Homework 3

① $f(x) = x^{780}$

Power rule , $x^n \rightarrow n \cdot x^{(n-1)}$

$\therefore f'(x) = 780 \cdot x^{(780-1)}$
$= \underline{780x^{779}}$

② $f(x) = x^3 - \frac{1}{x} + 4$

$- \frac{d}{dx}\left[x^3 - \frac{1}{x} + 4\right]$

$- \frac{d}{dx}(x^3) - \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}(4)$

$- 3x^2 + \frac{1}{x^2} + 0 \implies 3x^2 + \frac{1}{x^2}$

③ $f(x) = e^x - 3\sin x + x^3$

$- f'(x) = \frac{d}{dx}(e^x) - \frac{d}{dx}(3\sin x) + \frac{d}{dx}(x^3)$
$= e^x - 3\frac{d}{dx}(\sin x) + 3x^2$
$= e^x - 3\cos x + 3x^2$

$f''(x) = \frac{d}{dx}(e^x) - 3\frac{d}{dx}(\cos x) + 3\frac{d}{dx}(x^2)$
$= e^x + 3\sin x + 3 \cdot 2x \cdot \cancel{d}$
$= \underline{e^x + 3\sin x + 6x}$

④ $f(x) = (x+1)(3x+4)$
$= 3x^2 + 4x + 3x + 4$
$= 3x^2 + 7x + 4$

$f'(x) = \frac{d}{dx}(3x^2 + 7x + 4)$
$= 3\frac{d}{dx}(x^2) + 7\frac{d}{dx}(x) + \frac{d}{dx}(4)$
$= 3 \cdot 2x + 7 \cdot 1 + 0$
$= 6x + 7$

⑤ $f(x) = x \cdot e^x \cos x$
$- \frac{d}{dx}\left[x \cdot e^x \cdot \cos x\right]$
$- \frac{d}{dx}(x) \cdot e^x \cos x + x \cdot \frac{d}{dx}(e^x) \cdot \cos x + x \cdot e^x \cdot \frac{d}{dx}(\cos x)$
$- 1 \cdot e^x \cdot \cos x + x \cdot e^x \cdot \cos x + x \cdot e^x \cdot -\sin x$
$- e^x \cdot \cos x + x \cdot e^x \cdot \cos x - x \cdot e^x \cdot \sin x$

$- e^x\left[\cos x + x\cos x - \cancel{e} x \sin x\right]$

⑥ $f(x) = \cos^4 x$
$\frac{d}{dx}(\cos^4 x)$
$- 4\cos^3 x \cdot \frac{d}{dx}(\cos^4 x)$

$- 4\cos^3 x \cdot -\sin x$
$- -4\cos^3 x \cdot \sin x$

⑦ $f(x) = e^{\sin(x^2)}$

- $\frac{d}{dx}\left(e^{\sin(x^2)}\right)$
- $e^{\sin(x^2)} \cdot \frac{d}{dx}(\sin x^2)$
- $e^{\sin(x^2)} \cdot \cancel{2} \cos x^2 \cdot \frac{d}{dx}(x^2)$
- $e^{\sin(x^2)} \cdot \cos x^2 \cdot 2x$
- $e^{\sin(x^2)} \cdot \cos x^2 \cdot 2x$

⑧ $f(x) = x^{3/2} + \pi x^2 + 7$, $x = 4$ evaluated

$\frac{d}{dx}\left[x^{3/2} + \pi x^2 + 7\right]$

- $\frac{d}{dx}(x^{3/2}) + \frac{d}{dx}(\pi x^2) + \frac{d}{dx}(7)$
- $\cancel{\frac{3}{2}d}$ $\frac{3}{2} x^{1/2} + \pi \cdot \frac{d}{dx}(x^2) + 0$
- $\frac{3\sqrt{x}}{2} + 2\pi x$, $x = 4$ add value
- $\frac{3\sqrt{4}}{2} + 2\pi(4)$
- $\frac{3 \cdot 2}{2} + 8\pi$
- $\boxed{8\pi + 3}$

(9) $f(x) = \sin(x) \cdot e^{\cos(x)}, \quad x = \pi$

$- \cancel{\frac{d}{dx}\left(\sin(x)\right)} \quad \frac{d}{dx}\left(\sin(x) \cdot e^{(\cos(x))}\right)$

