DSCI 501: Homework 4

Taylor series expansion

Taylor series is widely used to approximate a function when you know all about the function's behavior (derivatives) at a particular point.

Taylor series of an infinitely differentiable function $f(x)$ at a point a can be represented as

$$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

1. Using Taylor expansion, linearize the function (to degree 1)

$$f(x) = e^{-2x}$$

about the point $x = 2$.

(a)

$$f(x) = \frac{1}{2e^4}x$$

(b)

$$f(x) = \frac{1}{e^4}[1 - 2(x-2)]$$

(c)

$$f(x) = \frac{1}{e^2}[1 - 2(x-2)]$$

(d)

$$f(x) = \frac{1}{e^4}[1 - 2(x^2-2)]$$

Newton-Raphson and Gradient descent method

Using our knowledge of differentiation, we know how to find the value of the function $f(x_0 + \delta x)$ by differentiating $f(x)$ at the point x_0.

$$f^{'}(x_0) = \frac{f(x_0 + \delta x) - f(x_0)}{\delta x}$$

$$f(x_0 + \delta x) = f(x_0) + f^{'}(x_0)\delta x$$

To find the root i.e., when $f(x_0 + \delta x) = 0$, we find the following equation.

$$\delta x = -\frac{f(x_0)}{f^{'}(x_0)}$$

But remember that the function is not a straight line so we will not exactly get the root. But we will get closer every step. However, repeating this step again and again, you will get the root and hence the solution to the function. Hence our new point will be:

$$x_{n+1} = x_n + \delta x.$$

The above method is basically Newton-Raphson method.

2. Using Newton-Raphson method, find the root of $y = x^3 - 2x + 2$ which is close to -2 i.e., start with an initial solution as -2. You may stop when $\delta x < 1e - 5$.. Note that $1e - 5 = 0.00001$

   (a) -1.6

   (b) -1.77

   (c) -1.7

   (d) -1.9

Remembering counting, permutations and combinations

3. Consider the sets $A = \{1, 2, 4, 6, 7\}$ and $B = \{1, 2, 3, 5, 7\}$. What is the intersection of A and B, $A \cap B$?

   (a) $A \cap B = \{1, 3, 5, 7\}$

   (b) $A \cap B = \{1, 2, 7\}$

   (c) $A \cap B = \{5, 7\}$

   (d) $A \cap B = \{1, 2, 5, 7\}$

4. Consider the sets $A = \{1, 2, 4, 6, 7\}$ and $B = \{1, 2, 3, 5, 7\}$. What is the union of A and B, $A \cup B$?

   (a) $A \cup B = \{1, 3, 5, 7\}$

   (b) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

   (c) $A \cup B = \{1, 5, 7\}$

   (d) $A \cup B = \{1, 2, 5, 7\}$

5. There are 25 students in the class. Each students should take at least one of the two foreign languages, French and German, and students are allowed to take both courses. It is known that 17 students signed up for the French language course and 14 students signed up for German language course. How many students signed up for both courses?

   (a) 5

(b) 6

(c) 4

(d) 10

6. How many integer numbers from 1 to 1000 are divisible by 2 or by 3?

   (a) 500

   (b) 600

   (c) 333

   (d) 667

7. How many integer numbers from 1 to 1000 are not divisible neither by 2, nor by 3?

   (a) 500

   (b) 600

   (c) 333

   (d) 667

8. In how many ways can one select a team of five students out of ten students?

   (a) 100

   (b) 126

   (c) 252

   (d) 248

9. In how many ways can one partition ten students into two teams of size five?

   (a) 126

   (b) 200

   (c) 252

   (d) 120

10. Using binomial theorem, find α_3 for the following expression

$$(3a - b)^7 = \alpha_0 a^7 + \alpha_1 a^6 b + \alpha_2 a^5 b^2 + \alpha_3 a^4 b^3 + \alpha_4 a^3 b^4 + \alpha_5 a^2 b^5 + \alpha_6 a b^6 + \alpha_7 b^7$$

    (a) -128

    (b) -10206

    (c) 20412

    (d) -2835

11. What is the number of 6-card hands with three hearts and three spades?

    (a) 81696

    (b) 81796

    (c) 312

    (d) 1796

Remembering probability and conditional probability

12. Consider two dice experiment where all 36 different pairs are equally probable. Calculate the probability of the event "the numbers are different for the two dice".

    (a) 10/36

    (b) 20/36

    (c) 30/36

    (d) 25/36

13. Consider the same experiment as above and now consider the sum of two numbers on the two dice. What is the most probable value of this sum?

    (a) 10

    (b) 12

    (c) 7

    (d) 6

14. Six people, including A,B, and C, form a queue in a random order. Consider the event "A is the first in the queue". What is its probability?

    (a) 1/3

    (b) 1/2

    (c) 1/6

    (d) 1/4

15. Same setting as above. What is the probability of the event "A precedes B in the queue"? Note that precede means A should come before B. For example, A precedes B both in ABCDE and ACDEB.

    (a) 1/3

    (b) 1/4

    (c) 1/2

(d) 1/6

16. Say you roll two dice. What is the probability that the first dice has 1 given the total sum is 5.

    (a) 1/36

    (b) 1/10

    (c) 1/4

    (d) 1/2