P = Prevalance

Traditional pooling

S = Pool size

chance of no positives in pool of size S

$(1-P)^S$

chance of retesting

$1 - (1-P)^S$

retests per pool

$S \cdot (1 - (1-P)^S)$

samples per pool

S

retest per sample

$1 - (1-P)^S$

Matrix pooling

F = Matrix size # 4x4, F=4

really it's pool size

Unequivocal results / configurations impossible when positive samples in matrix > F

All cases → Possibly equivocal cases

$$\sum_{i=2}^{F^2} (1-p)^{F^2-i} \cdot p^i \cdot {}_{F^2}C_i$$

→ equivocal cases

$$\cdot \frac{{}_{F^2}C_i - 2F \cdot {}_{F}C_i}{{}_{F^2}C_i}$$

I need average retests for any i

i = 2 is 4

i = 3 is $\frac{4x + 6y + 9z}{{}_{F^2}C_i - 2F \cdot {}_{F}C_i}$

${}_{F}C_i$ ways to arrange i positives in one pool

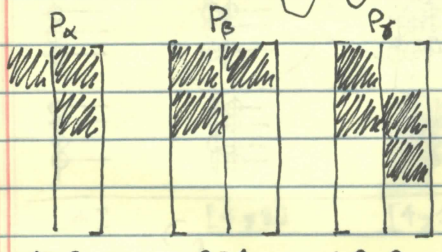
× 2 for the reflection equivalent probability in a square

$F \cdot {}_{F}C_i$ ~ ~ ~ ~ ~ with F pools

# of arrangements gives probability

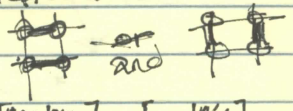
# of retests given by intersections.

1, 2, 3, 4	1,	5,	9,	13
5, 6, 7, 8	2,	6,	10,	14
9, 10, 11, 12	3,	7,	11,	15
13, 14, 15, 16	4,	8,	12,	16

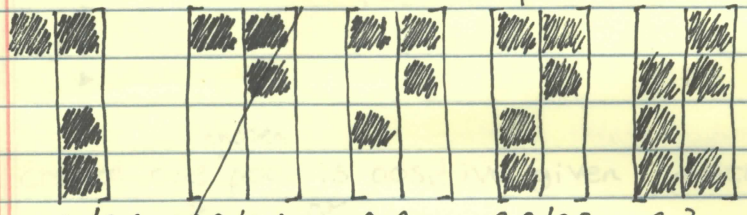


4 retests

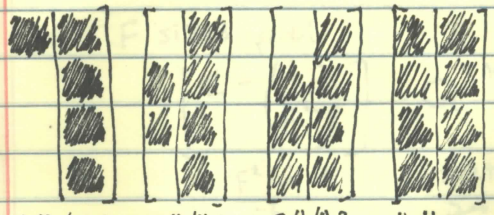
equivalency can be represented



1, 2 2, 1 2, 2



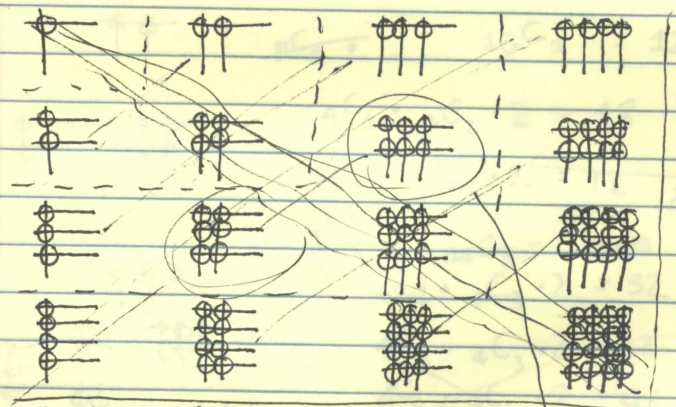
1, 3 / 3, 1 1, 2 / 2, 1 2, 2 3, 2 / 2, 3 3, 3



1, 4 / 4, 1 2, 4 / 4, 2 3, 4 / 4, 3 4, 4

2, 2
2, 3 / 3, 2
3, 3

F · FC_i



5 ${}_5C_2$

${}_5C_3$

${}_FC_2 * {}_FC_3 * 2$

chance one chosen pool is positive given i positives

~~p^i~~ ~~p^F~~

F^2 samples i positive samples

F sized pool

$$\left[1 - \left(\frac{i}{F^2}\right)\right] = Q^{\text{negative}}$$

$\frac{F^2 - i}{F^2}$ "

${}_{F^2}C_i =$

$\frac{12}{34} = Q \cdot (1-Q) \cdot Q \cdot (1-Q)$

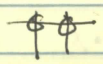
~~$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$~~

${}_4C_2 = 6$ $\frac{2}{4} \cdot \frac{2}{4}$ ~~$\frac{4}{16}$~~ $\frac{2}{4} \cdot \frac{2}{4} \cdot \frac{2}{4} \cdot \frac{2}{4} = \frac{1}{16}$

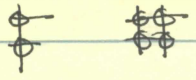
Should be $\frac{1}{6}$

F = 4

i = 2



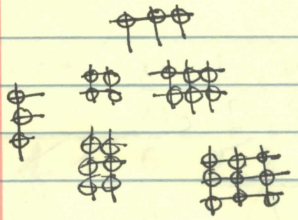
~~$_4C_1$~~ . $_{16}C_2 = 120$



$_4C_1 \cdot {}_4C_2 \cdot 2 = 48$ $\frac{48}{120} \cdot 0$ +

~~$_4C_2 \cdot {}_4C_2 \cdot 2 = 72$~~ $\frac{72}{120} \cdot 4$

$_{16}C_3 = 560$



$_4C_1 \cdot {}_4C_3 \cdot 2 = 32$

~~$_4C_3 \cdot {}_4C_3 \cdot 2 = 32$~~

~~$_4C_2 \cdot {}_4C_3 \cdot 2 = 48$~~

32

$x \cdot y \neq i$ $x \cdot y = i$ unequivocal

$x \cdot y = i$ retests $x \cdot y > i$ equivocal

$x \cdot y < i$ error

~~chance~~ distribution of configs * retest number

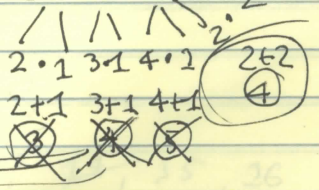
for i, $x \cdot y : [i, i^2]$

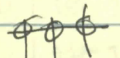
$x+y : [i+1, 2i]$

valid x and y is a factorization problem?

$i = 2$ $x \cdot y : [2, 4]$ $\{2, 3, 4\}$

$x+y : [3, 4]$





Things are easier if you don't use (1)