$- e^{\cos x} \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}\left(e^{\cos x}\right)$

$- e^{\cos x} \cdot \cos x + \sin x \cdot e \cdot \frac{d}{dx}(\cos x)$

$- e^{\cos x} \cdot \cos x + \sin x \cdot e \cdot -\sin x$

$- e^{\cos x} \cdot \cos x - e \cdot \sin^2 x$ $\quad \cos \pi = -1$

$\sin \pi = 0$

$- e^{-1} \cdot -1 - e \cdot 0$

$- \frac{-1}{e} - 0 \implies \frac{-1}{e}$

(10) $\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}$ for $f(x,y,z) = x^2y + y^2z + z^2x$

$$\frac{df}{dx} = \frac{d}{dx}\left[x^2y + y^2z + z^2x\right] = 2xy + z^2$$

$$\frac{df}{dy} = \frac{d}{dy}\left[x^2y + y^2z + z^2x\right] = \cancel{y^2}\,\cancel{z^2}\, x^2 + 2yz$$

$$\frac{df}{dz} = \frac{d}{dz}\left[x^2y + y^2z + z^2x\right] = y^2 + 2xz$$

(11) $f(x,y,z) = \cos(x) \cdot \sin(y) \cdot e^{2z}$,

$x(t) = t+1$, $y(t) = t-1$, $z\cancel{z}(t) = t^2$

$$\frac{d}{dx}\left[\cos(x) \cdot \sin(y) \cdot e^{2z}\right]$$

$$- \sin(y) \cdot e^{2z} \cdot \frac{d}{dx}\left[\cos(x)\right]$$

$$- \sin(y) \cdot e^{2z} \cdot -\sin x$$

$$- -e^{2z} \cdot \sin x \cdot \sin y \quad — \quad (1)$$

$$\frac{d}{dy}\left[\cos(x) \cdot \sin(y) \cdot e^{2z}\right]$$

$$- \cos(x) \cdot e^{2z} \cdot \frac{d}{dy}(\sin y)$$

$$- \cos(x) \cdot e^{2z} \cdot \cos(y).$$

$$- e^{2z} \cdot \cos x \cdot \cos y \quad — \quad (2)$$

$$- \frac{d}{dz}\left[\cos(x) \cdot \sin(y) \cdot e^{2z}\right]$$

$$- \cos x \cdot \sin y \cdot \frac{d}{dz}\left[e^{2z}\right]$$

$$- \cos x \cdot \sin y \cdot e^{2z} \cdot \frac{d}{dz}(2z)$$

$$- \cos x \cdot \cancel{co} \sin y \cdot e^{2z} \cdot 2$$

$$- 2e^{2z} \cdot \cos x \cdot \sin y \quad — \quad (3)$$

Combine ①, ② and ③

$= -e^{2z} \cdot \sin x \cdot \sin y + e^{2z} \cdot \cos x \cdot \cos y + 2e^{2z} \cdot \cos x \cdot \sin y$

$= e^{2z}[-\sin x \cdot \sin y + \cos x \cdot \cos y + 2 \cos x \cdot \sin y]$

Value of x, y, z in above

$= \cancel{2e^{2t^2}}$

$= e^{2t^2}[-\sin(t+1) \cdot \sin(t-1) + \cos(t+1) \cdot \cos(t-1) +$
$2 \cdot \cos(t+1) \cdot \sin(t-1)]$

⑫ Jacobian Row Vector $f(x,y) = 2x^2y + xy + 5$

$\frac{d}{dx}[2x^2y + xy + 5]$

$= \frac{d}{dx}(2x^2y) + \frac{d}{dx}(xy) + \frac{d}{dx}(5)$

$= y \cdot \frac{d}{dx}(2x^2) + y \cdot \frac{d}{dx}(x) + 0$

$= 4xy + y$ — ①

$\cancel{\frac{d}{dy}[2x^2y + xy + 5]}$

$$\frac{d}{dy}\left[2x^2y + xy + 5\right]$$

$$= \frac{d}{dy}(2x^2y) + \frac{d}{dxy}(xy) + \frac{d}{dy}(5)$$

$$= 2x^2 \cdot \frac{d}{dy}(y) + x \cdot \frac{d}{dy}(y) + 0$$

$$= 2x^2 + x \quad — ②$$

(13) $f(x, y) = x^2 + e^y e^z + \cos(x) \cdot \text{siz}(z)$

$$\frac{d}{dx}\left[x^2 + \cancel{y^z} e^y e^z + \cos(x) \cdot \sin(z)\right]$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(e^y e^z) + \frac{d}{dx}\left(\cos(x) \cdot \sin(z)\right)$$

