Variant 1

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 3 \\ -8 & 20 & -6 & 7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 8 & 4 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 15 & 1 & 8 \\ -3 & 7 & 8 \\ 9 & 3 & 8 \\ 3 & 5 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 13 \cdot x - 2 \cdot y + 12 \cdot z + 1 \cdot t = 5 \\ 9 \cdot x + 3 \cdot y + 8 \cdot z + 3 \cdot t = 1 \\ 7 \cdot x + 6 \cdot y + 8 \cdot z + 5 \cdot t = 8 \\ 5 \cdot x + 8 \cdot y + 4 \cdot z + 5 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 4) + y^2\,(12q + 1) + z^2\,(12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 2

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 1 & 2 \\ 6 & -7 & -18 & 20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 0 & 5 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 1 & 8 \\ 6 & -2 & 16 \\ 9 & 7 & -8 \\ 8 & 4 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 8 \cdot y + 6 \cdot z + 9 \cdot t = 9 \\ 7 \cdot x + 1 \cdot y + 8 \cdot z + 8 \cdot t = 6 \\ 4 \cdot x + 0 \cdot y + 5 \cdot z + 5 \cdot t = 7 \\ 10 \cdot x + 2 \cdot y + 11 \cdot z + 11 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 4x + 4$ find the best approximation with respect to the norm $\int_0^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q - 2) + y^2\,(8q + 1) + z^2\,(8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 3

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 2 \\ -8 & -2 & -2 & 13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 2 & 1 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -1 & 2 & 2 \\ 8 & -1 & 2 \\ 2 & 1 & 2 \\ 5 & 0 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 0 \cdot y + 2 \cdot z + 2 \cdot t = 9 \\ 9 \cdot x - 2 \cdot y + 3 \cdot z + 1 \cdot t = 0 \\ 9 \cdot x + 3 \cdot y + 7 \cdot z + 9 \cdot t = 0 \\ 1 \cdot x + 2 \cdot y + 1 \cdot z + 3 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 5].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 2) + y^2\,(12q + 1) + z^2\,(12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 4

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ -13 & -6 & 11 & -18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 1 & 2 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 4 & 1 \\ -3 & 6 & 5 \\ 12 & 3 & -1 \\ 2 & 5 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 8 \cdot y + 8 \cdot z + 4 \cdot t = 3 \\ 0 \cdot x + 9 \cdot y + 4 \cdot z + 9 \cdot t = 0 \\ 2 \cdot x + 5 \cdot y + 3 \cdot z + 7 \cdot t = 6 \\ 4 \cdot x + 1 \cdot y + 2 \cdot z + 5 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2(-12q + 1) + yz(6q - 4) + z^2(-12q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 5

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 3 \\ 20 & -16 & 11 & 10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 8 & 4 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -3 & -1 & -2 \\ 9 & 7 & 8 \\ 3 & 3 & 3 \\ 15 & 11 & 13 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 3 \cdot y + 3 \cdot z + 9 \cdot t = 9 \\ 9 \cdot x + 1 \cdot y + 9 \cdot z + 5 \cdot t = 2 \\ -1 \cdot x - 2 \cdot y + 2 \cdot z + 15 \cdot t = 8 \\ 7 \cdot x + 8 \cdot y + 4 \cdot z + 3 \cdot t = 2 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_1^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 6

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ 11 & 15 & 12 & -6 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 3 & 5 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & -3 & -3 \\ 2 & 0 & 3 \\ 1 & 3 & 9 \\ 0 & 6 & 15 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 3 \cdot y + 5 \cdot z + 4 \cdot t = 6 \\ 1 \cdot x + 3 \cdot y + 9 \cdot z + 2 \cdot t = 0 \\ 2 \cdot x + 3 \cdot y + 13 \cdot z + 0 \cdot t = 3 \\ 0 \cdot x + 8 \cdot y + 9 \cdot z + 8 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 + 4x + 4$ find the best approximation with respect to the norm $\int_1^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 6) + y^2\,(-8q + 1) + z^2\,(-8q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 7

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 2 & 3 \\ -13 & -6 & -18 & 5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 9 & 7 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 7 & 9 \\ 8 & 7 & 11 \\ 2 & 7 & 7 \\ -1 & 7 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 9 \cdot y + 7 \cdot z + 8 \cdot t = 9 \\ -3 \cdot x + 5 \cdot y + 7 \cdot z + 2 \cdot t = 5 \\ 2 \cdot x + 7 \cdot y + 7 \cdot z + 5 \cdot t = 8 \\ 2 \cdot x + 2 \cdot y + 9 \cdot z + 1 \cdot t = 5 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(2q + 6) + y^2\,(-4q + 1) + z^2\,(-4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 8

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 3 \\ 0 & -20 & 19 & 20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 4 & 0 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 13 & 15 & -2 \\ 3 & 3 & 4 \\ 8 & 9 & 1 \\ -2 & -3 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 9 \cdot y + 1 \cdot z + 3 \cdot t = 9 \\ 13 \cdot x + 14 \cdot y + 2 \cdot z - 2 \cdot t = 8 \\ 7 \cdot x + 8 \cdot y + 4 \cdot z + 5 \cdot t = 5 \\ 3 \cdot x + 4 \cdot y + 0 \cdot z + 8 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2(-4q + 1) + yz(2q + 2) + z^2(-4q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 9

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 2 \\ 17 & 18 & -3 & 17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 6 & 1 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 14 & 4 \\ 7 & 2 & 6 \\ 7 & 8 & 5 \\ 7 & -4 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 12 \cdot x + 10 \cdot y + 9 \cdot z + 9 \cdot t = 5 \\ 7 \cdot x + 8 \cdot y + 5 \cdot z + 7 \cdot t = 2 \\ 2 \cdot x + 6 \cdot y + 1 \cdot z + 5 \cdot t = 9 \\ 6 \cdot x + 0 \cdot y + 8 \cdot z + 1 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 5x + 5$ find the best approximation with respect to the norm $\int_{-2}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q + 6) + z^2\,(4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 10

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 0 & 1 & 2 & 3 \\ -6 & -17 & 2 & -8 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 6 & 2 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 13 & -2 \\ 4 & 8 & 2 \\ 5 & 3 & 6 \\ 6 & -2 & 10 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 0 \cdot y + 6 \cdot z + 8 \cdot t = 5 \\ 3 \cdot x + 6 \cdot y + 2 \cdot z + 1 \cdot t = 7 \\ 4 \cdot x + 8 \cdot y + 2 \cdot z + 5 \cdot t = 6 \\ 5 \cdot x + 10 \cdot y + 2 \cdot z + 9 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 3x + 4$ find the best approximation with respect to the norm $\int_{-2}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\,(2q + 6) + y^2\,(-4q + 1) + z^2\,(-4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 11

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 2 \\ 0 & 11 & 16 & 7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 7 & 4 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -2 & 14 & 10 \\ 4 & 4 & 4 \\ 7 & -1 & 1 \\ 1 & 9 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 1 \cdot y + 1 \cdot z + 6 \cdot t = 1 \\ -1 \cdot x + 1 \cdot y + 4 \cdot z - 3 \cdot t = 7 \\ 4 \cdot x + 4 \cdot y + 4 \cdot z + 1 \cdot t = 5 \\ 9 \cdot x + 7 \cdot y + 4 \cdot z + 5 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 - 5x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + xz\,(2q + 6) + y^2\,(-4q + 1) + z^2\,(-4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 12

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 2 & 3 \\ -18 & -15 & -3 & 9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 6 & 6 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 7 & 9 \\ 1 & 3 & 3 \\ 0 & 1 & 0 \\ 2 & 5 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} -3 \cdot x + 0 \cdot y + 0 \cdot z + 2 \cdot t = 2 \\ 1 \cdot x + 3 \cdot y + 3 \cdot z + 2 \cdot t = 2 \\ 5 \cdot x + 6 \cdot y + 6 \cdot z + 2 \cdot t = 1 \\ 5 \cdot x + 1 \cdot y + 9 \cdot z + 1 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 + 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q - 6) + z^2\,(4q - 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 13

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 1 \\ -1 & 12 & -5 & -4 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 0 & 3 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 15 & 0 & 12 \\ -6 & 12 & -6 \\ 8 & 4 & 6 \\ 1 & 8 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 0 \cdot y + 3 \cdot z + 0 \cdot t = 7 \\ 8 \cdot x + 8 \cdot y + 9 \cdot z + 2 \cdot t = 0 \\ 8 \cdot x + 4 \cdot y + 6 \cdot z + 1 \cdot t = 3 \\ 0 \cdot x + 5 \cdot y + 3 \cdot z + 4 \cdot t = 1 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 3x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-6) + z^2\,(4q-3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 14

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 2 & 3 \\ -11 & -10 & -11 & -3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 9 & 1 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 7 & 5 \\ 6 & 9 & 9 \\ 7 & 11 & 13 \\ 4 & 5 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 9 \cdot y + 1 \cdot z + 4 \cdot t = 0 \\ 1 \cdot x + 5 \cdot y + 9 \cdot z + 8 \cdot t = 9 \\ 5 \cdot x + 7 \cdot y + 5 \cdot z + 6 \cdot t = 8 \\ 8 \cdot x + 3 \cdot y + 0 \cdot z + 3 \cdot t = 8 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 3x - 5$ find the best approximation with respect to the norm $\int_{-1}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 2) + y^2\,(12q + 1) + z^2\,(12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 15

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ -5 & 9 & 3 & -2 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 2 & 2 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 5 & 3 \\ 3 & 7 & 2 \\ -1 & 9 & 1 \\ 11 & 3 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 1 \cdot y + 0 \cdot z + 0 \cdot t = 4 \\ 7 \cdot x + 8 \cdot y + 4 \cdot z + 5 \cdot t = 1 \\ 7 \cdot x + 2 \cdot y + 2 \cdot z + 1 \cdot t = 8 \\ 7 \cdot x + 5 \cdot y + 3 \cdot z + 3 \cdot t = 8 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 5x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 16

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 0 & 17 & -2 & -5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 4 & 5 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & -5 & -4 \\ 1 & 7 & 4 \\ 4 & 1 & 0 \\ -2 & 13 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 1 \cdot y + 0 \cdot z + 1 \cdot t = 3 \\ 7 \cdot x + 4 \cdot y + 5 \cdot z + 2 \cdot t = 6 \\ 1 \cdot x - 2 \cdot y - 5 \cdot z + 0 \cdot t = 2 \\ 5 \cdot x + 8 \cdot y + 5 \cdot z + 0 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q + 1) + yz\,(2q - 2) + z^2\,(-4q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 17

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 2 \\ -14 & -6 & -18 & 10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 5 & 2 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & -5 & 4 \\ 8 & 16 & 7 \\ 6 & 2 & 5 \\ 7 & 9 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 12 \cdot x + 13 \cdot y + 10 \cdot z + 11 \cdot t = 5 \\ 7 \cdot x + 9 \cdot y + 6 \cdot z + 6 \cdot t = 9 \\ 2 \cdot x + 5 \cdot y + 2 \cdot z + 1 \cdot t = 2 \\ 0 \cdot x + 6 \cdot y + 5 \cdot z + 6 \cdot t = 5 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 3x - 5$ find the best approximation with respect to the norm $\int_0^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment [0 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 6) + y^2\,(-12q + 1) + z^2\,(-12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 18

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 2 \\ 0 & 3 & 14 & -17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 7 & 4 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 11 & 14 & 3 \\ 9 & 9 & 5 \\ 5 & -1 & 9 \\ 7 & 4 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 14 \cdot x + 11 \cdot y + 6 \cdot z + 14 \cdot t = 0 \\ 4 \cdot x + 7 \cdot y + 4 \cdot z + 0 \cdot t = 9 \\ 2 \cdot x + 3 \cdot y + 0 \cdot z + 0 \cdot t = 3 \\ 9 \cdot x + 9 \cdot y + 5 \cdot z + 7 \cdot t = 2 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_0^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-2) + z^2\,(4q-1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 19

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 2 & 3 \\ 10 & 9 & 1 & -7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 0 & 2 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & -8 & 14 \\ 5 & 0 & 7 \\ 5 & 8 & 0 \\ 5 & 16 & -7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 0 \cdot y + 12 \cdot z + 2 \cdot t = 2 \\ 5 \cdot x + 0 \cdot y + 7 \cdot z + 5 \cdot t = 1 \\ 9 \cdot x + 6 \cdot y + 4 \cdot z + 1 \cdot t = 4 \\ 8 \cdot x + 0 \cdot y + 2 \cdot z + 8 \cdot t = 1 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 - 5x + 5$ find the best approximation with respect to the norm $\int_0^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 2) + y^2\,(12q + 1) + z^2\,(12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 20

