* DSCI 501 : Homework 1

1. $5a + b = -8$, where $a = 2$

$\rightarrow 5(2) + b = -8$

$\rightarrow 10 + b = -8$

$\rightarrow b = -18$

$\therefore$ $\boxed{d)\ a = 2,\ b = -18}$

2. $6a - 4b = 14$

$4a - 4b = 4 \qquad \times -1$

$\rightarrow 6a - \cancel{4b} = 14$

$-4a + \cancel{4b} = -4$

$2a = 10$

$\boxed{a = 5}$

$\therefore 6(5) - 4b = 14$

$\rightarrow 30 - 4b = 14$

$\rightarrow -4b = -16$

$\rightarrow \boxed{b = 4}$

$\therefore$ $\boxed{b)\ a = 5,\ b = 4}$

3. $2a + 3b = 18 \qquad \times -3$

$6a - 4b = 2$

$\rightarrow -\cancel{6a} - 9b = -54$

$\cancel{6a} - 4b = 2$

$-13b = -52$

$\boxed{b = 4}$

$2a + 3(4) = 18$

$2a + 12 = 18$

$2a = 6$

$a = 3$

$\therefore$ $\boxed{b)\ a = 3,\ b = 4}$

4. $3a - 2b + c = 7$ — ①
$a + b + c = 2$ — ②
$3a - 2b - c = 3$ — ③

① and ②

$3a - 2b + c = 7$
$a + b + c = 2$ $\times 2$

$\rightarrow$ $3a - \cancel{2b} + c = 7$
$2a + \cancel{2b} + 2c = 4$
$5a + 3c = 11$ — ④

② and ③

$a + b + c = 2$ $\times 2$
$3a - 2b - c = 3$

$\rightarrow$ $2a + \cancel{2b} + 2c = 4$
$3a - \cancel{2b} - c = 3$
$5a + c = 7$ — ⑤

④ and ⑤

$5a + 3c = 11$
$5a + c = 7$ $\times -1$

$\rightarrow$ $\cancel{5a} + 3c = 11$
$-\cancel{5a} - c = -7$
$2c = 4$
$\boxed{c = 2}$

~~Substitute c in~~

④ with value of c,

$5a + 3c = 11$

$\rightarrow 5a + 3(2) = 11$

$\rightarrow 5a + 6 = 11$

$\rightarrow 5a = 5 \rightarrow \boxed{a = 1}$

② with values of a and c,

$a + b + c = 2$

$\rightarrow 1 + b + 2 = 2$

$\rightarrow 3 + b = 2$

$\rightarrow \boxed{b = -1}$

$\therefore \boxed{b)\ a = 1,\ b = -1,\ c = 2}$

11.

$$s = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 3 \\ \sqrt{6} \end{bmatrix}$$

$\rightarrow |s| = \sqrt{2^2 + 1^2 + 4^2 + 3^2 + (\sqrt{6})^2}$

$|s| = \sqrt{4 + 1 + 16 + 9 + 6}$

$|s| = \sqrt{5 + 25 + 6}$

$|s| = \sqrt{36} \rightarrow \boxed{|s| = 6}$

12. $r = \begin{bmatrix} 3 \\ 2 \\ 8 \\ -5 \end{bmatrix}$ and $s = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{aligned} r \cdot s &= r_1 s_1 + r_2 s_2 + r_3 s_3 + r_4 s_4 \\ &= 3 \cdot 2 + 2 \cdot -1 + 8 \cdot 0 + -5 \cdot 1 \\ &= 6 - 2 + 0 - 5 \\ &= 6 - 7 \\ &= -1 \end{aligned}$$

$\boxed{r \cdot s = -1}$

13. Scalar Projection of v on u, $\frac{v \cdot u}{|u|}$

$v = \begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}, u = \begin{bmatrix} -6 \\ 5 \\ 10 \end{bmatrix}$

$\rightarrow \dfrac{v_1 u_1 + v_2 u_2 + v_3 u_3}{\sqrt{u_1^2 + u_2^2 + u_3^2}}$

$\rightarrow \dfrac{(0 \cdot -6) + (-4 \cdot 5) + (3 \cdot 10)}{\sqrt{(-6)^2 + (5)^2 + (10)^2}}$

$\rightarrow \dfrac{0 - 20 + 30}{\sqrt{36 + 25 + 100}} \rightarrow \dfrac{10}{\sqrt{}}$