$$= 2x + 0 + \sin(z) \cdot \frac{d}{dx}(\cos x)$$

$$= 2x + \sin(z) * -\sin(x)$$

$$= 2x - \sin(x) \cdot \sin(z)$$

$$= 2(0) - \sin(0) \cdot \sin(0) \rightarrow 0$$

$$= \frac{d}{dy}\left[x^2 + e^y e^z + \cos(x) \cdot \sin(z)\right]$$

$$= \frac{d}{dy}(x^2) + e^z \frac{d}{dy}(e^y) + \frac{d}{dy}(\cos x \cdot \sin z)$$

$$= 0 + e^{z+y} \cdot \frac{d}{dy}(y+z) + 0$$

$$= \cancel{\theta}\ e^{z+y} \cdot \frac{d}{dy}(y) \cdot \frac{d}{dy}(z)$$

$- e^{y+z}(1+0) \rightarrow e^{y+z} \rightarrow e^{0+0} \rightarrow e^0 \rightarrow 1$

$\frac{d}{dz}\left[x^2 + e^y e^z + \cos(x)\cdot\sin(z)\right]$

$- \frac{d}{dz}(x^2) + \frac{d}{dz}\left(e^{y+z}\right) + \frac{d}{dz}\left[\cos(x)\cdot\sin(z)\right]$

$- 0 + e^{y+z}\cdot\frac{d}{dz}(y+z) + \cos(x)\cdot\frac{d}{dz}(\sin z)$

$- e^{y+z}\cdot\left[\frac{d}{dz}(y) + \frac{d}{dz}(z)\right] + \cos(x)\cdot\cos(z)$

$- e^{y+z}[0+1] + \cos(x)\cdot\cos(z)$

$- e^{0+0} + \cos(0)\cdot\cos(0)$

$- e^0 + 1\cdot 1 \rightarrow 1+1 \rightarrow 2$

$\therefore f(x,y,z) = (0,1,2)$

(14) Point $(0, \pi)$

$u(x,y) = x^2y - \cos(x)\cdot\sin(y)$

$v(x,y) = e^{x+y}$

$\frac{du}{dx} = \frac{d}{dx}\left[x^2y - \cos(x)\cdot\sin(y)\right]$

$= \frac{d}{dx}\left[x^2y\right] - \frac{d}{dx}\left[\cos x\cdot\sin y\right]$

$= y\cdot\frac{d}{dx}(x^2) - \sin y\cdot\frac{d}{dx}(\cos x)$

$= 2xy - \sin y\cdot - \sin x$

$$\frac{du}{dx} = \cancel{d}\, 2xy + \sin x \cdot \sin y \qquad (x = 0, y = \pi)$$

$$= 2(0)(\pi) + \sin(0) \cdot \sin(\pi)$$

$$= 0 + 0 \cdot 0 = 0 + 0 = 0$$

$$\frac{du}{dy} = x^2 y - \cos(x) \cdot \sin(y)$$

$$= \frac{d}{dy}(x^2 y) - \frac{d}{dy}\left(\cos(x) \cdot \sin(y)\right)$$

$$= x^2 \cdot \frac{d}{dy}(y) - \cos(x) \cdot \frac{d}{dy}(\sin y)$$

$$= x^2 - \cos(x) \cdot \cos(y)$$

$$= 0 - 1 \cdot -1$$

$$= 0 + 1 = 1$$

$$\frac{dv}{dx} = \frac{d}{dx}\left[e^{x+y}\right]$$

$$= e^{x+y} \cdot \frac{d}{dx}(x + y)$$

$$= e^{x+y} \cdot \left[\frac{d}{dx}(x) + \frac{d}{dx}(y)\right]$$

$$= e^{x+y} \cdot (1 + 0)$$

$$= e^{x+y}$$

$$= e^{0+\pi} = e^{\pi}$$

Same for $\frac{dv}{dy} = \frac{du}{dx}$

(15) $f(x,y,z) = x \cdot y \cdot \cos(z) - \sin(x) \cdot e^y \cdot z^3$

$$\frac{d}{dx}\left[x \cdot y \cdot \cos(z) - \sin(x) \cdot e^y \cdot z^3\right]$$