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 3 \\ 16 & -4 & 1 & 14 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 5 & 2 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 9 & 5 \\ 4 & 0 & 5 \\ 1 & -9 & 5 \\ 10 & 18 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 14 \cdot x + 13 \cdot y + 8 \cdot z + 3 \cdot t = 4 \\ 7 \cdot x + 9 \cdot y + 5 \cdot z + 4 \cdot t = 6 \\ 0 \cdot x + 5 \cdot y + 2 \cdot z + 5 \cdot t = 3 \\ 4 \cdot x + 4 \cdot y + 9 \cdot z + 4 \cdot t = 3 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_1^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 21

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ -13 & 4 & -15 & -7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 8 & 5 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 7 & 8 \\ 1 & 13 & 15 \\ 1 & -5 & -6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 8 \cdot y + 7 \cdot z + 4 \cdot t = 3 \\ -5 \cdot x - 6 \cdot y - 3 \cdot z - 3 \cdot t = 4 \\ 1 \cdot x + 1 \cdot y + 1 \cdot z + 1 \cdot t = 6 \\ 7 \cdot x + 8 \cdot y + 5 \cdot z + 5 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_1^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 6) + y^2\,(-8q + 1) + z^2\,(-8q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 22

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 3 \\ 13 & 16 & 4 & -13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 8 & 4 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 8 & 8 \\ 5 & 12 & 8 \\ 1 & 4 & 8 \\ -1 & 0 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 8 \cdot y + 4 \cdot z + 7 \cdot t = 9 \\ 2 \cdot x + 8 \cdot y + 12 \cdot z - 5 \cdot t = 8 \\ 8 \cdot x + 5 \cdot y + 9 \cdot z + 0 \cdot t = 7 \\ 3 \cdot x + 8 \cdot y + 8 \cdot z + 1 \cdot t = 1 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_1^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 4) + y^2\,(-12q + 1) + z^2\,(-12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 23

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 2 & 3 \\ 17 & -10 & 4 & 11 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 2 & 2 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 13 & -3 \\ 9 & -2 & 12 \\ 5 & 8 & 2 \\ 7 & 3 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 4 \cdot y + 12 \cdot z + 10 \cdot t = 8 \\ 7 \cdot x + 3 \cdot y + 7 \cdot z + 5 \cdot t = 8 \\ 8 \cdot x + 2 \cdot y + 2 \cdot z + 0 \cdot t = 6 \\ 0 \cdot x + 1 \cdot y + 7 \cdot z + 6 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 4x - 6$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+2) + z^2\,(4q+1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 24

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 1 & 2 & 3 \\ 13 & -4 & -7 & 12 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 5 \\ 17 & 1 & -5 \\ 9 & 1 & 0 \\ -7 & 1 & 10 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 1 \cdot y + 5 \cdot z + 9 \cdot t = 2 \\ 2 \cdot x + 0 \cdot y + 7 \cdot z + 8 \cdot t = 3 \\ 1 \cdot x + 2 \cdot y + 10 \cdot z + 15 \cdot t = 5 \\ 1 \cdot x + 0 \cdot y + 0 \cdot z + 3 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x - 6$ find the best approximation with respect to the norm $\int_{-2}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 2) + y^2\,(12q + 1) + z^2\,(12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 25

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 3 \\ -19 & 5 & 17 & 1 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 9 & 5 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 2 & 8 & 5 \\ 8 & -2 & 13 \\ -1 & 13 & 1 \\ 5 & 3 & 9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 1 \cdot y + 9 \cdot z + 9 \cdot t = 3 \\ 2 \cdot x + 8 \cdot y + 5 \cdot z + 5 \cdot t = 0 \\ 1 \cdot x + 7 \cdot y + 5 \cdot z + 3 \cdot t = 4 \\ 3 \cdot x + 9 \cdot y + 5 \cdot z + 7 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 26

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 3 \\ -10 & -18 & -18 & -4 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 7 & 4 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 3 & 10 \\ 2 & 2 & 7 \\ 4 & 0 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 1 \cdot y + 4 \cdot z + 2 \cdot t = 9 \\ 2 \cdot x + 7 \cdot y + 4 \cdot z + 0 \cdot t = 1 \\ 3 \cdot x + 9 \cdot y + 1 \cdot z + 6 \cdot t = 5 \\ 4 \cdot x - 5 \cdot y + 4 \cdot z + 4 \cdot t = 1 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 3x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\left(-4q - 2\right) + y^2\left(8q + 1\right) + z^2\left(8q - 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 27

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 2 \\ -13 & 6 & -7 & 10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 3 & 2 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -1 & -3 & -1 \\ 3 & 5 & 7 \\ 5 & 9 & 11 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 2 \cdot y + 3 \cdot z + 9 \cdot t = 3 \\ 1 \cdot x + 3 \cdot y + 2 \cdot z + 6 \cdot t = 1 \\ 3 \cdot x + 5 \cdot y + 7 \cdot z + 1 \cdot t = 1 \\ 5 \cdot x + 7 \cdot y + 12 \cdot z - 4 \cdot t = 2 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_{-2}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 2) + y^2\,(8q + 1) + z^2\,(8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 28

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 3 \\ 8 & 10 & -18 & 7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 2 & 9 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & -3 & 4 \\ 9 & 9 & 1 \\ 7 & 1 & 3 \\ 8 & 5 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 9 \cdot y + 9 \cdot z + 4 \cdot t = 3 \\ 5 \cdot x + 2 \cdot y + 9 \cdot z + 0 \cdot t = 0 \\ 9 \cdot x + 0 \cdot y - 3 \cdot z + 16 \cdot t = 7 \\ 7 \cdot x + 1 \cdot y + 3 \cdot z + 8 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 4x - 6$ find the best approximation with respect to the norm $\int_{-1}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+2) + z^2\,(4q+1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 29

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 2 & 3 \\ -1 & 14 & -15 & -6 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 7 & 5 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 1 & 6 \\ 3 & 2 & 7 \\ -3 & 4 & 9 \\ 0 & 3 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 7 \cdot y + 5 \cdot z + 1 \cdot t = 9 \\ 7 \cdot x + 8 \cdot y + 7 \cdot z + 9 \cdot t = 6 \\ -2 \cdot x - 1 \cdot y + 11 \cdot z + 5 \cdot t = 7 \\ 0 \cdot x + 3 \cdot y + 8 \cdot z + 3 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 5x + 5$ find the best approximation with respect to the norm $\int_{-1}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+2) + z^2\,(4q+1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 30

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 17 & 18 & -9 & 9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 0 & 8 & 9 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 17 & -7 & 14 \\ -7 & 17 & -7 \\ 9 & 1 & 7 \\ 1 & 9 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 1 \cdot y + 7 \cdot z + 1 \cdot t = 6 \\ 9 \cdot x + 2 \cdot y + 6 \cdot z - 7 \cdot t = 1 \\ 9 \cdot x + 0 \cdot y + 8 \cdot z + 9 \cdot t = 8 \\ 5 \cdot x + 6 \cdot y + 5 \cdot z + 0 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_{-1}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 2) + y^2\,(-12q + 1) + z^2\,(-12q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 31

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 2 \\ 6 & 7 & 8 & -20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 0 & 9 & 9 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 7 & 0 \\ 7 & 9 & 0 \\ 9 & 10 & 0 \\ 5 & 8 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 16 \cdot y - 9 \cdot z + 5 \cdot t = 6 \\ 4 \cdot x + 8 \cdot y + 1 \cdot z + 7 \cdot t = 4 \\ 5 \cdot x + 8 \cdot y + 0 \cdot z + 7 \cdot t = 1 \\ 9 \cdot x + 0 \cdot y + 9 \cdot z + 9 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 4x - 6$ find the best approximation with respect to the norm $\int_{-1}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-8q + 1\right) + yz\left(4q + 2\right) + z^2\left(-8q + 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 32

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 20 & 6 & -1 & -5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 4 & 0 & 9 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 7 & 1 \\ 8 & -1 & 7 \\ 8 & 3 & 4 \\ 8 & 11 & -2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 13 \cdot x + 10 \cdot y + 2 \cdot z + 7 \cdot t = 1 \\ 3 \cdot x + 4 \cdot y + 0 \cdot z + 9 \cdot t = 8 \\ 3 \cdot x + 7 \cdot y + 0 \cdot z + 2 \cdot t = 9 \\ 8 \cdot x + 7 \cdot y + 1 \cdot z + 8 \cdot t = 8 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q + 1) + yz\,(2q + 6) + z^2\,(-4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 33

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 3 \\ -6 & 7 & -16 & 16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 6 & 6 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -1 & 16 & 3 \\ 7 & 2 & 9 \\ 11 & -5 & 12 \\ 3 & 9 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 2 \cdot y + 9 \cdot z + 3 \cdot t = 0 \\ 9 \cdot x + 6 \cdot y + 6 \cdot z + 4 \cdot t = 5 \\ 4 \cdot x + 7 \cdot y + 8 \cdot z + 6 \cdot t = 8 \\ 5 \cdot x - 2 \cdot y + 12 \cdot z + 2 \cdot t = 9 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 3x + 5$ find the best approximation with respect to the norm $\int_0^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-4) + z^2\,(4q-2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 34

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 2 & 3 \\ 14 & -17 & -19 & -9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 1 & 8 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 7 & 5 \\ 2 & 3 & 1 \\ 10 & 11 & 9 \\ -2 & -1 & -3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 5 \cdot y + 1 \cdot z + 4 \cdot t = 8 \\ 6 \cdot x + 7 \cdot y + 5 \cdot z + 2 \cdot t = 3 \\ 9 \cdot x + 13 \cdot y + 2 \cdot z - 2 \cdot t = 5 \\ 3 \cdot x + 1 \cdot y + 8 \cdot z + 6 \cdot t = 9 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 4x - 6$ find the best approximation with respect to the norm $\int_0^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 6) + y^2\,(-12q + 1) + z^2\,(-12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 35

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 1 & 2 & 3 \\ 10 & 0 & -3 & 0 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 1 & 4 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 2 & 7 \\ 0 & 4 & 1 \\ 0 & 3 & 4 \\ 0 & 5 & -2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 1 \cdot y + 4 \cdot z + 2 \cdot t = 4 \\ -4 \cdot x + 5 \cdot y + 4 \cdot z - 2 \cdot t = 7 \\ 8 \cdot x + 7 \cdot y + 3 \cdot z + 3 \cdot t = 3 \\ 0 \cdot x + 3 \cdot y + 4 \cdot z + 0 \cdot t = 8 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 5x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 5].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q - 2) + y^2\,(8q + 1) + z^2\,(8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 36

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 13 & -19 & 14 & 5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 4 \\ 8 & 1 & 1 \\ -2 & -1 & 7 \\ 13 & 2 & -2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 5 \cdot y + 8 \cdot z + 6 \cdot t = 3 \\ 1 \cdot x + 1 \cdot y + 1 \cdot z + 2 \cdot t = 8 \\ 5 \cdot x - 1 \cdot y + 7 \cdot z + 14 \cdot t = 5 \\ 3 \cdot x + 0 \cdot y + 4 \cdot z + 8 \cdot t = 5 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 5x - 6$ find the best approximation with respect to the norm $\int_1^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 6) + y^2\,(12q + 1) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 37

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 1 \\ -3 & -19 & -19 & 19 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 1 & 1 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 7 & 4 & 3 \\ -7 & -2 & -1 \\ 14 & 7 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 1 \cdot y + 7 \cdot z + 1 \cdot t = 2 \\ 1 \cdot x + 1 \cdot y + 1 \cdot z + 7 \cdot t = 4 \\ 13 \cdot x + 7 \cdot y + 5 \cdot z - 7 \cdot t = 9 \\ 7 \cdot x + 4 \cdot y + 3 \cdot z + 0 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 5x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 38

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 0 \\ -18 & -6 & 18 & -3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 1 & 5 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 4 & 8 \\ 2 & 6 & 1 \\ 4 & 8 & -6 \\ -2 & 2 & 15 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 2 \cdot y + 2 \cdot z + 9 \cdot t = 6 \\ -6 \cdot x + 7 \cdot y + 11 \cdot z + 0 \cdot t = 0 \\ 0 \cdot x + 4 \cdot y + 8 \cdot z + 2 \cdot t = 4 \\ 6 \cdot x + 1 \cdot y + 5 \cdot z + 4 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 5x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-4q + 1\right) + yz\left(2q + 4\right) + z^2\left(-4q + 2\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 39

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 11 & -13 & -4 & -6 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 8 & 1 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 6 & 8 \\ 0 & 6 & 7 \\ 0 & 6 & 9 \\ 0 & 6 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} -6 \cdot x + 4 \cdot y + 13 \cdot z - 4 \cdot t = 7 \\ 6 \cdot x + 6 \cdot y + 1 \cdot z + 7 \cdot t = 0 \\ 6 \cdot x + 8 \cdot y + 1 \cdot z + 4 \cdot t = 9 \\ 0 \cdot x + 6 \cdot y + 7 \cdot z + 0 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 3x + 4$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment [1 , 5].

