CS201A: Math for CS I/Discrete Mathematics

Midsem exam

Max marks:70
Time:120 mins.

23-Sep-2017

1. *Answer all 4 questions. The paper has 3 pages.*
2. **Please start each answer to a question on a fresh page. And keep answers of parts of a question together.**
3. *Just writing a number/final value/figure will not get you full credit. You must justify your answers.*
4. *You can consult* **only your own handwritten notes**. *Nothing else is allowed. Keep any electronic gadgets in your bag and the bag on or near the stage.*

1. Let A, B, C, D be sets. Define:

$$UX = (A \cup B) \times (C \cup D)$$
$$XU = (A \times C) \cup (B \times D)$$

Here is a proof that claims $XU = UX$.

$$\begin{aligned}
(x,y) \in XU &\iff (x,y) \in (A \times C) \cup (B \times D) && (1)\\
&\iff (x,y) \in ((A \times C) \text{ or } (B \times D)) && (2)\\
&\iff (x \in A \text{ and } y \in C) \text{ or } (x \in B \text{ and } y \in D) && (3)\\
&\iff (x \in A \text{ or } x \in B) \text{ and } (y \in C \text{ or } y \in D) && (4)\\
&\iff (x \in A \cup B) \text{ and } (y \in C \cup D) && (5)\\
&\iff (x,y) \in UX && (6)
\end{aligned}$$

(a) Give a counter example to show that the claim above is false.

(b) Indicate the line/lines in the proof that are erroneous and the actual error.

(c) What is the correct relation between UX and XU?

(d) Correct the proof given above to prove the claim made in (c). Do not rewrite the whole proof. Write only the changed lines.

[2,3,2,3=10]

2. (a) Let R be a binary relation on sets X and Y. We can define the inverse of R, written R^{-1}, as $y\, R^{-1}\, x$ holds iff $x\, R\, y$ holds, where $x \in X$ and $y \in Y$.
Fill the third column below with the weakest appropriate property such that each item is true:

No.	R^{-1} is	iff R is
1	Total	
2	an Injection	
3	a Surjection	
4	a Bijection	
5	a Function	

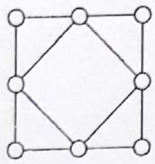
(b) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a total, injective function which is not a bijection. Give a concrete example of such a function.

(c) Let a be a positive integer. Argue geometrically using a figure that the cardinality of the interval $[0, a] \in \mathbb{R}$ is the same as the cardinality of $[0, 2a] \in \mathbb{R}$. Clearly indicate any results from elementary geometry that you use in your argument.

(d) Consider the sentence `"If this sentence is true then God exists"`. Analyse the paradoxical nature of the sentence.

[4×5=20]

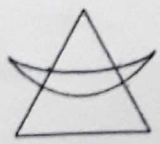
3. (a) Find the radius, diameter, girth and circumference of the graph in the figure below.



(b) How many *non-isomorphic* simple graphs with 4 nodes and 3 edges are possible? Draw them. (Note that a simple graph need not be connected.)

(c) The current midsem at IITK is over 7 days. How will you determine whether it is possible to schedule exams such that no student has more than two exams on the same day.

(d) Consider figure below which is a child's puzzle. The puzzle expects a child to start from any intersection point and trace each line or curved segment with a coloured pencil without raising the pencil or going over any line/curved segment more than once. Can a child solve the puzzle? Justify.



p → q
T T → T
T F F
F T T
F F T

(e) Characterize 2-critical and 3-critical graphs.

[5×4=20]

4. (a) Let $G = (V, E)$ be a simple graph with n nodes. Let $u, v \in V$ be non-adjacent nodes such that $deg(u) + deg(v) \geq n$. Construct graph $G' = (V, E')$ by adding (u, v) to E - that is $E' = E + (u, v)$. Argue that if G' has a Hamiltonian cycle then G has a Hamiltonian cycle.

(b) A bipartite graph $G = ((X, Y), E)$ is *degree constrained* when $\exists d > 0$ such that $deg(x) \geq d \geq deg(y)$ for all $x \in X$ and $y \in Y$.

i. Argue that: If $G = ((X, Y), E)$ is a degree constrained bi-partite graph then it has a matching that saturates X.

ii. A graph is *regular* when all nodes in the graph have the same degree. Supposing G is a regular, bi-partite graph then what can we say about a maximum matching of G? Justify your answer.

[10,(6,4)=20]