13. $u = \begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} -6 \\ 5 \\ 10 \end{bmatrix}$

Scalar Projection of v on u, $\frac{v \cdot u}{|u|}$

$\rightarrow \frac{v_1 u_1 + v_2 u_2 + v_3 u_3}{\sqrt{u_1^2 + u_2^2 + u_3^2}}$

$\rightarrow \frac{(-6 \cdot 0) + (5 \cdot -4) + (10 \cdot 3)}{\sqrt{0^2 + -4^2 + 3^2}}$

$\rightarrow \frac{0 - 20 + 30}{\sqrt{16+9}} \rightarrow \frac{10}{\sqrt{25}} \rightarrow \frac{10}{5} = \boxed{2}$

14. Vector Projection of v onto u, $\left(\frac{v \cdot u}{|u|^2}\right) \cdot u$

As we already have some values,

$\rightarrow \frac{v \cdot u}{|u|} \cdot \frac{u}{|u|}$, where $v \cdot u = 10$ and $|u| = 5$

$\rightarrow \frac{10}{5} \cdot \frac{u}{5}$

$\rightarrow \frac{2}{5} \begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0 \\ -8/5 \\ 6/5 \end{bmatrix}$

15. Consider 2 vectors, $u = [1 \ 2]$, $v = [3 \ 4]$

$$\|u+v\| = \|(1+3),(2+4)\|$$
$$= \|4, 6\|$$
$$= \sqrt{4^2+6^2}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$

$$\|u\|+\|v\| = \sqrt{1^2+2^2} + \sqrt{3^2+4^2}$$
$$= \sqrt{1+4} + \sqrt{9+16}$$
$$= \sqrt{5} + \sqrt{25}$$
$$= \sqrt{5}+5$$

$$\boxed{a) \ \|u+v\| \leq \|u\| + \|v\|}$$

16. $V_b = c_1 \cdot b_1 + c_2 b_2 + c_3 b_3 + c_4 b_4$
where c are coefficient

$$c_1 = \frac{v \cdot b_1}{b_1 \cdot b_1} = \frac{[1 \ 1 \ 2 \ 3] \cdot [1 \ 0 \ 0 \ 0]}{[1 \ 0 \ 0 \ 0] \cdot [1 \ 0 \ 0 \ 0]}$$
$$= \frac{1 \cdot 1 + 1 \cdot 0 + 2 \cdot 0 + 3 \cdot 0}{1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0}$$
$$= \frac{1}{1}$$

$$\boxed{c_1 = 1}$$

$$c_2 = \frac{v \cdot b_2}{b_2 \cdot b_2} = \frac{[1\ 1\ 2\ 3][0\ 2\ -1\ 0]}{[0\ 2\ -1\ 0][0\ 2\ -1\ 0]}$$

$$= \frac{1 \cdot 0 + 1 \cdot 2 + 2 \cdot -1 + 3 \cdot 0}{0 \cdot 0 + 2 \cdot 2 + -1 \cdot -1 + 0 \cdot 0}$$

$$= \frac{0 + 2 - 2 + 0}{0 + 4 + 1 + 0} = \frac{0}{5} = 0$$

$$\boxed{c_2 = 0}$$

$$c_3 = \frac{v \cdot b_3}{b_3 \cdot b_3} = \frac{[1\ 1\ 2\ 3][0\ 1\ 2\ 0]}{[0\ 1\ 2\ 0][0\ 1\ 2\ 0]}$$

$$= \frac{1 \cdot 0 + 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 0}{0 \cdot 0 + 1 \cdot 1 + 2 \cdot 2 + 0 \cdot 0}$$

$$= \frac{0 + 1 + 4 + 0}{0 + 1 + 4 + 0} = \frac{5}{5} = 1$$

$$\boxed{c_3 = 1}$$

$$c_4 = \frac{v \cdot b_4}{b_4 \cdot b_4} = \frac{[1\ 1\ 2\ 3][0\ 0\ 0\ 3]}{[0\ 0\ 0\ 3][0\ 0\ 0\ 3]}$$

$$= \frac{1 \cdot 0 + 1 \cdot 0 + 2 \cdot 0 + 3 \cdot 3}{0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 3 \cdot 3}$$

$$= \frac{0 + 0 + 0 + 9}{0 + 0 + 0 + 9} = \frac{9}{9} = 1$$

$$c_4 = 1$$

$$\therefore v_b = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$