Mathematical modeling for CVRP with twist constraint

For this problem an event is a location on the plane encoded by their coordinates (x,y). We have two sets of events; let's call them the set of deliveries and the set of pickups respectively. The goal of the model is to find a solution that:

- Give as outputs reasonably short route.
- Each event has a capacity. The vehicle performing the delivery also has a physical capacity limit and the capacity of events in the vehicle at any given time may not exceed this physical limit.
- A route is an ordering of events starting and ending with the "depot" event located at the origin (0,0).
- The length of a route is calculated as the sum of distances between each subsequent event. A delivery is loaded onto the vehicle at the first depot and dropped off at its specified location. A pickup, conversely, is loaded at its specified location and unloaded upon returning to the depot.
- The twist: the route should visit as many delivery events as possible but exactly one pickup event.

We consider the following inputs:

- C: The vehicle capacity.
- E : The set of event.
- 0 is the depot index.
- d_{ij} : The distance between the stop i and j $(i, j \in E)$.
- $p_i = \begin{cases} 1 \ if\ the\ site\ i\ is\ a\ pickup. \\ 0\ otherwise. \end{cases}$
- c_i event capacity $(i \in E)$.
- R routes set with the size equals to $\sum_{i \in E} p_i$.

To model this problem we need to define the following decision variables:

- M: is a big number.
- D_i^r : is the accumulated amount at site i by r $(r \in R)$.
- $x_{ij}^r = \begin{cases} 1 \ if\ the\ arc\ (i,j)\ is\ traveled\ by\ driver\ r. \\ 0\ otherwise. \end{cases}$

The objective functions and the constraints are then expressed as follow:

$$Minimize \sum_{r \in R} \sum_{i,j \in E \cup \{0\}} d_{ij} x_{ij}^r$$

$$\forall i \in S \qquad \sum_{r \in R} \sum_{j \in E \cup \{0\}} x_{ij}^r = 1 \qquad (1)$$

$$\forall r \in R, \forall i \in E \qquad \sum_{j \in E \cup \{0\}} x_{ij}^r = \sum_{j \in E \cup \{0\}} x_{ji}^r \qquad (2)$$

$$\forall r \in R, \forall i \in E \qquad D_i^r \leq C \qquad (3)$$

$$\forall r \in R, \forall i \in E \qquad 0 \leq D_i^r \qquad (4)$$

$$\forall r \in R, \forall i, j \in E (i \neq j) \qquad D_i^r - (1 - p_j) c_j + \left(1 + p_j - x_{ij}^r\right) M \geq D_j^r \qquad (5)$$

$$\forall r \in R, \forall i, j \in E (i \neq j) \qquad D_i^r + p_j c_j \leq D_j^r + \left(2 - p_j - x_{ij}^r\right) M \qquad (6)$$

The constraints (1) and (2) ensure that every site is visited one time. The vehicle capacity is respected thanks to inequalities (3) and (4) at every stop. (5) and (6) ensure the value of D_j^r in case of *j* event is a Delivery or Pickup respectively.