DSCI 501 - Homework 4

① $f(x) = e^{-2x}$, $x = 2$, order = 1

$\rightarrow f(x) = e^{-2x}$

$f'(x) = -2e^{-2x}$

$= \frac{-2}{e^{2x}}$

$f'(2) = \frac{-2}{e^{2(2)}}$

$= \frac{-2}{e^4}$

$f(x) \approx f(2) + f'(2)(x-2)$

$\approx e^{-4} - 2e^{-4}(x-2)$

$\approx \frac{1}{e^4} - \frac{2}{e^4}(x-2)$

$= \frac{1}{e^4}\left[1 - 2(x-2)\right]$

$= \frac{1}{e^4}\left[1-2(x-2)\right]$ (B)

② $y = x^3 - 2x + 2$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x) = x^3 - 2x + 2$

$f'(x) = 3x^2 - 2$

$x_0 = -2$

$\therefore f(-2) = (-2)^3 - 2(-2) + 2$

$= -8 + 4 + 2 = -2$

$f'(-2) = 3(-2)^2 - 2 = 3(4) - 2 = 12 - 2 = 10$

$x_1 = -2 - \frac{-2}{10} = -2 + 0.2 = -1.8$

$f(-1.8) = (-1.8)^3 - 2(-1.8) + 2$

$= -5.832 + 3.6 + 2$

$= -0.232$

$f'(1.8) = 3(-1.8)^2 - 2 = 7.72$

$x_2 = -1.8 - \left[\frac{-0.232}{7.72}\right]$

$= -1.8 + [0.03005]$

$= -1.76994$

$\approx \boxed{-1.77}$ (B)

③ $A = \{1, 2, 4, 6, 7\}$
$B = \{1, 2, 3, 5, 7\}$

$A \cap B = \{1, 2, 7\}$ → common (B)

④ $A = \{1, 2, 4, 6, 7\}$
$B = \{1, 2, 3, 5, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$
→ Unique elements (B)

⑤ French = $|F| = 17$
German = $|G| = 14$
Students = $|S| = 25$

$|F \cap G|$ → ?

$|F \cup G| = |F| + |G| - |F \cap G|$
→ $|F \cup G| = 17 + 14 - |F \cap G|$
→ $|F \cup G| = 31 - |F \cap G|$

$|F \cup G|$ can't exceed 25

∴ $31 - |F \cap G| \leq 25$
$31 - 25 \leq |F \cap G|$
$6 \leq |F \cap G|$

∴ (6) (b)

⑥ No. of 1000 divisible by 2 = $\frac{1000}{2} = 500$

No. of 1000 divisible by 3 = $\frac{1000}{3} = 333$

No. of 1000 divisible by 6 = $\frac{1000}{6} = 166$

Total = 500 + 333 − 166

= **667** (d)

⑦ 1000 not divisible by 2 and 3

= 1000 − 667 = **333** (c)

⑧ $^{10}C_5 = \frac{n!}{k!(n-k)!}$

$= \frac{10!}{5!\,5!}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5!}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$

$= 9 \times 7 \times 2 \times 2$

$= 63 \times 4$

$=$ **252**

(9) ${}^{10}C_5 = \boxed{252}$

(10) $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

$(3a-b)^7$, $a = 3a$, $b = -b$

$\alpha_3 = a^4 b^3$

$\therefore a^4b^3$ is $\binom{7}{3} = \frac{7!}{3!(7-3)!}$

$= \frac{7 \times 6 \times 5 \times 4!}{3! 4!}$

$= 35$

$\therefore \alpha_3 = -35$

$\therefore$ ~~$\alpha_3 *$~~ $\boxed{-2835}$ (d)

(11) 3 hearts out of 13 $= {}^{13}C_3$

3 spades out of 13 $= {}^{13}C_3$

${}^{13}C_3 * {}^{13}C_3 = \frac{13!}{3!\,10!} \times \frac{13!}{3!\,10!}$

$= \frac{13 \times 12^2 \times 11 \times 10!}{3! \times 10!} \times \frac{13 \times 12^2 \times 11 \times 10!}{3! \times 10!}$

$= \boxed{81796}$ (b)

(12) P(Diff nos.) = $\frac{\text{No. of favorable outcomes}}{\text{Total No. of outcomes}}$

$= \frac{30}{36}$

(12)

Sum	Ways to Obtain	Probability
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

$\therefore 6/36 = 1/6 \rightarrow$ (7) (c)

(14) P(A is first) = No. of arrangement when A is first / Total no. of possible arrangement

$= \frac{5!}{6!}$

$= \frac{5!}{6 \times 5!}$

$= \boxed{1/6}$ (c)

(15) No. of arrangements A preceeds B is $5 \times 4!$

P(A preceeds B) $= \frac{5 \times 4!}{6!}$

$= \frac{5 \times 4!}{6 \times 5 \times 4!}$

$= \frac{1}{6}$ (d)

(16)

First Die (1-6)	Second Die (1-6)	Probability
1	4	1/36
2	3	1/36
3	2	1/36
4	1	1/36

1/4 (c)