Limity

Obliczyć granice szeregu o wyrazie ogólnym

2.41

$$u_n = \sqrt{n+2} - \sqrt{n}$$
$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{n+2-n}{\sqrt{n+2}+\sqrt{n}} = \lim_{n\to\infty} \frac{2}{\sqrt{n}\left(\sqrt{1+\frac{2}{n}}+1\right)} = 0$$

2.45

$$\lim_{n\to\infty} 3n - \sqrt{9n^2+6n-15} = \lim_{n\to\infty} \frac{9n^2-9n^2-6n+15}{3n+\sqrt{9n^2+6n-15}} =$$
$$= \lim_{n\to\infty} \frac{n\left(-6+\frac{15}{n}\right)}{n\left(3+\sqrt{9+\frac{6}{n}-\frac{15}{n^2}}\right)} = \frac{-6}{6} = -1$$

2.46

$$\lim_{n\to\infty} \sqrt[3]{n^3+4n^2} - n = \lim_{n\to\infty} \frac{n^3+4n^2-n^3}{\left(\sqrt[3]{n^3+4n^2}\right)^2+n\sqrt[3]{n^3+4n^2}+n^2} =$$
$$\lim_{n\to\infty} \frac{4n^2}{n^2\left[\left(1+\frac{4}{n}\right)^{2/3}+\sqrt{1+\frac{4}{n}}+1\right]} = \frac{4}{3}$$

2.47

$$\lim_{n\to\infty} n\sqrt[3]{2} - \sqrt[3]{2n^3+5n^2-7} = \lim_{n\to\infty} \frac{2n^3-2n^3-5n^2+7}{2^{2/3}n^2+n\sqrt[3]{2}\sqrt[3]{2n^3+5n^2-7}+(2n^3+5n^2-7)^{2/3}} =$$
$$= \lim_{n\to\infty} \frac{-5n^2+7}{n^2\left[2^{2/3}+\sqrt[3]{2}\sqrt[3]{2+\frac{5}{n}-\frac{7}{n^3}}+\left(2+\frac{5}{n}-\frac{7}{n^3}\right)^{2/3}\right]} = -\frac{5}{\sqrt[3]{4}+\sqrt[3]{4}+\sqrt[3]{4}} = -\frac{5}{3\sqrt[3]{4}}$$

2.48

$$\lim_{n\to\infty} \frac{4^{n-1}-5}{2^{2n}-7} = \lim_{n\to\infty} \frac{2^{2n-2}-5}{2^{2n-2}\cdot 2^2-7} = \frac{1}{4}$$

2.50

$$\lim_{n\to\infty} \frac{3\cdot 2^{2n+2}-10}{5\cdot 4^{n-1}+3} = \lim_{n\to\infty} \frac{3\cdot 2^{2n+2}-10}{5\cdot 2^{2n-2}+3} = \frac{48}{5}$$

2.52

$$\lim_{n\to\infty} \frac{2^{n+1}-3^{n+2}}{3^{n+2}} = \lim_{n\to\infty} \frac{\left(\frac{2}{3}\right)^{n+1}-3}{3} = \lim_{n\to\infty} \left(\frac{2}{3}\right)^n - \frac{3}{3} = -1$$

2.53

$$\lim_{n\to\infty} \left(\frac{3}{2}\right)^n \frac{2^{n+1}-1}{3^{n+1}-1} = \lim_{n\to\infty} \frac{3^n \cdot 2^{n+1} - 3^n}{3^{n+1} \cdot 2^n - 2^n} = \lim_{n\to\infty} \frac{\left(2-\frac{1}{2^n}\right)}{3-\frac{1}{3^n}} = \frac{2}{3}$$

2.54

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \sqrt[n]{3^n + 2^n}$$
$$a_n = \sqrt[n]{3^n}$$
$$b_n = \sqrt[n]{2 \cdot 3^n}$$
$$\lim_{n\to\infty} a_n = 3$$
$$\lim_{n\to\infty} b_n = 3$$
$$a_n < u_n < b_n$$
na podstawie tw. o trzech ciagach $\lim_{n\to\infty} u_n = 3$

2.55

$$u_n = \sqrt[n]{10^n + 9^n + 8^n}$$
$$a_n = \sqrt[n]{10^n}$$
$$b_n = \sqrt[n]{3 \cdot 10^n}$$
$$\ldots$$
$$\lim_{n\to\infty} u_n = 10$$

2.56

$$u_n = \sqrt[n]{10^{100}} - \sqrt[n]{\frac{1}{10^{100}}} = \sqrt[n]{10^{100}} - \sqrt[-n]{10^{100}}$$
$$a_n = 0$$
$$b_n = \sqrt[n]{10^{100}}$$
$$a_n < u_n < b_n$$
$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$$
$$\lim_{n\to\infty} u_n = 0$$

2.57

$$u_n = \sqrt[n]{\left(\frac{2}{3}\right)^n + \left(\frac{3}{4}\right)^n}$$

$$a_n = \sqrt[n]{\left(\frac{3}{4}\right)^n}$$

$$b_n = \sqrt[n]{2 \cdot \left(\frac{3}{4}\right)^n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \frac{3}{4}$$

$$\lim_{n\to\infty} u_n = \frac{3}{4}$$

2.58

$$\lim_{n\to\infty} \frac{1+2+\cdots+n}{n^2} = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^2} = \lim_{n\to\infty} \frac{n^2+n}{2n^2} = \frac{1}{2}$$