$$= \frac{d}{dx}\left(x \cdot y \cdot \cos(z)\right) - \frac{d}{dx}\left(\sin(x) \cdot e^y \cdot z^3\right)$$

$$= y \cdot \cos(z) \cdot \frac{d}{dx}(x) - e^y \cdot z^3 \cdot \frac{d}{dx}(\sin(x))$$

$$= y \cdot \cos(z) - e^y \cdot z^3 \cdot \cos x$$

$$= y \cdot \cos(z) - \cos(x) \cdot e^y \cdot z^3 \quad \text{--- } (1)$$

$$\frac{d}{dy}\left[x \cdot y \cdot \cos(z)\right] - \frac{d}{dy}\left[\sin(x) \cdot e^y \cdot z^3\right]$$

$$= x \cdot \cos(z) \cdot \frac{d}{dy}(y) - \sin(x) \cdot z^3 \cdot \frac{d}{dy}(e^y)$$

$$= x \cdot \cos(z) - \sin(x) \cdot e^y \cdot z^3 \quad \text{--- } (2)$$

$$\frac{d}{dz}\left[x \cdot y \cdot \cos(z)\right] - \frac{d}{dz}\left[\sin(x) \cdot e^y \cdot z^3\right]$$

$$= x \cdot y \cdot \frac{d}{dz}\left[\cos(z)\right] - \sin(x) \cdot e^y \cdot \frac{d}{dz}(z^3)$$

$$= x \cdot y \cdot -\sin(z) - \sin(x) \cdot e^y \cdot 3z^2$$

$$= -xy\sin(z) - 3\sin(x) \cdot e^y \cdot z^2 \quad \text{--- } (3)$$

$$f'(x,y,z) = \begin{bmatrix} d/dx \\ d/dy \\ d/dz \end{bmatrix} = \begin{bmatrix} y \cdot \cos(z) - \cos(x) \cdot e^y \cdot z^3 \\ x \cdot \cos(z) - \sin(x) \cdot e^y \cdot z^3 \\ -xy \cdot \sin(z) - 3\sin(x) \cdot e^y \cdot z^2 \end{bmatrix}$$

$$\frac{d}{dx}\left[y \cdot \cos(z)\right] - \frac{d}{dx}\left[\cos(x) \cdot e^y \cdot z^3\right]$$

$$= 0 - e^y \cdot z^3 \cdot \frac{d}{dx}\left[\cos(x)\right]$$

$$= -e^y \cdot z^3 \cdot -\sin(x)$$

$$= \sin(x) \cdot e^y \cdot z^3 \quad — \quad 1 \Rightarrow 1$$

$$\frac{d}{dy}\left[y \cdot \cos(z)\right] - \frac{d}{dy}\left[\cos(x) \cdot e^y \cdot z^3\right]$$

$$= \cos(z) \cdot \frac{d}{dy}(y) - \cos(x) \cdot z^3 \cdot \frac{d}{dy}(e^y)$$

$$= \cos(z) - \cos(x) \cdot e^y \cdot z^3 \quad — \quad 2 \; 4 \Rightarrow 1|2$$

$$\frac{d}{dz}\left[y \cdot \cos(z)\right] - \frac{d}{dz}\left[\cos(x) \cdot e^y \cdot z^3\right]$$

$$= y \cdot \frac{d}{dz}\left[\cos(z)\right] - \cos(x) \cdot e^y \cdot \frac{d}{dz}\left[z^3\right]$$

$$= y \cdot -\sin(z) - \cos(x) \cdot e^y \cdot 3z^2$$

$$= -y \cdot \sin(z) - 3\cos(x) \cdot e^y \cdot z^2 \quad — \quad 3 \; 7 \Rightarrow 1|3$$

$$\frac{d}{dy}\left[x \cdot \cos(z)\right] - \frac{d}{dy}\left[\sin(x) \cdot e^y \cdot z^3\right]$$

$$= 0 - \sin(x) \cdot z^3 \cdot \frac{d}{dy}(e^y)$$

$$= -\sin(x) \cdot e^y \cdot z^3 \quad — \quad 5 \Rightarrow 2$$

$$\frac{d}{dz}\left[x \cdot \cos(z)\right] - \frac{d}{dz}\left[\sin(x) \cdot e^y \cdot z^3\right]$$