6. Find all the values of q such that the equation $2x^2 + xz\left(-4q + 2\right) + y^2\left(8q + 1\right) + z^2\left(8q + 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 40

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 3 \\ 18 & -19 & -12 & 3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 8 & 7 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 6 \\ 6 & 1 & 8 \\ -6 & 1 & 4 \\ 12 & 1 & 10 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 2 \cdot y + 8 \cdot z + 1 \cdot t = 6 \\ 0 \cdot x + 1 \cdot y + 6 \cdot z + 6 \cdot t = 2 \\ 1 \cdot x + 8 \cdot y + 7 \cdot z + 5 \cdot t = 4 \\ -1 \cdot x - 6 \cdot y + 5 \cdot z + 7 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 4) + y^2\,(-8q + 1) + z^2\,(-8q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 41

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 2 & 3 \\ 0 & -7 & 5 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 5 & 5 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 1 & 4 \\ 14 & -7 & 3 \\ 4 & 9 & 5 \\ -1 & 17 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 1 \cdot y + 4 \cdot z + 4 \cdot t = 7 \\ 4 \cdot x + 5 \cdot y + 1 \cdot z + 5 \cdot t = 8 \\ 9 \cdot x - 3 \cdot y + 3 \cdot z + 2 \cdot t = 8 \\ 9 \cdot x + 5 \cdot y + 5 \cdot z + 6 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 5x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(8q + 1) + yz\,(-4q - 2) + z^2\,(8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 42

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 3 \\ 1 & -12 & 8 & -3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 8 & 3 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 6 & 15 \\ 7 & 4 & 1 \\ 6 & 3 & -6 \\ 8 & 5 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 8 \cdot y + 3 \cdot z + 1 \cdot t = 9 \\ 9 \cdot x + 0 \cdot y - 1 \cdot z + 15 \cdot t = 3 \\ 2 \cdot x + 6 \cdot y + 5 \cdot z + 4 \cdot t = 4 \\ 7 \cdot x + 4 \cdot y + 1 \cdot z + 8 \cdot t = 8 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 3x + 4$ find the best approximation with respect to the norm $\int_{-2}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q + 1) + yz\,(2q + 4) + z^2\,(-4q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 43

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 2 & 3 \\ -4 & 20 & -10 & -17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 8 & 3 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 18 & -4 & -4 \\ 9 & 2 & 2 \\ 0 & 8 & 8 \\ -9 & 14 & 14 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 0 \cdot y + 5 \cdot z + 6 \cdot t = 7 \\ 9 \cdot x + 2 \cdot y + 2 \cdot z + 0 \cdot t = 9 \\ 8 \cdot x + 8 \cdot y + 3 \cdot z + 4 \cdot t = 2 \\ 10 \cdot x - 4 \cdot y + 1 \cdot z - 4 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 4x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q + 6) + z^2\,(4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 44

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 19 & -20 & 18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 9 & 6 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 12 & -3 & 11 \\ 6 & 7 & 7 \\ 3 & 12 & 5 \\ 9 & 2 & 9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 7 \cdot y + 0 \cdot z + 6 \cdot t = 2 \\ 6 \cdot x + 7 \cdot y + 7 \cdot z + 9 \cdot t = 6 \\ 10 \cdot x + 5 \cdot y + 8 \cdot z + 13 \cdot t = 6 \\ 2 \cdot x + 9 \cdot y + 6 \cdot z + 5 \cdot t = 9 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 4) + y^2\,(-8q + 1) + z^2\,(-8q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 45

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 2 & 3 \\ 15 & -5 & -20 & 3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 7 & 3 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -3 & 0 & 13 \\ 3 & 0 & 7 \\ 15 & 0 & -5 \\ 9 & 0 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 0 \cdot y + 1 \cdot z + 3 \cdot t = 2 \\ 18 \cdot x - 7 \cdot y - 1 \cdot z + 6 \cdot t = 6 \\ 0 \cdot x + 7 \cdot y + 3 \cdot z + 0 \cdot t = 5 \\ 7 \cdot x + 3 \cdot y + 9 \cdot z + 9 \cdot t = 1 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 5x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q - 4) + z^2\,(4q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 46

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 2 \\ -3 & 17 & 3 & -9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 5 & 7 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 1 & 5 \\ 6 & 0 & 9 \\ 10 & 2 & 1 \\ 4 & -1 & 13 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 11 \cdot x - 5 \cdot y + 11 \cdot z + 13 \cdot t = 2 \\ 6 \cdot x + 0 \cdot y + 9 \cdot z + 8 \cdot t = 7 \\ 1 \cdot x + 5 \cdot y + 7 \cdot z + 3 \cdot t = 3 \\ 0 \cdot x + 5 \cdot y + 0 \cdot z + 7 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 4x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 47

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 1 \\ -17 & 17 & -20 & 13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 0 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 2 & 8 & 4 \\ 3 & 3 & 5 \\ 1 & 13 & 3 \\ 4 & -2 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 5 \cdot y + 0 \cdot z + 1 \cdot t = 4 \\ 2 \cdot x + 8 \cdot y + 4 \cdot z + 3 \cdot t = 4 \\ 1 \cdot x + 11 \cdot y + 8 \cdot z + 5 \cdot t = 1 \\ 7 \cdot x + 5 \cdot y + 3 \cdot z + 9 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 3x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 48

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 2 & 3 \\ 9 & -2 & 16 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 9 & 2 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 5 & 9 \\ 1 & -3 & 9 \\ 1 & 1 & 9 \\ 1 & 9 & 9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 1 \cdot y + 9 \cdot z + 1 \cdot t = 9 \\ -3 \cdot x - 7 \cdot y + 16 \cdot z + 1 \cdot t = 3 \\ 5 \cdot x + 9 \cdot y + 2 \cdot z + 1 \cdot t = 8 \\ 4 \cdot x + 5 \cdot y + 1 \cdot z + 1 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 - 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 2].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 6) + y^2\,(-12q + 1) + z^2\,(-12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 49

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 3 \\ 10 & -17 & -3 & -12 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 2 & 2 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 3 & 2 \\ 9 & 1 & 6 \\ 6 & 2 & 4 \\ 0 & 4 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 8 \cdot y + 3 \cdot z + 1 \cdot t = 9 \\ 9 \cdot x + 2 \cdot y + 6 \cdot z - 1 \cdot t = 4 \\ 3 \cdot x + 2 \cdot y + 2 \cdot z + 7 \cdot t = 7 \\ 6 \cdot x + 2 \cdot y + 4 \cdot z + 3 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 - 5x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-8q + 1\right) + yz\left(4q - 2\right) + z^2\left(-8q - 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 50

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 3 \\ -12 & -20 & -6 & -13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 9 & 7 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 12 & -9 & 16 \\ 0 & 18 & -5 \\ 8 & 0 & 9 \\ 4 & 9 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 2 \cdot y + 1 \cdot z + 4 \cdot t = 8 \\ 8 \cdot x + 9 \cdot y - 3 \cdot z + 16 \cdot t = 1 \\ 0 \cdot x + 9 \cdot y + 7 \cdot z + 0 \cdot t = 3 \\ 4 \cdot x + 9 \cdot y + 2 \cdot z + 8 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q - 2) + y^2\,(8q + 1) + z^2\,(8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 51

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 3 \\ -10 & 13 & 14 & 17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 2 & 5 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -1 & -1 & -1 \\ 7 & 3 & 5 \\ 11 & 5 & 8 \\ 3 & 1 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 2 \cdot y + 5 \cdot z + 2 \cdot t = 3 \\ 7 \cdot x + 3 \cdot y + 5 \cdot z + 3 \cdot t = 9 \\ 1 \cdot x + 2 \cdot y + 5 \cdot z + 0 \cdot t = 0 \\ 13 \cdot x + 4 \cdot y + 5 \cdot z + 6 \cdot t = 5 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 5].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 6) + y^2\,(12q + 1) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 52

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 2 & 3 \\ -2 & -8 & 6 & 5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 6 & 9 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 2 & 6 \\ 7 & 14 & 0 \\ 10 & -4 & 9 \\ 8 & 8 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 6 \cdot y + 9 \cdot z + 3 \cdot t = 3 \\ 14 \cdot x + 10 \cdot y - 3 \cdot z + 15 \cdot t = 5 \\ 4 \cdot x + 2 \cdot y + 5 \cdot z + 5 \cdot t = 1 \\ 8 \cdot x + 8 \cdot y + 3 \cdot z + 9 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 3x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 6) + y^2\,(12q + 1) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 53

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 2 \\ -7 & 15 & 18 & 15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 6 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -3 & 0 & 4 \\ 2 & 3 & 5 \\ 12 & 9 & 7 \\ 7 & 6 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 4 \cdot y + 3 \cdot z + 4 \cdot t = 7 \\ 11 \cdot x + 7 \cdot y + 6 \cdot z - 1 \cdot t = 3 \\ 7 \cdot x + 6 \cdot y + 6 \cdot z + 2 \cdot t = 8 \\ 3 \cdot x + 5 \cdot y + 6 \cdot z + 5 \cdot t = 3 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_1^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-2) + z^2\,(4q-1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 54

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 2 \\ 15 & 17 & -5 & -20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 16 & 6 \\ 6 & 8 & 3 \\ 5 & 0 & 0 \\ 4 & -8 & -3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 7 \cdot y + 4 \cdot z + 5 \cdot t = 0 \\ 6 \cdot x + 8 \cdot y + 3 \cdot z + 5 \cdot t = 9 \\ 12 \cdot x + 16 \cdot y + 4 \cdot z + 10 \cdot t = 3 \\ 0 \cdot x + 0 \cdot y + 2 \cdot z + 0 \cdot t = 2 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x + 4$ find the best approximation with respect to the norm $\int_1^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 2) + y^2\,(12q + 1) + z^2\,(12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 55

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 1 & 2 & 3 \\ -14 & 14 & 19 & -13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 7 & 6 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 2 & 7 & 7 \\ 2 & 6 & 8 \\ 2 & 9 & 5 \\ 2 & 8 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 8 \cdot y + 6 \cdot z + 2 \cdot t = 3 \\ -3 \cdot x + 9 \cdot y + 6 \cdot z + 2 \cdot t = 1 \\ 8 \cdot x + 1 \cdot y + 1 \cdot z + 2 \cdot t = 6 \\ 7 \cdot x + 7 \cdot y + 6 \cdot z + 2 \cdot t = 9 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 3x - 6$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-4) + z^2\,(4q-2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 56

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 3 \\ -7 & 15 & 4 & 5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 0 & 6 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 0 & 3 \\ 9 & 18 & -3 \\ 8 & 9 & 0 \\ 6 & -9 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 2 \cdot y + 8 \cdot z + 3 \cdot t = 7 \\ 9 \cdot x + 0 \cdot y + 6 \cdot z + 3 \cdot t = 8 \\ 7 \cdot x + 0 \cdot y + 3 \cdot z + 8 \cdot t = 0 \\ 5 \cdot x + 0 \cdot y + 0 \cdot z + 13 \cdot t = 1 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 5x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 2) + y^2\,(-8q + 1) + z^2\,(-8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 57

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 3 \\ -20 & 18 & 9 & -11 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 0 & 6 & 9 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -6 & 5 & 18 \\ 0 & 6 & 9 \\ 6 & 7 & 0 \\ 12 & 8 & -9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} -7 \cdot x + 12 \cdot y + 12 \cdot z + 3 \cdot t = 9 \\ 0 \cdot x + 6 \cdot y + 9 \cdot z + 6 \cdot t = 8 \\ 7 \cdot x + 0 \cdot y + 6 \cdot z + 9 \cdot t = 4 \\ 8 \cdot x + 6 \cdot y + 3 \cdot z + 2 \cdot t = 2 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 - 4x - 6$ find the best approximation with respect to the norm $\int_{-2}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 6) + y^2\,(12q + 1) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 58

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 2 & 3 \\ 7 & -8 & 20 & 2 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 7 & 8 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 8 & 7 \\ 12 & 8 & 1 \\ 9 & 8 & 4 \\ 3 & 8 & 10 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 7 \cdot y + 8 \cdot z + 6 \cdot t = 4 \\ 9 \cdot x + 8 \cdot y + 4 \cdot z + 6 \cdot t = 4 \\ 0 \cdot x + 2 \cdot y + 8 \cdot z + 8 \cdot t = 4 \\ 10 \cdot x + 9 \cdot y + 0 \cdot z + 6 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 - 5x + 4$ find the best approximation with respect to the norm $\int_{-2}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 6) + y^2\,(-8q + 1) + z^2\,(-8q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 59

