Fleet Volume Assignment

What is the optimal assignment of parcels across the fleet portfolio subject to the datasets provided?

The Fleet Volume Assignment (FVA) problem is the problem of assigning a static portfolio of partner fleets to a fixed number of orders per delivery area, usually postal codes. On the one hand, the parcel to be delivered (Demand.csv file) in each area of a city, usually postal codes, is assumed to be given. On the other hand, a portfolio of partner fleets is assumed to be given which is static. Static means that no new fleets can be introduced. How should the fleets be assigned best?

Cost

First of all cost depend on the Cost per Parcel (CPP agreed on with each fleet. This cost is fixed until the next negotiation with the fleet during which we and the fleet agree on another, usually lower, CPP. Other than the CPP cost also depends on a fleet's productivity (ParcelsPerH.csv file). If for a given fleet with a fixed CPP the productivity of the fleet increases (more parcels per hour) this means that the fleet's margin increases. In this case, we renegotiate parcel rates with the fleet settling at a lower CPP.

Service

High service quality is constituted by a high Delivery Success Rate (DSR.csv file) and low percentage of delayed orders (Delayed.csv file). Additionally to service quality, a high share of green deliveries constitutes good service.

Risk

Risk comes in the three dimensions Fleet Diversification, Fleet Volume Constraints and Fleet Area Constraints. Fleet Diversification means that no fleet should receive more volume than a certain percentage of the total volume, usually 25%, to prevent dependency on individual fleets. Regarding Fleet Volume Constraints, each fleet has a certain capacity to deliver orders which is limited by the number of vehicles each fleet has (Fleets.csv file). Fleet Area Constraints emerge with fleets that do not serve certain sub-areas in the delivery area (FleetAreaConstraints.csv file). For example some fleets only serve the north of a city while other only serve the West.

Mathematical Modeling

The model considers the following inputs:

- F: Fleets set.
- P: PostCode set.
- $y_{ij} = \begin{cases} 1 \; if \; fleet \; j \in F \; is \; able \; to \; cover \; the \; area \; i \in P \\ 0 \; otherwise. \end{cases}$ *:* Fleet area constraint.
- Q_{ij}: The service quality of fleet $j \in F$ in area $i \in P$. This value (Saved in Service Qualtiy indicators.csv file) is calculated by taking into consideration the fleet area constraint ($Q_{ij} = 0$ if $y_{ij} = 0$), the delay rate D_{ij}, the delivery success rate DSR_{ij}, and the green rate G_{ij} considered as green capacity percentage from the max contractually volume.
$$Q_{ij} = y_{ij} \frac{DSR_{ij}\left(1 - D_{ij}\right)}{(1 - G_{ij})}$$
- D_i: The demanded amount by area i ($i \in P$) in parcel.
- C_j: The cost per parcel of the fleet j ($j \in F$).

- V_{max}^{j} : The max volume constraint of the fleet j. This value (Saved in Service Qualtiy indicators.csv file) is considered as a min of three values: 25% of the total volume, the fleet max capacity and the volume contractually agreed.
- V_{min}^{j} : The min volume constraint of the fleet j.
- y_{ij} : Fleet area constraint
- P_{ij} : The productivity rate of fleet j in area i.
- $x_{jk}^{i} = \begin{cases} 1 \; if Q_{ij} \geq Q_{ik}. \\ 0 \; otherwise. \end{cases}$

Also we need to define the following decision variable:

- N_{ij} : The delivered parcels in area i by fleet j.

The objective functions and the constraints are then expressed as follow:

$$Minimize \sum_{i \in P} \sum_{j \in F} C_j N_{ij} \qquad (*)$$

$$Maximize \sum_{i \in P} \sum_{j \in F} P_{ij} / N_{ij} \qquad (**)$$

$$\forall i \in P \qquad \sum_{j \in F} N_{ij} = D_i \qquad (1)$$

$$\forall j \in F \qquad \sum_{i \in P} N_{ij} \leq V_{max}^{j} \qquad (2)$$

$$\forall j \in F \qquad \sum_{i \in P} N_{ij} \geq V_{min}^{j} \qquad (3)$$

$$\forall i \in P, \forall j, k \in F \qquad N_{ik} \leq N_{ij} + \left(1 - x_{jk}^{i}\right) D_i \qquad (4)$$

$$\forall i \in P, \forall j \in F \qquad N_{ij} \leq y_{ij} D_i \qquad (5)$$

The first objective function (*) minimizes the total cost of all assignments, and the second function aims to maximize the productivity of all fleets in order to increase the renegotiation chance. If the company's negotiator feels that the negotiations may be easier/or harder with a fleet in comparison with another, which is a criteria hard to be modeled, then the negotiation focus on a subset $F' \subset F$ instread of the whole fleets. As a second method: we can give to every fleet j a weight $\alpha_j < 1$ with $\sum_{j \in F} \alpha_j = 1$. (**) will be then expressed as follow:

$$Maximize \sum_{i \in P} \sum_{j \in F} \alpha_j P_{ij} / N_{ij}$$

The constraints (2) and (3) ensure that the volume assignments respect the min and max constraint, and the constraint (1) guarantees the demands covertures. While (5) establish the fleet area constraint on the decision variables, the constraint (4) favors the fleets with high service quality in volumes assignment.

Heuristics approach

Assignment problems such as this one or for instance QAP (https://en.wikipedia.org/wiki/Quadratic_assignment_problem) are classified as NP-hard problem, so there is no known algorithm for solving this problem in polynomial time, and even small instances may require long time. Therefore researchers use all the time heuristics and meta-heuristics (https://en.wikipedia.org/wiki/Metaheuristic) algorithms to solve NP-hards. As instance genetic algorithm (https://en.wikipedia.org/wiki/Genetic_algorithm) is the most famous meta-heuristic.

Briefly, Genetic algorithms (GA) are adaptive methods inspired from the natural evolution of biological organisms. An initial population of individuals (chromosomes) is sorted according to their quality (fitness) and evolves through generations until satisfactory criteria of quality, a maximum number of iterations or time limits are reached. The chromosome encoding is proposed for our problem is as an array with a length equals to the fleets number multiplied by the areas number, the array contains in each case (gene) the area index assigned to a fleet or not as described in the following example:

Fleets	f1	f2	f3	f4	f5	f6	f7	f8	f9
Area1	0	1	0	1	0	0	0	0	0
Area2	0	1	1	0	0	0	0	0	0
…									
Area72	0	1	0	0	0	0	0	0	0

New array is formed from the chromosome but this time it goes to details. It contains the volume assigned to every fleet for every area as described as follow:

Fleets	f1	f2	f3	f4	f5	f6	f7	f8	f9
Area1	0	1289	0	1503	0	0	0	0	0
Area2	0	741	751	0	0	0	0	0	0
…									
Area72	0	31	0	0	0	0	0	0	0

The volume assignments respects the problem constraints especially the 4th one: to calculate N_{ij} we give to every fleet a weight that reflect is service quality equals to: $\pi_{ij} Q_{ij} \big/ \sum_{k \in F} \pi_{ik} Q_{ik}$ where π is the chromosome notation. Then N_{ij} is calculated as follow:

$$N_{ij} = D_i \frac{\pi_{ij} Q_{ij}}{\sum_{k \in F} \pi_{ik} Q_{ik}}$$