1) **The symmetric difference of two sets is a new set that contains every element that is in either set except the elements that are in both sets. For example, the symmetric difference of {3, 7, 2, 12, 9} and {8, 12, 4, 16, 7, 5} would be {3, 2, 9, 8, 4, 16, 5}. Prove that the family of regular languages is closed under symmetric difference.**

   According to the definition of symmetric difference, we can write the symmetric difference between S1 and S2 as (S1-S2) ∪ (S2 – S1). Now if we say that Ñ is the complement of N, then for any sets, we have (M-N) = (M∩ Ñ). In this case, we can say symmetric difference of S1 and S2 is (S1∩S̃2)∪(S2∩S̃1). Since regular language is closed when the expression is under intersection, union, and complementation, so the family of regular languages is closed under the symmetric difference.

2) **Prove that the language L = {w : $n_a(w) = n_b(w)$} is not regular.**

   First we suppose that language L is regular.

   So now we say that w = $a^m b^m$, w∈L. Consider the conditions of pumping lemma, w = xyz, if we separate them to x = a^{m-j}, y = a^j, z = b^m when $j \geq 1$, it satisfied $|xy| \leq m$, and $|y| \geq 1$. Now let i = 2, $w_i = xy^iz = xy^2z = a^{m-j}*(a^j)^2*b^m = a^{m-j}*a^{2*j}*b^m = a^{m+j}b^m$, which not belonging to L. Therefore, this condition is not satisfied the pumping lemma, so our assumption is not correct. Thus, language

$L = \{w : n_a(w) = n_b(w)\}$ is not regular.

3) **Prove that the language L with $\Sigma = \{a\}$, where $L = \{a^n$: n is a power of 2$\}$ is not regular.**

Suppose this language L is regular.

First we assume that $w = a^{2^\wedge p}$ where $p \geq 0$, and $w = xyz = 2^p \geq m$. Consider the conditions of pumping lemma, we can say that $x = a^j$, $y = a^k$, $z = a^{2^\wedge p\ -j-k}$, where $1 \leq k \leq m$. So, $|xy| = a^j * a^k = a^{j+k} \leq w$, because $w \geq m$, so $|xy| \leq m$. Also, y satisfied $|y| \geq 1$. Now consider i = 2, $w_i = xy^iz = a^j*(a^k)^2* a^{2^\wedge p\ -j-k} = a^{k+2p}$. In this case, k+2p must be power of 2, however, when k = 1, k+2p is not a power of 2 because k+2p must be an odd number. So, it doesn't satisfy the pumping lemma. Thus, our assumption is not correct, so the language L with $\Sigma = \{a\}$, where $L = \{a^n$: n is a power of 2$\}$ is not regular.