Limity

Obliczyć granice szeregu o wyrazie ogólnym

2.41

$$u_n=\sqrt{n+2}-\sqrt{n}\ \lim_{n o\infty}u_n=\lim_{n o\infty}rac{n+2-n}{\sqrt{n+2}+\sqrt{n}}=\lim_{n o\infty}rac{2}{\sqrt{n}\left(\sqrt{1+rac{2}{n}}+1
ight)}=0$$

2.45

$$egin{aligned} \lim_{n o\infty} 3n - \sqrt{9n^2 + 6n - 15} &= \lim_{n o\infty} rac{9n^2 - 9n^2 - 6n + 15}{3n + \sqrt{9n^2 + 6n - 15}} &= \\ &= \lim_{n o\infty} rac{n(-6 + rac{15}{n})}{n\left(3 + \sqrt{9 + rac{6}{n} - rac{15}{n^2}}
ight)} &= rac{-6}{6} &= -1 \end{aligned}$$

2.46

$$\lim_{n o\infty} \sqrt[3]{n^3+4n^2}-n=\lim_{n o\infty} rac{n^3+4n^2-n^3}{\left(\sqrt[3]{n^3+4n^2}
ight)^2+n\sqrt[3]{n^3+4n^2}+n^2}= \ \lim_{n o\infty} rac{4n^2}{n^2\left[\left(1+rac{4}{n}
ight)^{2/3}+\sqrt{1+rac{4}{n}}+1
ight]}=rac{4}{3}$$

2.47

$$\begin{split} &\lim_{n\to\infty} n\sqrt[3]{2} - \sqrt[3]{2n^3 + 5n^2 - 7} = \lim_{n\to\infty} \frac{2n^3 - 2n^3 - 5n^2 + 7}{2^{2/3}n^2 + n\sqrt[3]{2}\sqrt[3]{2n^3 + 5n^2 - 7} + (2n^3 + 5n^2 - 7)^{2/3}} = \\ &= \lim_{n\to\infty} \frac{-5n^2 + 7}{n^2 \left[2^{2/3} + \sqrt[3]{2}\sqrt[3]{2 + \frac{5}{n}} - \frac{7}{n^3} + \left(2 + \frac{5}{n} - \frac{7}{n^3}\right)^{2/3}\right]} = -\frac{5}{\sqrt[3]{4} + \sqrt[3]{4}} = -\frac{5}{\sqrt[3]{4}} \end{split}$$

2.48

$$\lim_{n \to \infty} \frac{4^{n-1} - 5}{2^{2n} - 7} = \lim_{n \to \infty} \frac{2^{2n-2} - 5}{2^{2n-2} \cdot 2^2 - 7} = \frac{1}{4}$$

2.50

$$\lim_{n o \infty} rac{3 \cdot 2^{2n+2} - 10}{5 \cdot 4^{n-1} + 3} = \lim_{n o \infty} rac{3 \cdot 2^{2n+2} - 10}{5 \cdot 2^{2n-2} + 3} = rac{48}{5}$$

2.52

$$\lim_{n o\infty}rac{2^{n+1}-3^{n+2}}{3^{n+2}}=\lim_{n o\infty}rac{\left(rac{2}{3}
ight)^{n+1}-3}{3}=\lim_{n o\infty}\left(rac{2}{3}
ight)^{n}-rac{3}{3}=-1$$

2.53

$$\lim_{n o \infty} \left(rac{3}{2}
ight)^n rac{2^{n+1}-1}{3^{n+1}-1} = \lim_{n o \infty} rac{3^n \cdot 2^{n+1}-3^n}{3^{n+1} \cdot 2^n - 2^n} = \lim_{n o \infty} rac{\left(2 - rac{1}{2^n}
ight)}{3 - rac{1}{3^n}} = rac{2}{3}$$

2.54

$$\lim_{n o \infty} u_n = \lim_{n o \infty} \sqrt[n]{3^n + 2^n}$$
 $a_n = \sqrt[n]{3^n}$ $b_n = \sqrt[n]{2 \cdot 3^n}$ $\lim_{n o \infty} a_n = 3$ $\lim_{n o \infty} b_n = 3$ $a_n < u_n < b_n$ na podstawie tw. o trzech ciagach $\lim_{n o \infty} u_n = 3$

2.55

$$egin{aligned} u_n &= \sqrt[n]{10^n + 9^n + 8^n} \ a_n &= \sqrt[n]{10^n} \ b_n &= \sqrt[n]{3 \cdot 10^n} \ & \dots \ \lim_{n o \infty} u_n &= 10 \end{aligned}$$

2.56

$$egin{align} u_n &= \sqrt[n]{10^{100}} - \sqrt[n]{rac{1}{10^{100}}} = \sqrt[n]{10^{100}} - \sqrt[n]{10^{100}} \ a_n &= 0 \ b_n &= \sqrt[n]{10^{100}} \ a_n &< u_n < b_n \ \lim_{n o \infty} a_n &= \lim_{n o \infty} b_n = 0 \ \lim_{n o \infty} u_n &= 0 \ \end{array}$$

2.57

$$egin{align} u_n &= \sqrt[n]{\left(rac{2}{3}
ight)^n + \left(rac{3}{4}
ight)^n} \ a_n &= \sqrt[n]{\left(rac{3}{4}
ight)^n} \ b_n &= \sqrt[n]{2\cdot\left(rac{3}{4}
ight)^n} \ \lim_{n o\infty}a_n &= \lim_{n o\infty}b_n = rac{3}{4} \ \lim_{n o\infty}u_n &= rac{3}{4} \ \end{aligned}$$

2.58

$$\lim_{n o\infty}rac{1+2+\cdots+n}{n^2}=\lim_{n o\infty}rac{rac{n(n+1)}{2}}{n^2}=\lim_{n o\infty}rac{n^2+n}{2n^2}=rac{1}{2}$$