position and three for orientation. In the case of a frame camera, one bundle represents the entire image. In the case of a linear sensor, each line defines a new bundle, theoretically with its own six elements of exterior orientation. In practice, due to near functional dependency among these numerous parameters, they are usually estimated with a much smaller set of independent parameters.

For a bundle of rays, the three elements of position fix the location of the vertex or center of perspective. This is point L in Fig. 4-5. The coordinates of point L are often referred to as the camera station or exposure station coordinates, and are expressed as

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$$L = \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} \tag{4-16}$$

With only this point of the bundle established, the rays themselves can still take any orientation in space. Analytical geometry tells us that three angles, or three independent parameters, are sufficient to describe the orientation or attitude of this bundle in the object space coordinate system. This is equivalent to saying that three independent parameters are necessary to define the rotation matrix that relates the object space and image space systems. See Section 4.5.1 and Appendix A for a summary of the different choices of three parameters that can be used to construct the 3×3 rotation matrix. The exterior orientation defines the relationship between the object and image space coordinate systems by the following equation:

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = kM \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix} \tag{4-17}$$

in which $(x, y, -f)$ are the image space coordinates, k is a scale factor, M is a 3×3 matrix containing the rotation parameters, and (X, Y, Z) represent the object point.

The standard approach to constructing M is by using three sequential rotations: ω about the X-axis, ϕ about the once-rotated Y-axis, and κ about the twice-rotated Z-axis.

$$M_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}$$

$$M_\phi = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \tag{4-18a}$$

$$M_\kappa = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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The total rotation matrix is then constructed as

$$M = M_\kappa M_\phi M_\omega \tag{4-18b}$$

$$M = \begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix}$$

The selection of the order of rotations, that is, the primary, secondary, and tertiary axes, is arbitrary, but it will affect the attitudes at which singularities occur. More details on the construction of rotation matrices are given in Section A.4 of Appendix A.