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 1 \\ 6 & -14 & 19 & 10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 7 & 4 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 1 & 8 \\ 16 & -4 & 9 \\ 0 & 6 & 7 \\ -8 & 11 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 2 \cdot y + 1 \cdot z + 0 \cdot t = 2 \\ 10 \cdot x - 5 \cdot y + 12 \cdot z - 2 \cdot t = 1 \\ 8 \cdot x + 1 \cdot y + 8 \cdot z + 0 \cdot t = 8 \\ 6 \cdot x + 7 \cdot y + 4 \cdot z + 2 \cdot t = 1 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 4x + 4$ find the best approximation with respect to the norm $\int_{-2}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q - 6) + z^2\,(4q - 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 60

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 2 \\ 18 & 5 & -16 & 14 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 4 & 2 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -1 & -8 & 8 \\ 14 & 16 & 2 \\ 4 & 0 & 6 \\ 9 & 8 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 0 \cdot y + 6 \cdot z + 9 \cdot t = 1 \\ 8 \cdot x + 4 \cdot y + 2 \cdot z + 8 \cdot t = 0 \\ 0 \cdot x - 4 \cdot y + 10 \cdot z + 10 \cdot t = 5 \\ 4 \cdot x + 8 \cdot y + 9 \cdot z + 4 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 61

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 2 \\ -3 & -16 & -11 & 7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 1 & 1 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 9 & 0 \\ 8 & 6 & 1 \\ 11 & 3 & 2 \\ 2 & 12 & -1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 1 \cdot y + 1 \cdot z + 4 \cdot t = 4 \\ 9 \cdot x + 4 \cdot y + 2 \cdot z + 0 \cdot t = 2 \\ 5 \cdot x + 9 \cdot y + 0 \cdot z + 8 \cdot t = 7 \\ 4 \cdot x + 17 \cdot y - 1 \cdot z + 12 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x - 6$ find the best approximation with respect to the norm $\int_{-1}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q - 2) + y^2\,(-12q + 1) + z^2\,(-12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 62

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 3 \\ 19 & -5 & 15 & -13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 7 & 3 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 17 & -5 \\ 3 & 9 & 1 \\ 0 & 1 & 7 \\ -3 & -7 & 13 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 4 \cdot y + 3 \cdot z + 4 \cdot t = 1 \\ 1 \cdot x + 7 \cdot y + 3 \cdot z + 7 \cdot t = 6 \\ 5 \cdot x + 11 \cdot y - 1 \cdot z - 7 \cdot t = 7 \\ 3 \cdot x + 9 \cdot y + 1 \cdot z + 0 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 3x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q - 6) + z^2\,(4q - 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 63

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 2 \\ -19 & 5 & -9 & -13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 9 & 7 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 9 \\ 9 & 15 & -5 \\ -3 & -6 & 16 \\ 5 & 8 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 9 \cdot y + 7 \cdot z + 8 \cdot t = 1 \\ 0 \cdot x + 8 \cdot y + 7 \cdot z + 3 \cdot t = 9 \\ 5 \cdot x + 8 \cdot y + 2 \cdot z + 1 \cdot t = 8 \\ 9 \cdot x + 7 \cdot y - 3 \cdot z - 6 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 4x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + xy\left(-6q - 2\right) + y^2\left(12q + 1\right) + z^2\left(12q - 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 64

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 1 & 2 \\ -11 & -18 & 14 & 19 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 9 & 2 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 16 & -4 & 13 \\ 9 & 2 & 9 \\ -5 & 14 & 1 \\ 2 & 8 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 7 \cdot y + 8 \cdot z + 11 \cdot t = 9 \\ 2 \cdot x + 8 \cdot y + 5 \cdot z + 9 \cdot t = 3 \\ 2 \cdot x + 9 \cdot y + 2 \cdot z + 7 \cdot t = 8 \\ 6 \cdot x + 3 \cdot y + 2 \cdot z + 1 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_0^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 4) + y^2\,(-12q + 1) + z^2\,(-12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 65

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 9 & 10 & 8 & 3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 8 & 8 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 14 & 6 & 9 \\ 2 & 8 & 7 \\ -4 & 9 & 6 \\ 8 & 7 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 8 \cdot y + 7 \cdot z + 8 \cdot t = 3 \\ -3 \cdot x + 8 \cdot y + 6 \cdot z + 15 \cdot t = 9 \\ 8 \cdot x + 3 \cdot y + 9 \cdot z + 3 \cdot t = 9 \\ 7 \cdot x + 8 \cdot y + 8 \cdot z + 1 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_0^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(2q + 4) + y^2\,(-4q + 1) + z^2\,(-4q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 66

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 2 & 3 \\ 9 & 1 & 9 & 18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 6 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 11 & 15 & 1 \\ -1 & -3 & 7 \\ 3 & 3 & 5 \\ 7 & 9 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 5 \cdot y + 6 \cdot z + 3 \cdot t = 1 \\ 11 \cdot x + 13 \cdot y + 0 \cdot z + 3 \cdot t = 6 \\ 8 \cdot x + 8 \cdot y + 6 \cdot z + 3 \cdot t = 1 \\ 7 \cdot x + 9 \cdot y + 3 \cdot z + 3 \cdot t = 2 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 67

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 3 \\ 10 & 4 & -4 & 3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 4 & 8 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 7 & 8 \\ 13 & 7 & 12 \\ 5 & 7 & 4 \\ 1 & 7 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 4 \cdot y + 8 \cdot z + 0 \cdot t = 8 \\ 11 \cdot x + 10 \cdot y + 8 \cdot z + 10 \cdot t = 8 \\ 9 \cdot x + 7 \cdot y + 8 \cdot z + 5 \cdot t = 0 \\ 7 \cdot x + 3 \cdot y + 5 \cdot z + 9 \cdot t = 1 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_0^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 4) + y^2\,(-8q + 1) + z^2\,(-8q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 68

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 1 \\ -9 & 18 & -8 & -16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 2 & 9 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -5 & 12 & 16 \\ 9 & 6 & 2 \\ 16 & 3 & -5 \\ 2 & 9 & 9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 3 \cdot y + 2 \cdot z + 8 \cdot t = 3 \\ -2 \cdot x + 16 \cdot y + 9 \cdot z + 14 \cdot t = 8 \\ 6 \cdot x + 2 \cdot y + 9 \cdot z + 4 \cdot t = 4 \\ 2 \cdot x + 9 \cdot y + 9 \cdot z + 9 \cdot t = 1 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_1^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(12q + 1) + yz\,(-6q + 4) + z^2\,(12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 69

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 2 \\ -18 & -1 & -7 & -16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 4 & 5 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & 1 & 9 \\ 9 & 8 & 4 \\ -1 & -6 & 14 \\ 14 & 15 & -1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 1 \cdot y + 9 \cdot z + 9 \cdot t = 7 \\ 8 \cdot x + 4 \cdot y + 5 \cdot z + 1 \cdot t = 1 \\ 0 \cdot x - 2 \cdot y + 13 \cdot z + 17 \cdot t = 7 \\ 2 \cdot x + 3 \cdot y + 4 \cdot z + 2 \cdot t = 2 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_1^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 6) + y^2\,(-8q + 1) + z^2\,(-8q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 70

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 2 \\ -12 & -13 & -1 & 18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 8 & 8 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & -4 & -2 \\ -1 & 11 & 13 \\ 2 & 6 & 8 \\ 5 & 1 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 8 \cdot y + 8 \cdot z + 6 \cdot t = 4 \\ 4 \cdot x - 6 \cdot y - 2 \cdot z - 2 \cdot t = 9 \\ 7 \cdot x + 3 \cdot y + 6 \cdot z + 3 \cdot t = 7 \\ 5 \cdot x + 1 \cdot y + 3 \cdot z + 2 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q + 1) + yz\,(2q - 2) + z^2\,(-4q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 71

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 0 \\ 17 & 11 & 7 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 8 & 1 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & -2 & 10 \\ -1 & 4 & 7 \\ 5 & 0 & 9 \\ 2 & 2 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x - 8 \cdot y + 17 \cdot z - 1 \cdot t = 3 \\ 2 \cdot x + 8 \cdot y + 1 \cdot z + 5 \cdot t = 1 \\ 5 \cdot x + 9 \cdot y + 5 \cdot z + 4 \cdot t = 0 \\ 5 \cdot x + 0 \cdot y + 9 \cdot z + 2 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 5x - 6$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment [1 , 5].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 4) + y^2\,(12q + 1) + z^2\,(12q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 72

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 3 \\ -9 & 4 & -1 & -10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 8 & 3 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 7 & 6 \\ -8 & 11 & 4 \\ 16 & -1 & 10 \\ 8 & 3 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 2 \cdot y + 4 \cdot z + 9 \cdot t = 1 \\ 3 \cdot x + 8 \cdot y + 3 \cdot z + 0 \cdot t = 6 \\ -3 \cdot x + 6 \cdot y + 9 \cdot z + 16 \cdot t = 2 \\ 0 \cdot x + 7 \cdot y + 6 \cdot z + 8 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_{-2}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-4q + 1\right) + yz\left(2q - 2\right) + z^2\left(-4q - 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 73

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 2 \\ 10 & 13 & 13 & 17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 1 & 1 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -2 & 7 & 3 \\ 2 & 4 & 2 \\ 10 & -2 & 0 \\ 6 & 1 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 4 \cdot y + 2 \cdot z + 6 \cdot t = 3 \\ 1 \cdot x + 1 \cdot y + 1 \cdot z + 5 \cdot t = 9 \\ 4 \cdot x + 8 \cdot y + 5 \cdot z + 7 \cdot t = 3 \\ 3 \cdot x + 7 \cdot y + 3 \cdot z + 7 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 - 4x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\left(-4q - 2\right) + y^2\left(8q + 1\right) + z^2\left(8q - 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 74

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 3 \\ -10 & 19 & 2 & 13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 6 & 1 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 7 & 6 \\ 3 & 8 & 8 \\ 0 & 5 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 4 \cdot y + 3 \cdot z + 6 \cdot t = 7 \\ -5 \cdot x + 6 \cdot y + 7 \cdot z - 3 \cdot t = 1 \\ 1 \cdot x + 6 \cdot y + 4 \cdot z + 2 \cdot t = 2 \\ 7 \cdot x + 6 \cdot y + 1 \cdot z + 7 \cdot t = 5 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 3x + 4$ find the best approximation with respect to the norm $\int_{-2}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\,(2q + 6) + y^2\,(-4q + 1) + z^2\,(-4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 75

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 2 & 3 \\ -19 & -5 & -9 & -20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 1 & 0 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 10 & 9 & 1 \\ -5 & 6 & 1 \\ 5 & 8 & 1 \\ 0 & 7 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 7 \cdot y + 1 \cdot z + 5 \cdot t = 8 \\ 3 \cdot x + 6 \cdot y + 1 \cdot z + 7 \cdot t = 5 \\ 8 \cdot x + 1 \cdot y + 0 \cdot z + 0 \cdot t = 5 \\ -8 \cdot x + 13 \cdot y + 2 \cdot z + 10 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + xy\,(2q + 4) + y^2\,(-4q + 1) + z^2\,(-4q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 76

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 2 & 3 \\ -5 & -7 & 1 & -2 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 2 & 7 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 2 & 2 \\ 2 & 5 & 7 \\ 8 & -1 & -3 \\ -1 & 8 & 12 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 5 \cdot y + 7 \cdot z + 5 \cdot t = 5 \\ 1 \cdot x + 3 \cdot y + 2 \cdot z + 8 \cdot t = 9 \\ 2 \cdot x + 2 \cdot y + 7 \cdot z + 8 \cdot t = 8 \\ 2 \cdot x + 8 \cdot y + 7 \cdot z + 2 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 4x + 4$ find the best approximation with respect to the norm $\int_{-1}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\left(-6q - 4\right) + y^2\left(12q + 1\right) + z^2\left(12q - 2\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 77

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 3 \\ 3 & 9 & -6 & 11 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 5 & 4 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 7 & 5 \\ 4 & -7 & -3 \\ 10 & 14 & 9 \\ 6 & 0 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 5 \cdot y + 4 \cdot z + 6 \cdot t = 4 \\ 3 \cdot x + 6 \cdot y + 3 \cdot z + 5 \cdot t = 6 \\ 5 \cdot x - 5 \cdot y - 2 \cdot z + 10 \cdot t = 0 \\ 6 \cdot x + 0 \cdot y + 1 \cdot z + 8 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_{-1}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(12q + 1) + yz\,(-6q + 6) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 78

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 14 & 2 & 20 & 0 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 6 & 8 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 2 & 6 \\ 5 & 2 & 11 \\ 8 & 2 & -4 \\ 7 & 2 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 1 \cdot y + 8 \cdot z + 9 \cdot t = 6 \\ 12 \cdot x - 2 \cdot y - 6 \cdot z + 6 \cdot t = 1 \\ 7 \cdot x + 2 \cdot y + 1 \cdot z + 6 \cdot t = 9 \\ 2 \cdot x + 6 \cdot y + 8 \cdot z + 6 \cdot t = 8 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_{-1}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 79

