Norms and Inner Products

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The goal of this note is to explain the relationship between inner products and norms. Along the way, we discuss some of the complexity functional analysis has over (finite-dimensional) linear algebra, and go over some concepts that break down in finite-dimensional spaces.

1 Definitions

Let V be a k-vector space (not necessarily finite-dimensional). A **norm** on V is a function $\|\cdot\| : V \to \mathbb{R}$ such that

1. (**Subadditivity**/**Triangle Inequality**):
$$\|u + v\| \leq \|u\| + \|v\|, \text{ for all } u, v \in V.$$
2. (**Absolute Homogeneity**):
$$\|\lambda v\| = |\lambda| \|v\|, \text{ for all } \lambda \in k, v \in V.$$
3. (**Positive-Definiteness**): For all $v \in V$, if
$$\|v\| = 0,$$
then $v = 0$.

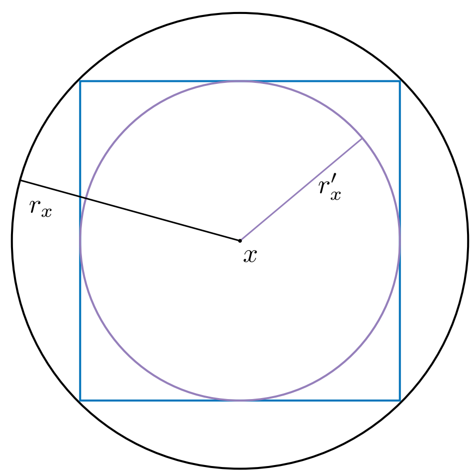

Figure 1: Transition functions determine how sections translate on a G-bundle.