	0.831	0.534	0.556	0.958	0.611	0.556	0.910	0.317
Chess	0.959	0.946	0.956	0.970	0.849	0.850	0.924	0.957
Tic-tac-toe	0.982	0.975	0.979	0.972	0.677	0.666	0.968	0.857

Classification: ADE				
Dataset	Ens3	Ens3P	Ens5	Ens5P
Car	0.761	0.797	0.855	0.963
Obesity	0.446	0.902	0.798	0.906
Chess	0.871	0.965	0.912	0.967
Tic-tac-toe	0.982	0.982	0.975	0.947

istic

Informatique
Électronique

