Homework 2

① Rotation @ $30° = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$

② $6A + 4B + 2C = 56$ - Week 1
$4A + 2B + 4C = 46$ - Week 2
$3A + 3B + 3C = 45$ - Week 3

$$\begin{bmatrix} 6 & 4 & 2 \\ 4 & 2 & 4 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 56 \\ 46 \\ 45 \end{bmatrix}$$

③
$$\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 4 & 2 & 4 & 46 \\ 3 & 3 & 3 & 45 \end{array}\right]$$

- $R_1 \to R_1/2$, $R_2 \to R_2/2$, $R_3 \to R_3/3$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 28 \\ 2 & 1 & 2 & 23 \\ 1 & 1 & 1 & 15 \end{array}\right]$$

- $R_2 \to R_2 - 2R_3$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 28 \\ 0 & -1 & 0 & \\ 1 & 1 & 1 & 15 \end{array}\right]$$

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③ $\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 4 & 2 & 4 & 46 \\ 3 & 3 & 3 & 45 \end{array}\right]$

~~$3(4) - \frac{2}{3}(6) = 12$~~

- $R_2 \to 3R_2 - 2/3\,R_1$

$\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 0 & -2/3 & 8/3 & 26/3 \\ 3 & 3 & 3 & 45 \end{array}\right]$

- $R_2 \to R_2 * 3$

$\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 0 & -2 & 8 & 26 \\ 3 & 3 & 3 & 45 \end{array}\right]$

- $R_2 \to R_2 / -2$

$\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 0 & 1 & -4 & -13 \\ 3 & 3 & 3 & 45 \end{array}\right]$

- $R_3 \to 2R_3 - R_1$

$\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 0 & 1 & -4 & -13 \\ 0 & 2 & 4 & 34 \end{array}\right]$

- $R_3 \to R_3 - 2R_2$

$\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & 12 & 60 \end{array}\right]$

$4 - 7(-4)$

$4 - (-3)$

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④ As

: a+

- $R_3 \to R_3/12$

$$\left[\begin{array}{ccc|c} 6 & 4 & 2 & 56 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$\frac{8}{12} = \frac{2}{3}$

$\frac{112}{12} \frac{56}{6} \frac{28}{3}$

- $R_1 \to R_1 * 2$

$$\left[\begin{array}{ccc|c} 12 & 8 & 4 & 112 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$

$\frac{1}{3} - \frac{4}{3} = \frac{-3}{3} = -1$

$\frac{28}{3} - \left(\frac{-13}{3}\right) \quad \frac{15}{3} = 5$

$\frac{28+13}{3} =$

- $R_1 \to R_1/12$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 28/3 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

- ~~$R_1 \to R_1 * 3$~~ $= R_1 \to R_1 + \frac{1}{3} R_2$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 28 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

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$$\therefore \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ -13 \\ 5 \end{bmatrix}$$

④ As $\boxed{c = 5}$

$\therefore 1b - 4c = -13 \to b - 4(5) = -13$

$\to b - 20 = -13$

$\to \boxed{b = 7}$

$\therefore a + b - c = 5 \to a + 7 - 5 = 5$

$\to a + 2 = 5$

$\to \boxed{a = 3}$

⑦ $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} (1\times-1)+(-1\times1) & (1\times1)+(-1\times-1) \\ (-1\times-1)+(1\times1) & (-1\times1)+(1\times-1) \end{bmatrix}$

$= \begin{bmatrix} -1-1 & 1+1 \\ 1+1 & -1-1 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

⑧ $\left[\begin{array}{ccc|ccc} 6 & 4 & 2 & 1 & 0 & 0 \\ 4 & 2 & 4 & 0 & 1 & 0 \\ 3 & 3 & 3 & 0 & 0 & 1 \end{array}\right]$

$R_2 \rightarrow R_2 - 2/3\,R_1$, $R_3 \rightarrow R_3 - \frac{1}{2}R_1$

$\left[\begin{array}{ccc|ccc} 6 & 4 & 2 & 1 & 0 & 0 \\ 0 & -2/3 & 8/3 & -2/3 & 1 & 0 \\ 0 & 1 & 2 & -1/2 & 0 & 1 \end{array}\right]$

