CS 3100, Fall 2020, Assignment 10 – 100 pts total

Please fill the notebook `u1234567_asg10_Prob1234.ipynb` within `20_Mapping_Redn/` to typeset your proofs.

1. (LT, 25%) Please solve these PCP instances. **Note that our PCP solver requires that all cells be used at least once.**

   (a) (5 pts) Solve the PCP instance below by using the PCP tool:
   `pcp_solve([('110', '1'), ('1', '0'), ('0', '110')], OWN_INSTALL)`.
   Report the solution length.

   (b) (5 pts) Solve the PCP instance below by using the PCP tool:
   `pcp_solve([('110', '1'), ('1', '0'), ('0', '110'), ('0','1') ], OWN_INSTALL)`.
   This is the above hard instance with one more tile added (namely `('0','1')`).
   You will observe that the solution becoming shorter.

   (c) (5 pts) Solve the PCP instance below by hand, and then verify using the PCP solver:
   `pcp_solve([('1', '0'), ('101', '1'), ('0', '110')], OWN_INSTALL)`.

   (d) (10 pts) Come up with your own hard PCP instance, in that the system must have less than six (6) tiles, but the solution must be at least 30 steps long (a growth factor of five or more). Present your instance in the notebook. If you cannot find a hard instance meeting these specifications, list two of your best attempts (in terms of exhibiting the highest observed growth factor) and report it (i.e., ratio of **solution length** divided by **instance length** or number of tiles).
   `pcp_solve( [ YOUR, TILES, HERE ], OWN_INSTALL )`.

Answer for these part in `u1234567_asg10_Prob1234.ipynb`.

2. (AR, 25%)

   (a) (15%, adapting Theorem 15.3 for $Halt_{TM}$) Adapt the proof of Theorem 15.3, Page 233, to a very similar proof that spells out that $Halt_{TM}$ is not recursive.

$$Halt_{TM} = \{\langle M, w\rangle \ : \ M \text{ is a TM and } w \text{ its input and } M \text{ halts on } w\}$$

   Please imitate the proof of 15.3 step by step, making the necessary changes:

Answer for this part in `u1234567_asg10_Prob1234.ipynb`.

(b) (10%, adapting the proof in Figure 15.10 for CFL_{TM}) Adapt the proof in Figure 15.10 to argue that CFL_{TM} is not recursive.

$$CFL_{TM} = \{\langle M \rangle \ : \ M \text{ is a TM whose language is context} - \text{free.}\}$$

Answer for this part in `u1234567_asg10_Prob1234.ipynb`.

3. (XL, SV 50%) Do Problem 5, Page 232, Exercise 15.2.3 which asks you to perform a mapping reduction from the PCP to the CFG Ambiguity problem, and thus argue that if there is a decider for the CFG Ambiguity problem, then there will be a decider for PCP also.

The languages involved in this exercise are now formally defined: PCP is defined on Page 229, Section 15.2, and reproduced below:
Given any alphabet Σ such that $|\Sigma| > 1$, consider the *tile alphabet* $\mathcal{T} \subseteq \Sigma^+ \times \Sigma^+$. Define

$$PCP = \{S \ : \ S \text{ is a finite sequence of elements over } \mathcal{T} \text{ that has a solution}\}.$$

Also, define

$$Amb = \{\langle G \rangle \ : \ G \text{ is a } CFG \text{ that has } \geq 1 \text{ parse for some } w \in \Sigma^*\}$$

Here is how we can build a mapping reduction from PCP to Amb; please fill in missing steps (if any) and argue that the mapping reduction actually works (achieves its purpose).

Let

$$A = w_1, w_2, \ldots, w_n$$

and

$$B = x_1, x_2, \ldots, x_n$$

be two lists of words over a finite alphabet Σ. Let $a_1, a_2, \ldots, a_n$ be symbols that do not appear in any of the w_i or x_i. Let G be a CFG

$$(\{S, S_A, S_B\}, \ \Sigma \cup \{a_1, \ldots, a_n\}, \ P, \ S),$$

where P contains the productions

$S \to S_A$,
$S \to S_B$,
For $1 \leq i \leq n$, $S_A \to w_i S_A a_i$,
For $1 \leq i \leq n$, $S_A \to w_i a_i$,
For $1 \leq i \leq n$, $S_B \to x_i S_B a_i$, and
For $1 \leq i \leq n$, $S_B \to x_i a_i$.

Now, argue that G is ambiguous if and only if the PCP instance (A, B) has a solution (thus, we may view the process of going from (A, B) to G as the desired mapping reduction).

(*Note: I'm using a different font for A and B of Figure 15.7 to avoid name-clashes with A and B used in the PCP problem.* I'll in fact call them A and B, below.)

(a) (28 points, SV, for the $x \in \mathsf{A} \Rightarrow f(x) \in \mathsf{B}$ part) Study Definition 15.5 and Figure 15.7 which defines *mapping reduction*. Then, answer these in `u1234567_asg10_Prob1234.ipynb`:

   i. (3 points) What are A and B in our case? What is f?

   ii. (25 points) Argue in neat bulleted steps that $x \in \mathsf{A} \Rightarrow f(x) \in \mathsf{B}$. Be thorough in your steps, showing what $x \in \mathsf{A}$ means, what $f(x) \in \mathsf{B}$ means, and how the whole proof works.

(b) (22 points, XL, for the $x \notin \mathsf{A} \Rightarrow f(x) \notin \mathsf{B}$ part)

Argue in neat bulleted steps that $x \in \mathsf{A} \Rightarrow f(x) \in \mathsf{B}$. Be thorough in your steps, showing what $x \notin \mathsf{A}$ means, what $f(x) \notin \mathsf{B}$ means, and how the whole proof works.