$$= x \cdot \frac{d}{dz}\left[\cos(z)\right] - \sin(x) \cdot e^y \cdot \frac{d}{dz}(z^3)$$

$$= x \cdot - \sin(z) - \sin(x) \cdot e^y \cdot 3z^2$$

$$= -x \cdot \sin(z) - 3\sin(x) \cdot e^y \cdot z^2 \quad — \quad 6\ 8 \Rightarrow 2$$

$$\frac{d}{dz}\left[-x \cdot y \cdot \sin(z)\right] - \frac{d}{dz}\left[3 \cdot \sin(x) \cdot e^y \cdot z^2\right]$$

$$= -xy \cdot \frac{d}{dz}(\sin(z)) - 3\sin(x) \cdot e^y \cdot \frac{d}{dz}(z^2)$$

$$= -xy \cdot \cos(z) - 3 \cdot \sin(x) \cdot e^y \cdot 2z$$

$$= -xy \cdot \cos(z) - 6\sin(x) \cdot e^y \cdot z \quad — \quad 9$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \checkmark$$

(18) $f(x_1, x_2) = \cos(x_1) \cdot \sin(x_2)$

$x_1(u_1, u_2) = 2u_1^2 + 3u_2^2 - u_2$

$x_2(u_1, u_2) = 2u_1 - 5u_2^3$

$u_1(t) = e^{t/2}$

$u_2(t) = e^{-2t}$

$$\frac{df}{dx} = \frac{d}{dx_1}\left[\cos(x_1) \cdot \sin(x_2)\right]$$
$$= \sin(x_2) \cdot \frac{d}{dx_1}\left[\cos(x_1)\right]$$
$$= \sin(x_2) \cdot -\sin(x_1)$$
$$= -\sin(x_1) \cdot \sin(x_2)$$

$$= \frac{d}{dx_2}\left[\cos(x_1) \cdot \sin(x_2)\right]$$
$$= \cos(x_1) \cdot \frac{d}{dx_2}\left[\sin(x_2)\right]$$
$$= \cos(x_1) \cdot \cos(x_2)$$

$$\frac{dx_1}{du_1} = \frac{d}{du_1}\left[2u_1^2 + 3u_2^2 - u_2\right]$$
$$= \frac{d}{du_1}(2u_1^2) + \frac{d}{du_1}(3u_2^2) - \frac{d}{du_1}(u_2)$$
$$= 4u_1 + 0 - 0 \quad = 4u_1$$

$$\frac{dx_1}{du_2} = \frac{d}{du_2}\left[2u_1^2 + 3u_2^2 - u_2\right]$$
$$= 0 + 6u_2 - 1$$
$$= 6u_2 - 1$$

$$\frac{dx_2}{du_1} = \frac{d}{du_1}\left[2u_1 - 5u_2^3\right]$$
$$= 2 - 0 \quad = 2$$

$$\frac{dx_2}{du_2} = \frac{d}{du_2}\left[2u_1\right] - \frac{d}{du_2}\left[5u_2^3\right]$$
$$= 0 - 15u_2^2$$
$$= \cancel{3} - 15u_2^2$$

$$\frac{du_1}{dt} = \frac{d}{dt}\left[e^{t/2}\right]$$
$$= e^{t/2} \cdot \frac{d}{dt}\left(\frac{t}{2}\right)$$
$$= e^{t/2} \cdot \frac{1}{2} \cdot \frac{d}{dt}(t)$$
$$= \frac{e^{t/2}}{2}$$

$$\frac{du_2}{dt} = \frac{d}{dt}\left[e^{-2t}\right]$$
$$= e^{-2t} \cdot \frac{d}{dt}(-2t)$$
$$= e^{-2t} \cdot -2 \cdot \frac{d}{dt}(t)$$
$$= -2e^{-2t}$$

$$\therefore \frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{du} \cdot \frac{du}{dt}$$

$$\Rightarrow \begin{bmatrix} -\sin(x_1)\cdot\sin(x_2) & \cos(x_1)\cdot\cos(x_2) \end{bmatrix} \begin{bmatrix} 4u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} \frac{e^{t/2}}{2} \\ -2e^{-2t} \end{bmatrix}$$