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ -1 & 0 & 0 & 13 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 7 & 9 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -1 & 0 & 5 \\ 5 & 9 & 11 \\ 3 & 6 & 9 \\ 1 & 3 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 6 \cdot y + 9 \cdot z + 1 \cdot t = 5 \\ 4 \cdot x + 1 \cdot y + 1 \cdot z + 4 \cdot t = 8 \\ 3 \cdot x + 7 \cdot y + 9 \cdot z + 4 \cdot t = 5 \\ 3 \cdot x + 5 \cdot y + 9 \cdot z - 2 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 3x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 80

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 2 & 3 \\ -7 & -8 & 6 & 19 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 2 & 3 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 10 & -3 & 6 \\ 5 & 2 & 4 \\ -5 & 12 & 0 \\ 0 & 7 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 2 \cdot y + 3 \cdot z + 4 \cdot t = 1 \\ 8 \cdot x + 0 \cdot y + 8 \cdot z + 8 \cdot t = 6 \\ 3 \cdot x + 2 \cdot y + 5 \cdot z - 4 \cdot t = 8 \\ 5 \cdot x + 2 \cdot y + 4 \cdot z + 0 \cdot t = 8 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 3x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 81

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 2 \\ -12 & -2 & -1 & 8 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 6 & 8 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 9 & 3 \\ 0 & 7 & 9 \\ 0 & 8 & 6 \\ 0 & 10 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 9 \cdot y + 3 \cdot z + 0 \cdot t = 0 \\ -8 \cdot x + 12 \cdot y - 2 \cdot z - 2 \cdot t = 7 \\ 3 \cdot x + 7 \cdot y + 1 \cdot z + 7 \cdot t = 6 \\ 8 \cdot x + 6 \cdot y + 8 \cdot z + 2 \cdot t = 2 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 2) + y^2\,(-12q + 1) + z^2\,(-12q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 82

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 2 & 3 \\ 16 & 17 & -13 & -8 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 3 & 5 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & -1 & -3 \\ 0 & 14 & 15 \\ 2 & 4 & 3 \\ 1 & 9 & 9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 3 \cdot y + 5 \cdot z + 4 \cdot t = 1 \\ 1 \cdot x + 9 \cdot y + 9 \cdot z + 2 \cdot t = 2 \\ -2 \cdot x + 15 \cdot y + 13 \cdot z + 0 \cdot t = 0 \\ 7 \cdot x + 2 \cdot y + 0 \cdot z + 5 \cdot t = 2 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_0^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 83

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 1 & 2 & 3 \\ -1 & -9 & -9 & -18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 3 & 2 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 5 & 3 \\ 11 & 2 & 3 \\ 7 & 4 & 3 \\ 9 & 3 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 3 \cdot y + 2 \cdot z + 2 \cdot t = 2 \\ 1 \cdot x + 0 \cdot y + 3 \cdot z + 3 \cdot t = 6 \\ 7 \cdot x + 4 \cdot y + 3 \cdot z + 9 \cdot t = 9 \\ 11 \cdot x + 5 \cdot y + 4 \cdot z + 16 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(8q + 1) + yz\,(-4q - 6) + z^2\,(8q - 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 84

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 2 & 3 \\ -8 & 20 & -5 & 2 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 8 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 7 & 9 \\ 11 & 14 & 17 \\ 5 & 0 & 1 \\ 2 & -7 & -7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 1 \cdot y + 8 \cdot z + 5 \cdot t = 1 \\ 6 \cdot x + 1 \cdot y + 6 \cdot z + 2 \cdot t = 7 \\ 8 \cdot x + 7 \cdot y + 9 \cdot z + 5 \cdot t = 2 \\ 16 \cdot x + 13 \cdot y + 10 \cdot z + 5 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 4x + 4$ find the best approximation with respect to the norm $\int_1^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\left(-12q + 1\right) + yz\left(6q - 6\right) + z^2\left(-12q - 3\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 85

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 1 & 3 \\ -18 & -11 & 20 & -2 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 2 & 9 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 4 & 4 \\ 0 & 9 & 2 \\ -6 & 14 & 0 \\ 12 & -1 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 2 \cdot y + 9 \cdot z + 2 \cdot t = 0 \\ 6 \cdot x + 4 \cdot y + 4 \cdot z + 0 \cdot t = 6 \\ 3 \cdot x + 6 \cdot y - 1 \cdot z - 8 \cdot t = 8 \\ 9 \cdot x + 2 \cdot y + 9 \cdot z + 8 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 5x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 4) + y^2\,(-12q + 1) + z^2\,(-12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 86

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 2 & 3 \\ -12 & 10 & 17 & -6 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 1 & 5 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 8 & 1 \\ -5 & 14 & -6 \\ 10 & -4 & 15 \\ 5 & 2 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 1 \cdot y + 5 \cdot z + 2 \cdot t = 5 \\ 0 \cdot x + 2 \cdot y + 2 \cdot z + 2 \cdot t = 8 \\ 2 \cdot x + 3 \cdot y + 11 \cdot z - 2 \cdot t = 7 \\ 5 \cdot x + 2 \cdot y + 8 \cdot z + 0 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 3x + 4$ find the best approximation with respect to the norm $\int_1^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(2q+2) + y^2\,(-4q+1) + z^2\,(-4q+1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 87

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 2 \\ -16 & 12 & 8 & 10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 1 & 4 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 6 & 8 \\ 3 & 2 & 1 \\ 3 & 10 & 15 \\ 3 & -2 & -6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 6 \cdot y + 8 \cdot z + 3 \cdot t = 2 \\ 9 \cdot x + 8 \cdot y + 5 \cdot z + 5 \cdot t = 0 \\ 4 \cdot x + 11 \cdot y + 12 \cdot z + 6 \cdot t = 9 \\ 2 \cdot x + 1 \cdot y + 4 \cdot z + 0 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 3x - 6$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q+1) + yz\,(2q-2) + z^2\,(-4q-1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 88

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 2 & 3 \\ -16 & 19 & -7 & 0 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 7 & 4 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -1 & -2 & 8 \\ 14 & 10 & 5 \\ 9 & 6 & 6 \\ 4 & 2 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 16 \cdot x + 5 \cdot y + 8 \cdot z + 8 \cdot t = 7 \\ 2 \cdot x + 4 \cdot y + 5 \cdot z + 2 \cdot t = 8 \\ 9 \cdot x + 6 \cdot y + 6 \cdot z + 4 \cdot t = 8 \\ 2 \cdot x + 7 \cdot y + 4 \cdot z + 0 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 3x + 4$ find the best approximation with respect to the norm $\int_{-2}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q - 2) + y^2\,(-12q + 1) + z^2\,(-12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 89

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ -10 & 1 & -14 & -2 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 6 & 2 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 5 & 6 \\ 11 & 5 & 6 \\ 3 & 5 & 6 \\ -1 & 5 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 9 \cdot y + 8 \cdot z + 0 \cdot t = 3 \\ 3 \cdot x + 5 \cdot y + 6 \cdot z + 7 \cdot t = 8 \\ 5 \cdot x + 6 \cdot y + 2 \cdot z + 0 \cdot t = 5 \\ 1 \cdot x + 4 \cdot y + 10 \cdot z + 14 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 3x + 5$ find the best approximation with respect to the norm $\int_{-2}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-4q + 1\right) + yz\left(2q + 2\right) + z^2\left(-4q + 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 90

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 2 \\ 8 & -2 & 9 & 18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 7 & 4 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 3 & 2 \\ 10 & 0 & -3 \\ 2 & 6 & 7 \\ -2 & 9 & 12 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 3 \cdot y + 2 \cdot z + 2 \cdot t = 7 \\ 2 \cdot x + 7 \cdot y + 4 \cdot z + 0 \cdot t = 3 \\ 6 \cdot x - 1 \cdot y + 0 \cdot z + 4 \cdot t = 1 \\ 6 \cdot x + 7 \cdot y + 4 \cdot z + 0 \cdot t = 5 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 3x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\left(-6q + 6\right) + y^2\left(12q + 1\right) + z^2\left(12q + 3\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 91

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 1 & 2 & 3 \\ -6 & -8 & -10 & -1 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 3 & 0 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 5 & 1 \\ 9 & 9 & 3 \\ 13 & 13 & 5 \\ 1 & 1 & -1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 7 \cdot y + 2 \cdot z + 17 \cdot t = 2 \\ 5 \cdot x + 5 \cdot y + 1 \cdot z + 9 \cdot t = 4 \\ 9 \cdot x + 3 \cdot y + 0 \cdot z + 1 \cdot t = 4 \\ 3 \cdot x + 8 \cdot y + 1 \cdot z + 2 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_{-2}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 4) + y^2\,(8q + 1) + z^2\,(8q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 92

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 1 & 3 \\ -15 & 19 & -15 & -16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 7 & 1 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 9 & 1 \\ 9 & 13 & -5 \\ 7 & 5 & 7 \\ 6 & 1 & 13 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 7 \cdot y + 1 \cdot z + 8 \cdot t = 8 \\ 8 \cdot x + 9 \cdot y + 1 \cdot z + 7 \cdot t = 9 \\ 5 \cdot x + 4 \cdot y + 2 \cdot z + 9 \cdot t = 2 \\ 11 \cdot x + 11 \cdot y + 1 \cdot z + 6 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 4x + 5$ find the best approximation with respect to the norm $\int_{-1}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q + 1) + yz\,(2q - 4) + z^2\,(-4q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 93

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 0 \\ 4 & -5 & 19 & -12 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 8 & 7 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 7 & 7 \\ -4 & -5 & 9 \\ 11 & 13 & 6 \\ 1 & 1 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 11 \cdot x + 6 \cdot y + 7 \cdot z + 1 \cdot t = 8 \\ 8 \cdot x + 4 \cdot y + 5 \cdot z + 4 \cdot t = 4 \\ 1 \cdot x + 8 \cdot y + 7 \cdot z + 1 \cdot t = 3 \\ 6 \cdot x + 7 \cdot y + 7 \cdot z + 1 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_{-1}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q + 1) + yz\,(2q - 6) + z^2\,(-4q - 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 94

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 2 \\ 3 & -17 & -16 & -16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 3 & 4 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 7 & 1 \\ 7 & 7 & 3 \\ -7 & 7 & -1 \\ 14 & 7 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 7 \cdot y + 8 \cdot z + 0 \cdot t = 3 \\ -7 \cdot x + 11 \cdot y - 2 \cdot z + 9 \cdot t = 3 \\ 7 \cdot x + 3 \cdot y + 4 \cdot z + 5 \cdot t = 0 \\ 0 \cdot x + 7 \cdot y + 1 \cdot z + 7 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 3x - 5$ find the best approximation with respect to the norm $\int_{-1}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-12q + 1\right) + yz\left(6q + 6\right) + z^2\left(-12q + 3\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 95

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 2 & 3 \\ -14 & 13 & -10 & -17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 1 & 1 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & 6 & 1 \\ 7 & 4 & -1 \\ -2 & 10 & 5 \\ 1 & 8 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 8 \cdot y + 3 \cdot z + 4 \cdot t = 2 \\ -4 \cdot x + 15 \cdot y + 5 \cdot z + 5 \cdot t = 9 \\ 7 \cdot x + 9 \cdot y + 3 \cdot z + 8 \cdot t = 7 \\ 6 \cdot x + 1 \cdot y + 1 \cdot z + 3 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 4x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+4) + z^2\,(4q+2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 96

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 1 & 3 \\ -19 & -2 & 4 & -3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 3 & 1 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 8 & 3 \\ 10 & 10 & 0 \\ 0 & 6 & 6 \\ -5 & 4 & 9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 6 \cdot y + 6 \cdot z + 5 \cdot t = 0 \\ 8 \cdot x + 1 \cdot y + 9 \cdot z + 5 \cdot t = 1 \\ -8 \cdot x + 9 \cdot y + 11 \cdot z + 8 \cdot t = 4 \\ 8 \cdot x + 3 \cdot y + 1 \cdot z + 2 \cdot t = 2 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 3x + 5$ find the best approximation with respect to the norm $\int_0^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 4) + y^2\,(-12q + 1) + z^2\,(-12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 97

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 1 \\ 19 & -13 & -14 & 5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 7 & 6 & 9 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 9 & 1 \\ -3 & 5 & 13 \\ 3 & 7 & 7 \\ 15 & 11 & -5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 9 \cdot y + 1 \cdot z + 3 \cdot t = 9 \\ 11 \cdot x + 11 \cdot y - 4 \cdot z - 3 \cdot t = 8 \\ 7 \cdot x + 7 \cdot y + 6 \cdot z + 9 \cdot t = 5 \\ 2 \cdot x + 6 \cdot y + 7 \cdot z + 9 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 3x - 5$ find the best approximation with respect to the norm $\int_0^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 2) + y^2\,(8q + 1) + z^2\,(8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 98

