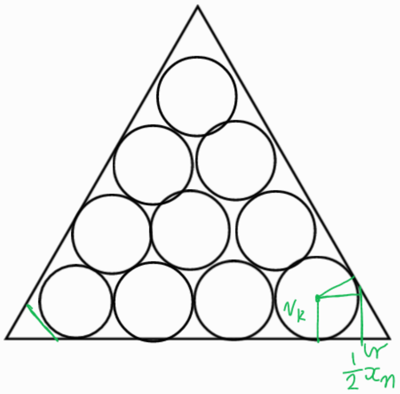



$$S = \frac{a^2\sqrt{3}}{4}$$

$$S_{K_n} = k_n \cdot \pi \cdot r_k^2$$

$$r_k = \frac{a - x_n}{2n}$$

$$S_{K_n} = \frac{n(n+1)}{2} \cdot \pi \cdot r_k^2 = \frac{\pi \cdot \cancel{n}(n+1)(a - x_n)^2}{8n^{\cancel{2}}}$$

$$\lim_{n\to\infty} \frac{S_{K_n}}{S} = \lim_{n\to\infty} \frac{\frac{\pi(n+1)(a - \overset{\to 0}{x_n})^2}{8n}}{\frac{a^2\sqrt{3}}{4}} = \lim_{n\to\infty} \frac{\pi(n+1)\cancel{a^2}}{2\cancel{a^2}n\sqrt{3}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{3}\pi n + \sqrt{3}\pi}{6n} = \frac{\pi\sqrt{3}}{6}$$

b) $$\lim_{n\to\infty} \frac{3n^5+5n-3}{n^2+2n} = \lim_{n\to\infty} \frac{\cancel{n^5}^{\,3}\left(3+\frac{5}{n^4}-\frac{3}{n^5}\right)}{\cancel{n^2}\left(1+\frac{2}{n}\right)} = \infty$$

c) $$\lim_{n\to\infty} \frac{3n^2+5n-3}{n^4+2n} = \lim_{n\to\infty} \frac{n^2\left(3+\frac{5}{n}-\frac{3}{n^2}\right)}{n^2\left(n^2+\frac{2}{n}\right)} = 0$$

d) $$\lim_{n\to\infty} \frac{4n^3+n+3}{5n^3+4n^2} = \lim_{n\to\infty} \frac{\cancel{n^3}\left(4+\frac{1}{n^2}+\frac{3}{n^3}\right)}{\cancel{n^3}\left(5+\frac{4}{n}\right)} = \frac{4}{5}$$

g) $$\lim_{n\to\infty} \frac{(n+5)^{1000}}{n^{1000}+2n} = \lim_{n\to\infty} \frac{\cancel{n^{1000}}\left(1+\frac{5}{n}\right)^{1000}}{\cancel{n^{1000}}\left(1+\frac{2}{n^{999}}\right)} = 1$$

h) $$\lim_{n\to\infty} \frac{(n+5)^{1000}}{n^{1001}+2n} = \lim_{n\to\infty} \frac{\cancel{n^{1000}}\left(1+\frac{5}{n}\right)^{1000}}{\cancel{n^{1000}}\left(n+\frac{2}{n^{1000}}\right)} = 0$$

b) $\lim\limits_{n\to\infty} \sqrt{2n^2+n} - n = \lim\limits_{n\to\infty} \dfrac{2n^2+n-n^2}{\sqrt{n^2+n}+n} = \lim\limits_{n\to\infty} \dfrac{n^2+n}{n\sqrt{1+\frac{1}{n}}+n} = \infty$

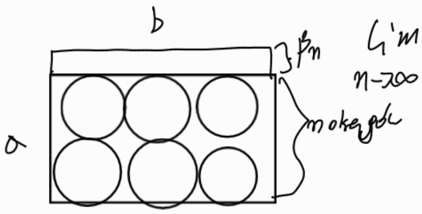
e) $\lim\limits_{n\to\infty} \left(\sqrt{n+4} - \sqrt{n}\right) = \lim\limits_{n\to\infty} \dfrac{n+4-n}{\sqrt{n+4}+\sqrt{n}} = \lim\limits_{n\to\infty} \dfrac{4}{\sqrt{n}\left(\sqrt{1+\frac{4}{n}}+\sqrt{1}\right)} = 0$

f) $\lim\limits_{n\to\infty} \left(\sqrt{2n+5} - \sqrt{n+3}\right) = \lim\limits_{n\to\infty} \dfrac{2n+5-n-3}{\sqrt{2n+5}+\sqrt{n+3}} = \lim\limits_{n\to\infty} \dfrac{n+2}{\sqrt{n}\left(\sqrt{2+\frac{5}{n}}+\sqrt{1+\frac{3}{n}}\right)} = \infty$

a) $\lim\limits_{n\to\infty} \dfrac{4n^2+9n-3-4n^2}{\sqrt{4n^2+9n-3}+4n} = \dfrac{\cancel{n}\left(9-\frac{3}{n}\right)}{\cancel{n}\left(\sqrt{4+\frac{9}{n}-\frac{3}{n^2}}+4\right)} = \dfrac{9}{4}$

d) $\lim\limits_{n\to\infty} \dfrac{n^2+7n+5-n^2-3n-2}{\sqrt{n^2+7n+5}+\sqrt{n^2+3n+2}} = \lim\limits_{n\to\infty} \dfrac{\cancel{n}\left(4+\frac{3}{n}\right)}{\cancel{n}\left(\sqrt{1+\frac{7}{n}+\frac{5}{n^2}}+\sqrt{1+\frac{3}{n}+\frac{2}{n^2}}\right)} = 2$