$R_3 \rightarrow -3/2\,R_2 + R_3$

$\left[\begin{array}{ccc|ccc} 6 & 4 & 2 & 1 & 0 & 0 \\ 0 & -2/3 & 8/3 & -2/3 & 1 & 0 \\ 0 & 2 & -2 & 1/2 & -3/2 & 1 \end{array}\right]$

$R_1 \rightarrow -3/2\,R_2 + R_1$

$\left[\begin{array}{ccc|ccc} 0 & -2/3 & 8/3 & -2/3 & 1 & 0 \\ 0 & 2 & -2 & 1/2 & -3/2 & 1 \end{array}\right]$

$3 - \frac{1}{2}(2)$

$3 - 1 = (2)$

$0 - \frac{2}{3}(1) = -2/3$

$0 - \frac{1}{2}(1) = -1/2$

$\frac{-3}{2}(0) + 0 = 0$

$\frac{-3}{2}\left(\frac{-2}{3}\right) + 1 = 2$

$\frac{-3}{2}\left(\frac{8}{3}\right) + 2 = -2$

$\frac{-3}{2}\left(\frac{-2}{3}\right) - \frac{1}{2} = \frac{1}{2}$

$\frac{-3}{2}(1) + 0 = \frac{-3}{2}$

$\frac{-3}{2}(0) + 6 = 6$

$\frac{-3}{2}\left(\frac{-2}{3}\right) + 4 = 5$

$\frac{-3}{2}\left(\frac{8}{3}\right) + 2 = -2$

3)

LU decomposition of $\begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$

$$L = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$$

- $R_2 \rightarrow R_2 - (-1)R_1$

$$(-1)\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 6 & -4 & 0 \end{bmatrix}$$

$-3-(-1)3 = -3+3 = 0$
$5-(-1)-7 = 5-7 = -2$
$1-(-1)-2 = 1-2 = -1$

- $R_3 \rightarrow R_3 - (2)R_1$

$$(2)\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 10 & 4 \end{bmatrix}$$

$6-2(3) = 6-6 = 0$
$-4-2(-7) = -4+14 = 10$
$0-2(-2) = 0+4 = 4$

- $R_3 \rightarrow R_3 - (-5)R_2$

$$(-5)\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$10-(-5)(-2) = 10-10 = 0$
$4-(-5)(-1) = 4-5 = -1$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$2\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix} + 1\begin{vmatrix}1 & 1\\ 0 & 2\end{vmatrix} + 0$$

$$\rightarrow 2(4-1) + 1(2-0) + 0 \rightarrow 2(3) + 2)$$

6 + 2

(15)
$$\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & 1\\ 0 & 1 & 2\end{bmatrix}$$

$$-\lambda^3 + 6\lambda^2 - \left(2\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix} + 2\begin{vmatrix}2 & 0\\ 0 & 2\end{vmatrix} + 2\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}\right) + 4 = 0$$

6 + 4 + 6

~~$-\lambda^3 + 6\lambda^2 - (2(2) + 2(4)$~~

$$-\lambda^3 + 6\lambda^2 - 10\lambda + 4 = 0$$

$$\rightarrow -(\lambda - 2)(\lambda^2 - 4\lambda + 2) = 0$$

$$\rightarrow (2-\lambda)(\lambda^2 - 4\lambda + 2) = 0$$

$2 - \lambda = 0 \rightarrow \lambda = 2$

$\lambda^2 - 4\lambda = -2$

$\lambda^2 - 4\lambda + 4 = -2 + 4$

$\lambda^2 - 4\lambda + 4 = 2$

$(\lambda + 2)^2 = \pm 2$

$\lambda + 2 = \pm\sqrt{2}$

$\therefore \lambda = \pm\sqrt{2} - 2$

$\therefore \lambda = 2 + \sqrt{2}$

$\lambda = 2 - \sqrt{2}$

$\lambda = 2 + \sqrt{2}$

$\lambda = 2 - \sqrt{2}$

$\lambda = 2$

(2)

$\therefore \frac{df}{dy}$