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 2 & 3 \\ -2 & -11 & 1 & -9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 2 & 9 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & 3 & 7 \\ 0 & 1 & 2 \\ 8 & 5 & 12 \\ -4 & -1 & -3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 4 \cdot y + 5 \cdot z - 5 \cdot t = 7 \\ 1 \cdot x + 8 \cdot y + 5 \cdot z + 5 \cdot t = 7 \\ 1 \cdot x + 2 \cdot y + 9 \cdot z + 5 \cdot t = 3 \\ 4 \cdot x + 3 \cdot y + 7 \cdot z + 0 \cdot t = 0 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 4) + y^2\,(-8q + 1) + z^2\,(-8q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 99

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ 20 & 14 & -5 & 16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 4 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 8 & 16 \\ 4 & 2 & 0 \\ 5 & 5 & 8 \\ 3 & -1 & -8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 10 \cdot y + 12 \cdot z + 6 \cdot t = 7 \\ 5 \cdot x + 5 \cdot y + 8 \cdot z + 4 \cdot t = 1 \\ 2 \cdot x + 0 \cdot y + 4 \cdot z + 2 \cdot t = 6 \\ 1 \cdot x + 7 \cdot y + 8 \cdot z + 3 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_0^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-6) + z^2\,(4q-3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 100

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 2 & 3 \\ 19 & 8 & 8 & -5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 4 & 7 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & -4 & 2 \\ 8 & 4 & 4 \\ 10 & 8 & 5 \\ 6 & 0 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 0 \cdot y + 3 \cdot z + 8 \cdot t = 8 \\ 4 \cdot x + 4 \cdot y + 7 \cdot z + 3 \cdot t = 3 \\ 0 \cdot x + 7 \cdot y + 0 \cdot z + 0 \cdot t = 6 \\ 8 \cdot x - 4 \cdot y - 1 \cdot z + 13 \cdot t = 1 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 + 4x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-4q + 1\right) + yz\left(2q + 6\right) + z^2\left(-4q + 3\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 101

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 2 & 3 \\ 2 & -2 & -2 & -16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 2 & 4 & 9 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 3 & 0 \\ 6 & 2 & 2 \\ 5 & 1 & 4 \\ 8 & 4 & -2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 8 \cdot y + 6 \cdot z + 4 \cdot t = 0 \\ 2 \cdot x + 2 \cdot y + 4 \cdot z + 9 \cdot t = 1 \\ 7 \cdot x + 3 \cdot y + 0 \cdot z + 6 \cdot t = 6 \\ 12 \cdot x + 4 \cdot y - 4 \cdot z + 3 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 4) + y^2\,(8q + 1) + z^2\,(8q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 102

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 2 & 3 \\ -9 & -13 & -7 & -9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 8 & 5 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 1 & 13 \\ 2 & 4 & 8 \\ 1 & 7 & 3 \\ 0 & 10 & -2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 7 \cdot y + 3 \cdot z + 2 \cdot t = 0 \\ -2 \cdot x + 6 \cdot y + 1 \cdot z - 3 \cdot t = 2 \\ 5 \cdot x + 5 \cdot y + 9 \cdot z + 4 \cdot t = 6 \\ 4 \cdot x + 8 \cdot y + 5 \cdot z + 7 \cdot t = 4 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 4x - 6$ find the best approximation with respect to the norm $\int_1^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-2) + z^2\,(4q-1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 103

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 3 \\ 16 & -4 & -18 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 0 & 2 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -3 & 1 & -7 \\ 5 & 7 & 7 \\ 9 & 10 & 14 \\ 1 & 4 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 0 \cdot y + 2 \cdot z + 5 \cdot t = 8 \\ 5 \cdot x + 7 \cdot y + 7 \cdot z + 1 \cdot t = 5 \\ 6 \cdot x + 14 \cdot y + 12 \cdot z - 3 \cdot t = 1 \\ 7 \cdot x + 1 \cdot y + 9 \cdot z + 0 \cdot t = 8 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q - 6) + z^2\,(4q - 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 104

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 2 \\ -2 & -9 & -20 & -18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 4 & 0 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 8 & 7 \\ 3 & -2 & 1 \\ 0 & 13 & 10 \\ 2 & 3 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 5 \cdot y + 6 \cdot z + 3 \cdot t = 9 \\ 3 \cdot x + 4 \cdot y + 0 \cdot z + 2 \cdot t = 8 \\ 1 \cdot x + 8 \cdot y + 7 \cdot z + 2 \cdot t = 2 \\ -1 \cdot x + 12 \cdot y + 14 \cdot z + 2 \cdot t = 8 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 - 5x + 4$ find the best approximation with respect to the norm $\int_{-2}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(12q + 1) + yz\,(-6q + 6) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 105

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 2 \\ -19 & -13 & 5 & 14 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 8 & 1 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 7 & 8 \\ 0 & 7 & 1 \\ -9 & 7 & -6 \\ 18 & 7 & 15 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} -7 \cdot x + 6 \cdot y + 1 \cdot z + 12 \cdot t = 1 \\ 7 \cdot x + 8 \cdot y + 1 \cdot z + 6 \cdot t = 0 \\ 7 \cdot x + 7 \cdot y + 9 \cdot z + 9 \cdot t = 6 \\ 0 \cdot x + 7 \cdot y + 1 \cdot z + 9 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_{-2}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q - 2) + y^2\,(-12q + 1) + z^2\,(-12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 106

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 3 \\ -12 & 1 & -17 & -17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 8 & 8 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -2 & -1 & 8 \\ 10 & 11 & 8 \\ 2 & 3 & 8 \\ 6 & 7 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 7 \cdot y + 8 \cdot z + 2 \cdot t = 4 \\ 3 \cdot x + 3 \cdot y + 7 \cdot z + 5 \cdot t = 4 \\ 9 \cdot x + 6 \cdot y + 8 \cdot z - 1 \cdot t = 4 \\ 3 \cdot x + 8 \cdot y + 8 \cdot z + 5 \cdot t = 1 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 4x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 4) + y^2\,(-12q + 1) + z^2\,(-12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 107

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 2 \\ 11 & 20 & 4 & 0 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 4 & 9 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -8 & 10 & 8 \\ 0 & 9 & 6 \\ 16 & 7 & 2 \\ 8 & 8 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 7 \cdot y + 2 \cdot z + 2 \cdot t = 9 \\ 8 \cdot x + 4 \cdot y + 9 \cdot z + 6 \cdot t = 0 \\ 0 \cdot x + 9 \cdot y + 6 \cdot z + 8 \cdot t = 9 \\ -8 \cdot x + 14 \cdot y + 3 \cdot z + 10 \cdot t = 5 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 - 4x + 4$ find the best approximation with respect to the norm $\int_{-2}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q + 1) + yz\,(2q + 6) + z^2\,(-4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 108

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 2 & 3 \\ -14 & -17 & -12 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 5 & 4 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 2 & 13 & 4 \\ 4 & 3 & 6 \\ 5 & -2 & 7 \\ 3 & 8 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 5 \cdot y + 4 \cdot z + 3 \cdot t = 3 \\ 0 \cdot x + 1 \cdot y + 8 \cdot z + 3 \cdot t = 4 \\ 4 \cdot x + 3 \cdot y + 6 \cdot z + 3 \cdot t = 0 \\ 1 \cdot x + 3 \cdot y + 4 \cdot z + 1 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-6) + z^2\,(4q-3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 109

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 1 & 3 \\ 4 & 1 & -2 & -10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 5 & 0 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & 11 & 5 \\ 6 & 8 & 5 \\ 8 & 5 & 5 \\ 10 & 2 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 9 \cdot y + 0 \cdot z + 6 \cdot t = 9 \\ 6 \cdot x + 8 \cdot y + 5 \cdot z + 8 \cdot t = 6 \\ 5 \cdot x + 5 \cdot y + 0 \cdot z + 2 \cdot t = 2 \\ 7 \cdot x + 11 \cdot y + 10 \cdot z + 14 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 3x - 6$ find the best approximation with respect to the norm $\int_{-1}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 2) + y^2\,(8q + 1) + z^2\,(8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 110

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 2 \\ 4 & 16 & 0 & -10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 8 & 4 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 15 & 3 & 15 \\ 9 & 4 & 8 \\ 3 & 5 & 1 \\ -3 & 6 & -6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 8 \cdot y + 4 \cdot z + 4 \cdot t = 9 \\ 3 \cdot x + 5 \cdot y + 1 \cdot z + 9 \cdot t = 5 \\ 2 \cdot x + 2 \cdot y - 2 \cdot z + 14 \cdot t = 2 \\ 6 \cdot x + 8 \cdot y + 4 \cdot z + 2 \cdot t = 5 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_{-1}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 4) + y^2\,(-8q + 1) + z^2\,(-8q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 111

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 2 & 3 \\ 14 & 4 & 2 & 0 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 8 & 6 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} -3 & 3 & 15 \\ 12 & 6 & -6 \\ 2 & 4 & 8 \\ 7 & 5 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 8 \cdot y + 6 \cdot z + 4 \cdot t = 8 \\ 10 \cdot x + 2 \cdot y - 4 \cdot z + 0 \cdot t = 9 \\ 7 \cdot x + 5 \cdot y + 1 \cdot z + 2 \cdot t = 9 \\ 8 \cdot x + 6 \cdot y + 2 \cdot z + 6 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 4x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + xz\left(-4q + 4\right) + y^2\left(8q + 1\right) + z^2\left(8q + 2\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 112

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 19 & 13 & -13 & 1 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 1 & 7 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 5 & 7 \\ 3 & 7 & 1 \\ 4 & 6 & 4 \\ 2 & 8 & -2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 6 \cdot y + 4 \cdot z + 3 \cdot t = 0 \\ 9 \cdot x + 0 \cdot y + 0 \cdot z + 8 \cdot t = 2 \\ 1 \cdot x + 11 \cdot y + 1 \cdot z + 1 \cdot t = 4 \\ 7 \cdot x + 1 \cdot y + 7 \cdot z + 5 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 - 5x + 5$ find the best approximation with respect to the norm $\int_0^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 2) + y^2\,(8q + 1) + z^2\,(8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 113

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 2 \\ 3 & -19 & -6 & 9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 0 & 1 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & -3 & -4 \\ 7 & 1 & 0 \\ 5 & 5 & 4 \\ 3 & 9 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 5 \cdot y + 4 \cdot z + 7 \cdot t = 0 \\ 9 \cdot x + 10 \cdot y + 7 \cdot z + 11 \cdot t = 2 \\ 2 \cdot x + 8 \cdot y + 1 \cdot z + 7 \cdot t = 1 \\ 1 \cdot x + 0 \cdot y + 1 \cdot z + 3 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 5x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 4) + y^2\,(-8q + 1) + z^2\,(-8q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 114

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 3 \\ -14 & 4 & 6 & -2 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 6 & 8 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 6 \\ -7 & -3 & 5 \\ 9 & 7 & 7 \\ 17 & 12 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 7 \cdot y + 7 \cdot z + 1 \cdot t = 3 \\ 2 \cdot x + 6 \cdot y + 8 \cdot z + 1 \cdot t = 9 \\ 16 \cdot x + 8 \cdot y + 6 \cdot z + 1 \cdot t = 8 \\ 8 \cdot x + 1 \cdot y + 2 \cdot z + 5 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 + 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-8q + 1) + yz\,(4q - 2) + z^2\,(-8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 115

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 2 & 3 \\ 6 & 9 & 20 & 12 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 2 & 9 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 13 & 0 & 2 \\ 3 & 0 & 2 \\ 8 & 0 & 2 \\ -2 & 0 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 0 \cdot y + 2 \cdot z + 3 \cdot t = 6 \\ 8 \cdot x + 0 \cdot y + 0 \cdot z + 6 \cdot t = 6 \\ 16 \cdot x - 2 \cdot y - 5 \cdot z + 0 \cdot t = 8 \\ 0 \cdot x + 2 \cdot y + 9 \cdot z + 6 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 3x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 5].