2.93



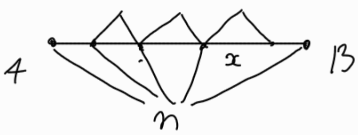
$\lim\limits_{n\to\infty} \dfrac{S_{k_n}}{S} =$

$r_k = \dfrac{a}{2n} \Rightarrow m = \dfrac{(b-\beta n)}{\frac{a}{n}} = \dfrac{n(b-\beta n)}{a}$

$S_{k_n} = k_n \cdot \pi r_k^2 = n \cdot m \cdot \pi r_k^2 = \pi \dfrac{\cancel{n^2}(b-\beta n)}{\cancel{a}} \cdot \dfrac{a^{\cancel{2}}}{4\cancel{n^2}} = \dfrac{\pi a(b-\beta n)}{4}$

$\lim\limits_{n\to\infty} \dfrac{S_{k_n}}{S} = \lim\limits_{n\to\infty} \dfrac{\pi a(b-\beta n \to 0)}{4ab} = \dfrac{\pi}{4}$

2.94



$S_n = (n-2)\dfrac{x^2\sqrt{3}}{4} = (n-2)\dfrac{\left(\frac{AB}{n}\right)^2\sqrt{3}}{4} =$

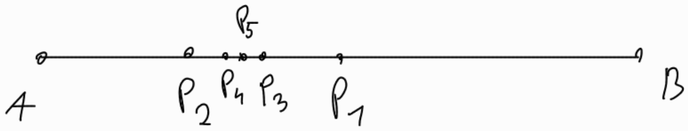
$= (n-2)\dfrac{AB^2\sqrt{3}}{4n^2}$

$\lim\limits_{n\to\infty} S_n = 0$

$P_n = (n-2)\cdot\dfrac{3\sqrt{3}\cdot x}{2} = (n-2)\dfrac{3\sqrt{3}}{2}\cdot\dfrac{AB}{n}$

$\lim\limits_{n\to\infty} P_n = 3\cdot\dfrac{\cancel{n}\left(AB-\frac{2AB}{n}\to 0\right)}{\cancel{n}} = AB\cdot 3$

2.95




$$P_1 = A + \frac{AB}{2} = \frac{AB}{2}$$

$$P_2 = P_1 - \frac{AP_1}{2} = \frac{AB}{2} - \frac{AB}{4} = \frac{AB}{4}$$

$$P_3 = P_2 + \frac{P_1P_2}{2} = \frac{AB}{4} + \frac{AB}{8} = \frac{3}{8}AB$$

$$P_4 = P_3 - \frac{P_2P_3}{2} = \frac{3}{8}AB - \frac{1}{16}AB = \frac{5}{16}AB$$

$$P_5 = \frac{5}{16}d + \frac{1}{32}d = \frac{11}{32}d$$

$$P_6 = \frac{11}{32}d - \frac{1}{64}d = \frac{21}{64}d$$

$\vdots$

$$P_n = \begin{cases} a_1 - b_1 + a_2 - b_2 + \ldots + a_n - b_n & \text{dla } n \text{ parzystego} \\ a_1 - b_1 + a_2 - b_2 + \ldots + a_n & \text{dla } n \text{ nieparzystego} \end{cases}$$

$a_1 = \frac{1}{2}d \qquad q_a = \frac{1}{4}$

$b_1 = \frac{1}{4}d \qquad q_b = \frac{1}{4}$

$$AP_n = S_{a_n} - S_{b_n} = a_1 \cdot \frac{1-q_a^n}{1-q_a} - b_1 \cdot \frac{1-q_b^n}{1-q_b} = \frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}} \cdot \left(\frac{1}{2}d - \frac{1}{4}d\right) =$$

$$= \frac{4-\left(\frac{1}{4}\right)^{n-1}}{3} \cdot \frac{1}{4}d$$

$$\lim_{n\to\infty} AP_n = \frac{d}{3}$$

2.96

$$Q_t^{(n)} = \left(Q_0 + \frac{t}{n}\right)^n$$

$$\lim_{n\to\infty} Q_t^{(n)} = \lim_{n\to\infty}\left[\left(Q_0 + \frac{kt}{n}\right)^{\frac{n}{kt}}\right]^{kt} = e^{kt}$$