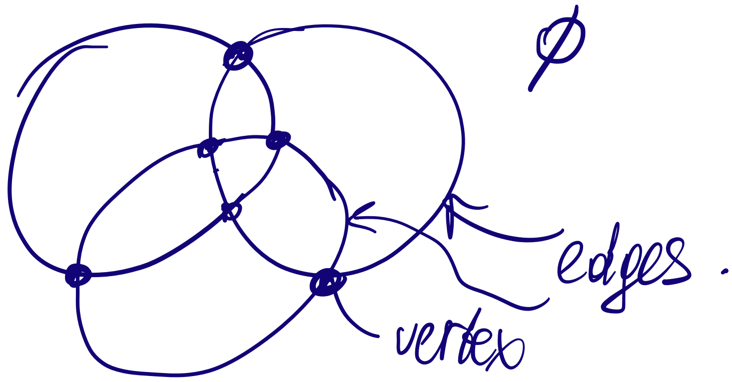
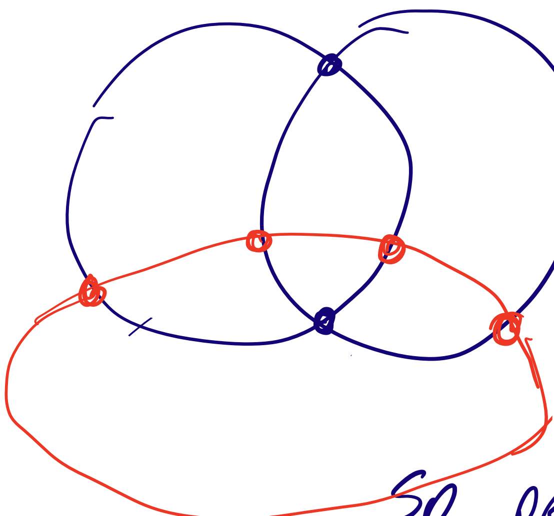
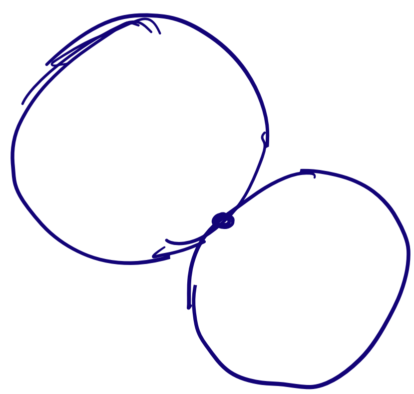
Venn diagram



- $|\mathcal{Y}| = |2^{[n]}| = 2^n$ regions (including ∅) ⇒ Need $\boxed{F_n \geq 2^n}$
- Otoh, $\boxed{V_n = \binom{n}{2} = \frac{1}{2}(n^2 - n)}$ since each vertex is at the intersection of 2 circles. (We are "economic"). $\Rightarrow V_4 = 6$
- Finally, to estimate E we use a fact specific to circles: adding a circle we add .



or



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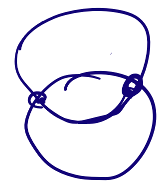
so generate ≤ 2 edges.

Adding a circle into a diagram with $n-1$ ones, we get new arcs by splitting the previous ones, plus partitioning the new circle into at most $2(n-1)$ arcs :($2(n-1)$ intersection points on it).

$$E_n \le 2E_{n-1} + 2(n-1)$$



$E_1 = 1$

$$= 2(n-1) + 2^2(n-2) + \dots + 2^{n-2}E_2$$

In particular, $E_3 \le 2E_2 + 2\cdot 2 = 12$

$E_2 \le 4$ $\quad E_4 \le 2E_3 + 2\cdot 3 = 30$

⊙ By Euler's formula,

$$F_4 \le 2 - V_4 + E_4 = 2 - 6 + 30 = 26.$$

But also $F_4 \ge 2^4 = 16$.

$\Rightarrow$ Cannot do with 5 sets.

To do better lets <u>compute</u> V_n similarly to F_n:

$$V_n = V_{n-1} + 2(n-1) \Rightarrow V_4 = V_3 + 6 = V_2 + 10 = 12.$$

$\Rightarrow F_4 \le 2 - 12 + 30 = 20.$ Still not enough!

$\boxed{n=5:}$ $F_5 \geq 2^5 = 32.$

But also

$V_5 = V_4 + 2\cdot 4 = 20$

$E_5 \leq 2E_4 + 2\cdot 4 = 38.$

$\Rightarrow F_5 \leq 2 - 20 + 38 = 20.$