6. Find all the values of q such that the equation $2x^2 + xy\,(2q + 2) + y^2\,(-4q + 1) + z^2\,(-4q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 116

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 3 \\ 16 & -1 & 12 & 20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 3 & 0 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & -8 & 11 \\ 4 & 16 & -1 \\ 4 & 0 & 7 \\ 4 & 8 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x - 3 \cdot y + 14 \cdot z + 0 \cdot t = 4 \\ 4 \cdot x + 0 \cdot y + 7 \cdot z + 4 \cdot t = 5 \\ 8 \cdot x + 3 \cdot y + 0 \cdot z + 8 \cdot t = 5 \\ 2 \cdot x + 8 \cdot y + 5 \cdot z + 0 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 - 5x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 2].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-8q + 1) + yz\,(4q + 6) + z^2\,(-8q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 117

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 3 \\ -7 & -7 & -11 & 4 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 1 & 9 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 6 & 1 \\ 3 & 9 & 8 \\ -3 & 3 & -6 \\ 6 & 12 & 15 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 1 \cdot y + 9 \cdot z + 4 \cdot t = 7 \\ 0 \cdot x + 17 \cdot y + 7 \cdot z - 4 \cdot t = 8 \\ 3 \cdot x + 9 \cdot y + 8 \cdot z + 0 \cdot t = 3 \\ 6 \cdot x + 6 \cdot y + 2 \cdot z + 5 \cdot t = 1 \end{cases}$$

5. For the polynomial $x^3 + 3x^2 - 4x + 5$ find the best approximation with respect to the norm $\int_1^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 6) + y^2\,(-12q + 1) + z^2\,(-12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 118

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 3 \\ 4 & 15 & -8 & 12 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 3 & 0 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 5 & 13 & 0 \\ 5 & 3 & 6 \\ 5 & -2 & 9 \\ 5 & 8 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 3 \cdot y + 12 \cdot z + 2 \cdot t = 9 \\ 5 \cdot x + 3 \cdot y + 6 \cdot z + 5 \cdot t = 4 \\ 2 \cdot x + 7 \cdot y + 8 \cdot z + 6 \cdot t = 5 \\ 8 \cdot x + 3 \cdot y + 0 \cdot z + 8 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_1^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(6q - 2) + y^2\,(-12q + 1) + z^2\,(-12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 119

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 1 & 2 \\ 0 & 3 & -5 & -11 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 7 & 9 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & 11 & 8 \\ 4 & 6 & 7 \\ 4 & -4 & 5 \\ 4 & 1 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 7 \cdot y + 9 \cdot z + 0 \cdot t = 5 \\ 4 \cdot x + 1 \cdot y + 6 \cdot z + 4 \cdot t = 9 \\ 2 \cdot x - 5 \cdot y + 3 \cdot z + 8 \cdot t = 5 \\ 6 \cdot x + 0 \cdot y + 8 \cdot z + 7 \cdot t = 9 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 4x - 5$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 4) + y^2\,(12q + 1) + z^2\,(12q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 120

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 1 & 3 \\ 15 & -13 & -18 & 14 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 1 & 2 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 14 & -6 \\ 3 & 4 & 8 \\ 4 & -1 & 15 \\ 2 & 9 & 1 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 7 \cdot y + 2 \cdot z + 6 \cdot t = 5 \\ 3 \cdot x + 4 \cdot y + 8 \cdot z + 2 \cdot t = 8 \\ -3 \cdot x + 7 \cdot y + 14 \cdot z + 1 \cdot t = 6 \\ 9 \cdot x + 1 \cdot y + 2 \cdot z + 3 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 3x - 6$ find the best approximation with respect to the norm $\int_{-2}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + xz\,(2q + 6) + y^2\,(-4q + 1) + z^2\,(-4q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 121

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 3 \\ -2 & -13 & -11 & 9 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 2 & 7 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 9 & 0 \\ 6 & 5 & 1 \\ 3 & 1 & 2 \\ 0 & -3 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 1 \cdot x + 2 \cdot y + 7 \cdot z + 2 \cdot t = 8 \\ 6 \cdot x + 5 \cdot y + 1 \cdot z + 3 \cdot t = 6 \\ 11 \cdot x + 8 \cdot y - 5 \cdot z + 4 \cdot t = 0 \\ 9 \cdot x + 4 \cdot y + 3 \cdot z + 8 \cdot t = 9 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\left(-6q - 2\right) + y^2\left(12q + 1\right) + z^2\left(12q - 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 122

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 2 \\ 11 & -17 & -11 & -5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 0 & 8 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 9 & 0 \\ 5 & 9 & 8 \\ 1 & 9 & -8 \\ 7 & 9 & 16 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 3 \cdot y + 8 \cdot z + 9 \cdot t = 1 \\ 1 \cdot x + 18 \cdot y + 8 \cdot z - 1 \cdot t = 7 \\ 5 \cdot x + 9 \cdot y + 8 \cdot z + 3 \cdot t = 0 \\ 9 \cdot x + 0 \cdot y + 8 \cdot z + 7 \cdot t = 5 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x + 5$ find the best approximation with respect to the norm $\int_{-2}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q - 6) + z^2\,(4q - 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 123

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 1 & 3 \\ -20 & 6 & 4 & -6 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 5 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 6 & 10 \\ 8 & -3 & -5 \\ 8 & 3 & 5 \\ 8 & 0 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 6 \cdot y + 5 \cdot z + 4 \cdot t = 3 \\ 8 \cdot x + 0 \cdot y + 0 \cdot z + 8 \cdot t = 8 \\ 13 \cdot x - 5 \cdot y - 5 \cdot z + 14 \cdot t = 5 \\ 3 \cdot x + 5 \cdot y + 5 \cdot z + 2 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 5x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q-2) + z^2\,(4q-1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 124

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 3 \\ 1 & 2 & 1 & -4 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 6 & 5 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & 10 & 11 \\ 6 & 5 & 6 \\ 8 & 0 & 1 \\ 10 & -5 & -4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 11 \cdot x - 6 \cdot y - 3 \cdot z + 12 \cdot t = 9 \\ 5 \cdot x + 6 \cdot y + 5 \cdot z + 0 \cdot t = 6 \\ 5 \cdot x + 1 \cdot y + 6 \cdot z + 0 \cdot t = 1 \\ 8 \cdot x + 0 \cdot y + 1 \cdot z + 6 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 4x + 4$ find the best approximation with respect to the norm $\int_{-1}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 4) + y^2\,(12q + 1) + z^2\,(12q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 125

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ -20 & -16 & -15 & -17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 4 & 5 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 2 & -7 & 8 \\ 6 & 7 & 0 \\ 8 & 14 & -4 \\ 4 & 0 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 12 \cdot x + 10 \cdot y - 5 \cdot z + 5 \cdot t = 6 \\ 9 \cdot x + 1 \cdot y + 4 \cdot z + 3 \cdot t = 7 \\ 6 \cdot x + 7 \cdot y + 0 \cdot z + 4 \cdot t = 4 \\ 0 \cdot x + 4 \cdot y + 5 \cdot z + 3 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 4x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 126

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 3 \\ 12 & 1 & 12 & 3 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 0 & 7 & 9 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 0 & 5 & 0 \\ 0 & -3 & 8 \\ 0 & 9 & -4 \\ 0 & 1 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 1 \cdot y + 4 \cdot z + 0 \cdot t = 7 \\ -5 \cdot x + 2 \cdot y + 1 \cdot z - 9 \cdot t = 0 \\ 8 \cdot x + 2 \cdot y + 2 \cdot z + 7 \cdot t = 1 \\ 5 \cdot x + 0 \cdot y + 7 \cdot z + 9 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 + 4x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(2q + 4) + y^2\,(-4q + 1) + z^2\,(-4q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 127

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 0 & 2 \\ 15 & 9 & 4 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 3 & 8 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 11 & 1 & 4 \\ 9 & 3 & 3 \\ 5 & 7 & 1 \\ 7 & 5 & 2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 1 \cdot y + 0 \cdot z + 9 \cdot t = 9 \\ 7 \cdot x + 5 \cdot y + 2 \cdot z + 9 \cdot t = 5 \\ 11 \cdot x + 7 \cdot y - 4 \cdot z + 13 \cdot t = 0 \\ 3 \cdot x + 3 \cdot y + 8 \cdot z + 5 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 - 4x + 4$ find the best approximation with respect to the norm $\int_{-1}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 2) + y^2\,(8q + 1) + z^2\,(8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 128

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 4 & 5 & -7 & -18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 8 & 9 & 2 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 10 & 9 & 12 \\ 7 & -3 & 0 \\ 8 & 1 & 4 \\ 9 & 5 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 1 \cdot y + 4 \cdot z + 9 \cdot t = 8 \\ 5 \cdot x + 8 \cdot y + 9 \cdot z + 2 \cdot t = 7 \\ 6 \cdot x + 6 \cdot y + 6 \cdot z + 8 \cdot t = 0 \\ 11 \cdot x - 6 \cdot y - 1 \cdot z + 16 \cdot t = 0 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_0^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 2) + y^2\,(12q + 1) + z^2\,(12q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 129

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 15 & 0 & -5 & -5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 4 & 6 & 0 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 4 & 4 \\ 13 & 1 & 4 \\ 9 & 2 & 4 \\ 5 & 3 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 5 \cdot y + 6 \cdot z + 6 \cdot t = 1 \\ 3 \cdot x + 4 \cdot y + 6 \cdot z + 0 \cdot t = 8 \\ 9 \cdot x + 2 \cdot y + 4 \cdot z + 5 \cdot t = 7 \\ 15 \cdot x + 0 \cdot y + 2 \cdot z + 10 \cdot t = 8 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 + 3x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 3].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-4q + 1\right) + yz\left(2q - 2\right) + z^2\left(-4q - 1\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 130

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 2 & 3 \\ 14 & -4 & -9 & -11 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 9 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 8 & 8 \\ 10 & 14 & 16 \\ 4 & 2 & 0 \\ 1 & -4 & -8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 8 \cdot y + 8 \cdot z + 4 \cdot t = 5 \\ 2 \cdot x + 0 \cdot y + 9 \cdot z + 6 \cdot t = 3 \\ 4 \cdot x + 9 \cdot y + 0 \cdot z + 9 \cdot t = 9 \\ 12 \cdot x + 16 \cdot y + 7 \cdot z + 2 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 - 5x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\,(-8q + 1) + yz\,(4q - 2) + z^2\,(-8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 131

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 3 \\ 15 & 15 & 7 & -11 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 6 & 9 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 6 & 8 \\ 9 & 0 & 4 \\ 9 & 9 & 10 \\ 9 & 3 & 6 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 5 \cdot y + 5 \cdot z + 6 \cdot t = 7 \\ 15 \cdot x + 6 \cdot y + 7 \cdot z + 15 \cdot t = 7 \\ 9 \cdot x + 6 \cdot y + 8 \cdot z + 9 \cdot t = 7 \\ 3 \cdot x + 6 \cdot y + 9 \cdot z + 3 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_0^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+6) + z^2\,(4q+3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 132

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 1 \\ 17 & -7 & -5 & -17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 4 & 1 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 9 & 4 \\ 1 & 1 & 4 \\ 5 & 5 & 4 \\ -3 & -3 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 5 \cdot x + 4 \cdot y + 8 \cdot z + 3 \cdot t = 1 \\ 1 \cdot x + 1 \cdot y + 4 \cdot z + 5 \cdot t = 5 \\ -3 \cdot x - 2 \cdot y + 7 \cdot z + 7 \cdot t = 5 \\ 5 \cdot x + 4 \cdot y + 1 \cdot z + 3 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 - 4x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q - 4) + y^2\,(12q + 1) + z^2\,(12q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 133

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 3 \\ 12 & -7 & 8 & 12 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 0 & 8 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & -5 & 0 \\ -3 & 16 & 0 \\ 0 & 9 & 0 \\ 3 & 2 & 0 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 0 \cdot y + 8 \cdot z + 5 \cdot t = 1 \\ 3 \cdot x + 2 \cdot y + 0 \cdot z + 0 \cdot t = 0 \\ 6 \cdot x + 0 \cdot y + 2 \cdot z + 3 \cdot t = 4 \\ -3 \cdot x + 4 \cdot y - 8 \cdot z - 5 \cdot t = 4 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 3x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(2q + 4) + y^2\,(-4q + 1) + z^2\,(-4q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 134

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 0 & 1 & 2 \\ 17 & 4 & 0 & 8 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 8 & 8 & 4 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 5 & 8 \\ 12 & 6 & 14 \\ 0 & 4 & 2 \\ -6 & 3 & -4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 4 \cdot y + 2 \cdot z + 6 \cdot t = 7 \\ -5 \cdot x + 0 \cdot y - 4 \cdot z + 8 \cdot t = 5 \\ 5 \cdot x + 8 \cdot y + 8 \cdot z + 4 \cdot t = 3 \\ 2 \cdot x + 1 \cdot y + 2 \cdot z + 5 \cdot t = 7 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 4x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [1 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q + 6) + y^2\,(-12q + 1) + z^2\,(-12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 135

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 0 & 1 \\ 11 & 2 & -7 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 5 & 2 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 12 & 8 & 13 \\ 7 & 7 & 9 \\ -3 & 5 & 1 \\ 2 & 6 & 5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 7 \cdot y + 9 \cdot z + 2 \cdot t = 7 \\ 4 \cdot x + 3 \cdot y + 2 \cdot z + 7 \cdot t = 3 \\ 6 \cdot x + 5 \cdot y + 2 \cdot z + 8 \cdot t = 2 \\ 8 \cdot x + 9 \cdot y + 16 \cdot z - 4 \cdot t = 0 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x - 5$ find the best approximation with respect to the norm $\int_1^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 6) + y^2\,(-8q + 1) + z^2\,(-8q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 136

