521 M7410 –Adjustment and Analysis of Spatial Information

Fall Semester 2015

Homework No. 5

handed out Thursday, October 29, 2015
due Thursday, November 05, 2015, 09:10 **Name:** ____________________

LSQ for Nonlinear Models

1. Adjust the following problem with a non-linear I.O. approach, assuming that $\boldsymbol{p},\boldsymbol{q}$ are observables of equal precision ($\sigma_l = \pm 1.0\boldsymbol{cm}$), and $\boldsymbol{P},\boldsymbol{Q}$ are given constants.

Model: $\begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{bmatrix} = \sigma \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{bmatrix} + \begin{bmatrix} \boldsymbol{t}_p \\ \boldsymbol{t}_q \end{bmatrix}$

Observations (unit: meters):

P, Q	p, q	P, Q	p, q
10, 20	16.6791, 16.1734	-30, 98	8.4674, 103.4796
23, 71	47.6718, 58.7223	21, -10	15.7592, -15.7964
60, 45	72.4188, 20.8377	-23, -8	-24.3569, 2.3997

1) Find LSQ estimates of all parameters and observables. By the way, how do you decide their initial values?
2) Compute $\hat{\sigma}_0$, $\Sigma_{\Delta\Delta}$, Σ_{vv}, and $\Sigma_{\hat{l}\hat{l}}$.
3) Plot $(\Delta\boldsymbol{X})_i$, $(\boldsymbol{v}^T\boldsymbol{P}\boldsymbol{v})_i$, and $(\boldsymbol{v}^T\boldsymbol{P}\boldsymbol{v})_i / (\boldsymbol{v}^T\boldsymbol{P}\boldsymbol{v})_{i-1}$ as functions of iteration number $\boldsymbol{i}$ = 1~10. Discuss behaviors of these functions.

2. If the model in 1 becomes:

$$\begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ -\boldsymbol{b} & \boldsymbol{a} \end{bmatrix} \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{bmatrix} + \begin{bmatrix} \boldsymbol{t}_p \\ \boldsymbol{t}_q \end{bmatrix}$$

Show (with detailed mathematical steps) that the solutions (i.e., parameter and uncertainty estimates) can be derived from those obtained in 1.

Your (individual) final report should contain (use A4 papers):

- this page as the cover sheet
- source code(s) and outputs; do not forget to add your name and lots of comment cards to the source listing (% ....)
- input and output files from program [input/output values used and calculated], if any
- plots, including captions on axes, title, your name, LB#/HM#, course title, date (if any)
- derivation and description of formulas used, accompanied by figures where applicable
- evidence of computational accuracy
- discussion of results