Week 4 Wednesday Worksheet

Valentino Aceves

April 24, 2024

Hypothesis Testing for β_1

$$H_0 : \beta_1 = c \quad \text{vs.} \quad H_1 : \beta_1 \neq c$$

In hypothesis testing, we make an assumption or initial guess about what the slope could be. Most of the time, we test $\beta_1 = 0$ because this would mean there is no relationship between X and Y.

Under the null hypothesis, we have the test statistic

$$\hat{t} = \frac{\hat{b}_1 - \beta_1}{SE(\hat{b}_1)} \sim t(d.f. = n-2)$$

β_1 is interpretable into "real world" values (eg. salary), however $\hat{t}$ is not. $\hat{t}$ is β_1 transformed to be interpretable in the "t-world" with respect to what we are testing it against. Think of $\hat{t}$ as β_1 but in the "t-world" instead of the "real world". Its also basically a z-score.

Let $\alpha =$ significance level. Let $t_c = t_{(1-\frac{\alpha}{2}, n-2)}$ be the $(1 - \frac{\alpha}{2})$ quantile of the $t(n-2)$ distribution. For two-sided hypothesis test, that is

$$P(t(n-2) > t_c) = P(t(n-2) < -t_c) = \frac{\alpha}{2}$$

For two sided tests, we **reject the null hypothesis** when our t-statistic is "more extreme" than the t-value or when the p-value is less than our significance level.

$$|\hat{t}| > t_c \quad \longleftrightarrow \quad P(|t(n-2)| > |\hat{t}|) = p < \alpha$$

Interpretation

How do we make sense of p-value? By formal definition, p-value is the probability of drawing another sample and compute another estimate β_1 such that this new β_1 is at least as "extreme" as the one you actually computed/observed using your current sample, assuming the null is true. In other words: Assuming our null hypothesis is true, p is the probability of drawing a sample to give us our $\hat{b}_1$. p is $\hat{t}$ in the "probability world".

We reject the null hypothesis when $p < \alpha$. This is saying that we drew a sample that was so extreme, it has less than an α% chance of occurring if we assume the null hypothesis is true.

We **want** to prove ourselves wrong with hypothesis testing. If $p > \alpha$, we **fail to reject** the null hypothesis.

t_c is to $\hat{t}$ as α is to p. t_c and α are the threshold to reject the null hypothesis but in the "t-world" and "probability world" respectively. $\hat{t}$ and p is the regression's $\hat{b}_1$ but transformed to be interpretable in the "probability world" and "t-world" respectively. Mathematically, p is the probability that anything greater $\hat{t}$ were to occur in the $t(n-2)$ distribution (t-world). α is the probability that anything greater than t_c were to occur in the $t(n-2)$ distribution.

$$P(|t(n-2)| > |\hat{t}|) = p \qquad\qquad P(t(n-2) > t_c) = P(t(n-2) < -t_c) = \frac{\alpha}{2}$$

Confidence Interval

$$[\hat{b}_1 - t_c \cdot SE(\hat{b}_1)\ ,\ \hat{b}_1 + t_c \cdot SE(\hat{b}_1)]$$

where $t_c = t_{(1-\frac{\alpha}{2}, n-2)}$. If β_1 is outside of the confidence interval, we would reject the null hypothesis.

Null Hypothesis	Alternative Hypothesis	Critical Value t_c	Rejection Rule
$H_0 : \beta = c$	$H_1 : \beta \neq c$	$t_c(1-\alpha/2, m)$	$\hat{t} < -t_c$ or $\hat{t} > t_c$
$H_0 : \beta = c$	$H_1 : \beta > c$	$t_c(1-\alpha, m)$	$\hat{t} > t_c$
$H_0 : \beta = c$	$H_1 : \beta < c$	$t_c(1-\alpha, m)$	$\hat{t} < -t_c$
Test statistic		confidence level	
$\hat{t} = \frac{\hat{b}-c}{\widehat{se}(b)}$		α	

Hypothesis Testing Using p-Values

Null Hypothesis	Alternative Hypothesis	Which Tail?	p-value Calculation	Rejection Rule
$H_0 : \beta = c$	$H_1 : \beta \neq c$	Two-tail test	$p = P(\lvert t(d.f)\rvert > \lvert\hat{t}\rvert)$	$p < \alpha$
$H_0 : \beta = c$	$H_1 : \beta > c$	Right-tail test	$p = P(t(d.f) > \hat{t})$	$p < \alpha$
$H_0 : \beta = c$	$H_1 : \beta < c$	Left-tail test	$p = P(t(d.f) < \hat{t})$	$p < \alpha$
Test statistic depends on c $\hat{t} = \frac{\hat{b}-c}{\hat{S}e(b)}$		Depends on H_1 $(\neq, >, <)$	$d.f. = N - 2$	Depends on p and α