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 2 \\ 3 & 3 & 1 & 7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 7 & 4 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 8 & 7 \\ 6 & 7 & 7 \\ 5 & 6 & 7 \\ 4 & 5 & 7 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 6 \cdot x + 7 \cdot y + 7 \cdot z + 5 \cdot t = 8 \\ 4 \cdot x + 4 \cdot y + 0 \cdot z + 7 \cdot t = 2 \\ 6 \cdot x + 7 \cdot y + 10 \cdot z + 3 \cdot t = 1 \\ 6 \cdot x + 7 \cdot y + 4 \cdot z + 7 \cdot t = 0 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 3x - 6$ find the best approximation with respect to the norm $\int_{-2}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 2].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 6) + y^2\,(12q + 1) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 137

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & -1 & 0 & 3 \\ -6 & 10 & -1 & -15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 2 & 5 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 3 & 2 \\ 8 & 3 & -5 \\ 8 & 3 & 16 \\ 8 & 3 & 9 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 3 \cdot y + 9 \cdot z + 8 \cdot t = 5 \\ 13 \cdot x + 4 \cdot y + 13 \cdot z + 11 \cdot t = 4 \\ 0 \cdot x + 4 \cdot y + 4 \cdot z + 4 \cdot t = 9 \\ 3 \cdot x + 2 \cdot y + 5 \cdot z + 5 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 + 3x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_{-2}^{3} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(-6q + 6) + y^2\,(12q + 1) + z^2\,(12q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 138

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 0 & 2 & 3 \\ 19 & 17 & 19 & -18 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 3 & 7 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 9 & 5 & 3 \\ 3 & 11 & 13 \\ 6 & 8 & 8 \\ 12 & 2 & -2 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 7 \cdot x + 13 \cdot y + 9 \cdot z + 13 \cdot t = 1 \\ 6 \cdot x + 8 \cdot y + 8 \cdot z + 9 \cdot t = 9 \\ 7 \cdot x + 2 \cdot y + 7 \cdot z + 1 \cdot t = 4 \\ 5 \cdot x + 3 \cdot y + 7 \cdot z + 5 \cdot t = 6 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_{-2}^{4} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 4].

6. Find all the values of q such that the equation $2x^2 + xz\,(6q - 2) + y^2\,(-12q + 1) + z^2\,(-12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 139

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 1 & 3 \\ 10 & 10 & 3 & -19 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 8 & 2 & 6 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 7 & 8 & 2 \\ 8 & 2 & 9 \\ 9 & -4 & 16 \\ 6 & 14 & -5 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 2 \cdot y + 9 \cdot z + 7 \cdot t = 0 \\ 8 \cdot x + 2 \cdot y + 6 \cdot z + 7 \cdot t = 7 \\ 8 \cdot x + 2 \cdot y + 12 \cdot z + 7 \cdot t = 9 \\ 1 \cdot x + 5 \cdot y + 2 \cdot z + 1 \cdot t = 5 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 + 3x - 5$ find the best approximation with respect to the norm $\int_{-2}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-2 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-4q + 1\right) + yz\left(2q - 4\right) + z^2\left(-4q - 2\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 140

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & 2 & 3 \\ -7 & 17 & -3 & 20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 1 & 4 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 4 & 0 & 9 \\ 6 & 10 & -7 \\ 5 & 5 & 1 \\ 3 & -5 & 17 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 0 \cdot y + 9 \cdot z + 5 \cdot t = 1 \\ 7 \cdot x + 4 \cdot y + 6 \cdot z + 2 \cdot t = 1 \\ 5 \cdot x + 1 \cdot y + 4 \cdot z + 8 \cdot t = 8 \\ 3 \cdot x - 1 \cdot y + 14 \cdot z + 2 \cdot t = 3 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_{-1}^{2} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 2].

6. Find all the values of q such that the equation $2x^2 + xz\,(-4q + 6) + y^2\,(8q + 1) + z^2\,(8q + 3) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 141

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 2 \\ -5 & 14 & 8 & 17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 9 & 8 & 1 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 7 & -3 \\ 4 & 6 & 3 \\ 2 & 5 & 9 \\ 0 & 4 & 15 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 3 \cdot y - 2 \cdot z + 3 \cdot t = 1 \\ 5 \cdot x + 9 \cdot y + 8 \cdot z + 1 \cdot t = 8 \\ 4 \cdot x + 6 \cdot y + 3 \cdot z + 2 \cdot t = 9 \\ 9 \cdot x + 6 \cdot y + 9 \cdot z + 8 \cdot t = 1 \end{cases}$$

5. For the polynomial $-2x^3 + 2x^2 + 4x + 4$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(2q + 4) + y^2\,(-4q + 1) + z^2\,(-4q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 142

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -1 & 1 & 2 \\ -2 & -17 & -2 & 15 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 3 & 2 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 8 & -2 \\ 4 & 0 & 8 \\ 6 & -4 & 13 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 4 \cdot x + 3 \cdot y + 2 \cdot z + 7 \cdot t = 7 \\ 4 \cdot x - 3 \cdot y + 14 \cdot z - 3 \cdot t = 0 \\ 4 \cdot x + 0 \cdot y + 8 \cdot z + 2 \cdot t = 9 \\ 3 \cdot x + 9 \cdot y + 9 \cdot z + 6 \cdot t = 8 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 - 5x + 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [-1 , 4].

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q + 1) + yz\,(-2q + 4) + z^2\,(4q + 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 143

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 2 \\ 5 & 14 & -12 & -20 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 9 & 2 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ -3 & -7 & -9 \\ 9 & 7 & 9 \\ 15 & 14 & 18 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 0 \cdot y + 0 \cdot z + 9 \cdot t = 6 \\ 9 \cdot x + 0 \cdot y + 6 \cdot z + 3 \cdot t = 2 \\ 7 \cdot x + 9 \cdot y + 2 \cdot z + 8 \cdot t = 1 \\ -1 \cdot x - 9 \cdot y - 2 \cdot z + 10 \cdot t = 9 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 - 5x - 5$ find the best approximation with respect to the norm $\int_{-1}^{5} |f(x)|dx$ by a polynomial of degree 2 on a line segment [-1 , 5].

6. Find all the values of q such that the equation $2x^2 + y^2\left(-8q + 1\right) + yz\left(4q + 4\right) + z^2\left(-8q + 2\right) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 144

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -2 & 0 & 1 & 3 \\ 9 & -10 & -5 & -10 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 5 & 5 & 2 & 6 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 5 & 5 \\ 2 & 1 & 0 \\ 1 & -3 & -5 \\ 4 & 9 & 10 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 1 \cdot y + 0 \cdot z + 3 \cdot t = 4 \\ 0 \cdot x + 9 \cdot y + 3 \cdot z + 3 \cdot t = 5 \\ -1 \cdot x - 3 \cdot y - 2 \cdot z + 0 \cdot t = 8 \\ 5 \cdot x + 5 \cdot y + 2 \cdot z + 6 \cdot t = 1 \end{cases}$$

5. For the polynomial $-2x^3 - 3x^2 - 5x + 5$ find the best approximation with respect to the norm $\int_0^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + xz\,(6q - 2) + y^2\,(-12q + 1) + z^2\,(-12q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 145

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & -2 & -1 & 0 \\ 12 & 14 & -8 & 16 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 8 & 4 & 7 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 8 & 4 & 5 \\ 0 & 2 & 8 \\ 16 & 6 & 2 \\ -8 & 0 & 11 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 8 \cdot x + 4 \cdot y + 5 \cdot z + 0 \cdot t = 9 \\ 8 \cdot x + 7 \cdot y + 3 \cdot z + 2 \cdot t = 7 \\ 14 \cdot x + 0 \cdot y + 6 \cdot z - 7 \cdot t = 2 \\ 2 \cdot x + 8 \cdot y + 4 \cdot z + 7 \cdot t = 3 \end{cases}$$

5. For the polynomial $x^3 - 4x^2 - 5x - 5$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 3].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 2) + y^2\,(-8q + 1) + z^2\,(-8q - 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 146

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ 19 & 11 & 1 & 5 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 3 & 7 & 5 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 6 & 4 \\ -2 & 10 & 5 \\ 7 & -2 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 3 \cdot y + 7 \cdot z + 5 \cdot t = 0 \\ 0 \cdot x + 9 \cdot y + 1 \cdot z + 3 \cdot t = 4 \\ 1 \cdot x + 6 \cdot y + 4 \cdot z + 4 \cdot t = 7 \\ 9 \cdot x + 2 \cdot y + 6 \cdot z + 1 \cdot t = 8 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 - 5x - 6$ find the best approximation with respect to the max-norm by a polynomial of degree 2 on a line segment [0 , 4].

6. Find all the values of q such that the equation $2x^2 + xy\,(4q - 4) + y^2\,(-8q + 1) + z^2\,(-8q - 2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 147

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ 2 & -5 & -3 & -6 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 1 & 2 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 3 & 6 & 7 \\ 9 & 6 & 1 \\ 15 & 6 & -5 \\ -3 & 6 & 13 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 0 \cdot x + 11 \cdot y + 12 \cdot z + 15 \cdot t = 1 \\ 6 \cdot x + 1 \cdot y + 2 \cdot z + 3 \cdot t = 2 \\ 3 \cdot x + 6 \cdot y + 7 \cdot z + 9 \cdot t = 6 \\ 1 \cdot x + 3 \cdot y + 4 \cdot z + 1 \cdot t = 7 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 - 5x + 5$ find the best approximation with respect to the norm $\int_0^5 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 0\ ,\ 5\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(4q+1) + yz\,(-2q+2) + z^2\,(4q+1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 148

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 1 & 2 & 3 \\ -12 & -15 & 18 & 7 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 6 & 2 & 5 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 10 & 12 & 12 \\ 8 & 6 & 2 \\ 9 & 9 & 7 \\ 7 & 3 & -3 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 9 \cdot x + 9 \cdot y + 7 \cdot z + 8 \cdot t = 6 \\ 6 \cdot x + 2 \cdot y + 5 \cdot z + 8 \cdot t = 3 \\ 5 \cdot x + 7 \cdot y + 6 \cdot z + 3 \cdot t = 4 \\ 12 \cdot x + 16 \cdot y + 9 \cdot z + 8 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 - 3x^2 + 4x + 5$ find the best approximation with respect to the norm $\int_1^2 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 2\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 149

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -1 & 1 & 2 & 3 \\ 18 & -7 & 14 & 11 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 1 & 3 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 6 & 2 & 0 \\ 11 & 2 & -2 \\ -4 & 2 & 4 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 2 \cdot x + 5 \cdot y + 7 \cdot z + 3 \cdot t = 6 \\ 0 \cdot x + 4 \cdot y + 3 \cdot z + 9 \cdot t = 5 \\ 2 \cdot x + 0 \cdot y + 1 \cdot z + 3 \cdot t = 5 \\ 1 \cdot x + 2 \cdot y + 2 \cdot z + 6 \cdot t = 6 \end{cases}$$

5. For the polynomial $x^3 + 2x^2 + 3x + 4$ find the best approximation with respect to the norm $\int_1^3 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 3\ ]$.

6. Find all the values of q such that the equation $2x^2 + xy\,(4q + 2) + y^2\,(-8q + 1) + z^2\,(-8q + 1) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.

Variant 150

1. Find an interpolation polynomial in the Lagrange form that passes through the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} -3 & 1 & 2 & 3 \\ 20 & 10 & 19 & -17 \end{bmatrix}$$

2. Find a (parametric) equation defining the Bezier curve defined by the four points whose coordinates form the columns of the matrix

$$P = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 1 & 6 & 8 \end{bmatrix}$$

Plot the points and the curve on the coordinate plane.

3. Find a full rank decomposition and the pseudoinverse of the matrix

$$A = \begin{bmatrix} 6 & 0 & -6 \\ 7 & 3 & 1 \\ 9 & 9 & 15 \\ 8 & 6 & 8 \end{bmatrix}$$

4. Find the minimal length least squares solution of the system of linear equations

$$\begin{cases} 3 \cdot x + 1 \cdot y + 6 \cdot z + 8 \cdot t = 4 \\ 8 \cdot x + 6 \cdot y + 8 \cdot z + 7 \cdot t = 3 \\ 2 \cdot x + 7 \cdot y + 4 \cdot z + 3 \cdot t = 0 \\ 13 \cdot x + 11 \cdot y + 10 \cdot z + 6 \cdot t = 5 \end{cases}$$

5. For the polynomial $-2x^3 - 4x^2 + 3x - 5$ find the best approximation with respect to the norm $\int_1^4 |f(x)|dx$ by a polynomial of degree 2 on a line segment $[\ 1\ ,\ 4\ ]$.

6. Find all the values of q such that the equation $2x^2 + y^2\,(-4q+1) + yz\,(2q-4) + z^2\,(-4q-2) = 1$ defines a unit circle with respect to some norm? Find the value of this norm fro the vector $(1, 1, 1)$ as a function of q.