Mathematical modeling of VRP with stationary charging

We consider the following inputs:

- R: Set of routes (or vehicles).
- C: Vehicle capacity.
- W: Vehicle weight.
- S : Set of stops
- S': Set of charging points including the depot with index 0.
- d_{ij}: The distance between the stop i and j ($i, j \in S \cup S'$).
- V: average speed of vehicle (= 60 KM/h)
- t_{ij} travelling time of arc (i, j) is calculated in term of d_{ij} , average moving speed and speed deviation factor $T_\alpha(t)$ with the following expression:

$$t_{ij} = {d_{ij}}\Big/{V * (1 - T_\alpha(t))}$$

$T_\alpha(t)$ is changing in term of days of the week and time range of the day as described in the rush hours table.

- I_{bat}: Battery capacity.
- $I_{min} = \alpha\, I_{bat}$ min capacity should a battery reach. α mostly takes 20%
- CSR: Energy consumption rate in arc (i, j) depend on distance and the loaded and empty weight of the vehicle.
- a_i, b_i: are time windows of the stop i. If i is a charging station $a_i = a_0$, $b_i = b_0$
- c_i : Demand of customer (or stop) i $(i \in S \cup S')$. A charging station has no demand, so $c_i = 0$
- A_i : Service time at stop i $(i \in S \cup S')$ per unit. Time to reload the battery is the service time of a charging station (1.5 min/KWH).
- C_f: fixed cost = driver cost per day.
- C_v: variable cost related to the cost of electricity charging in stations other than the depot.
- C_0: variable cost related to the cost of electricity in depot, it is cheaper that stations.
- M: is a large enough number.

To model this problem, we need to define the following decision variables:

- D_i^r : is the accumulated weight of a vehicle at stop i by r ($r \in R$).
- w_i^r: is the timestamp at the stop i by r ($r \in R$).
- w^r: is the timestamp at the end of rout r ($r \in R$).
- I_i^r: State of charging at stop i.
- $x_{ij}^r = \begin{cases} 1\ if\ the\ arc\ (i, j)\ is\ traveled\ by\ route\ r. \\ 0\ otherwise. \end{cases}$
- $y^r = \begin{cases} 1\ if\ route\ (or\ vehicle)\ is\ used. \\ 0\ otherwise \end{cases}$

$$Minimize\ C_f \sum_{r \in R} y^r + C_v \sum_{\substack{i \in S' \\ i \neq 0}} \sum_{r \in R} (I_{bat} - I_i^r) + C_0 \sum_{r \in R} (I_{bat} - I_0^r)$$

Minimization of total cost: driver's payment + reloading the batteries

$\forall i \in S$

$$\sum_{j \in S \cup S'} \sum_{r \in R} x_{ji}^r = 1 \qquad (1)$$

Every stop is visited on time (charging stations are excluded here)

$\forall i \in S \cup S', \forall r \in R$ $$\sum_{j \in S \cup S'} x_{ij}^r = \sum_{j \in S \cup S'} x_{ji}^r \quad (2)$$

$\forall r \in R$ $$y^r = \sum_{\substack{j \in S \cup S' \\ j \neq 0}} x_{j0}^r \quad (3)$$

$\forall i \in S \cup S', \forall r \in R$ $$D_i^r \leq C + W \quad (4)$$

$\forall i \in S \cup S', \forall r \in R$ $$D_i^r \geq c_i + W \quad (5)$$

Vehicle capacity constraint.

$\forall r \in R, \forall i, j \in S \cup S', j \neq 0$ $$D_i^r - D_j^r \geq c_i + \left(1 - x_{ij}^r\right)(C + W) \quad (6)$$

Accumulated weight of vehicle changed according to the loaded weight at every stop.

$\forall r \in R, \forall i \in S, j \in S \cup S', j \neq 0$ $$w_i^r + A_i c_i + t_{ij} \leq w_j^r + \left(1 - x_{ij}^r\right)M \quad (7)$$

$\forall r \in R, \forall i \in S', j \in S \cup S', j \neq 0$ $$w_i^r + A_i(I_{bat} - I_i^r) + t_{ij} \leq w_j^r + \left(1 - x_{ij}^r\right)M \quad (8)$$

$\forall r \in R, \forall i \in S \cup S'$ $$w_i^r \geq a_i \quad (9)$$

$\forall r \in R, \forall i \in S \cup S'$ $$w_i^r \leq b_i \quad (10)$$

Time windows constraints

$\forall r \in R$ $$w^r \leq b_0 \quad (11)$$

$\forall r \in R, \forall i \in S \cup S'$ $$w_i^r + A_i c_i + t_{i0} \leq w^r + (1 - x_{i0}^r)M \quad (12)$$

$\forall r \in R, \forall i \in S, j \in S \cup S', j \neq 0$ $$I_i^r - CSR\, D_i^r d_{ij} \geq I_j^r + \left(1 - x_{ij}^r\right)I_{bat} \quad (13)$$

Charging rate change according to the travelled distance, the vehicle weight, and either the arc (i, j) is travelled with wireless charging option or not. Also, if the stop i is a charging point it is assumed that the battery is fully charged.

$\forall r \in R, \forall i \in S', j \in S \cup S'$ $$I_{bat} - CSR\, D_i^r d_{ij} \geq I_j^r + \left(1 - x_{ij}^r\right)I_{bat} \quad (14)$$

$\forall i \in S, \forall r \in R$ $$I_i^r \geq I_{min} \quad (15)$$

State of charging rate must respect the safe rates in every customer

$\forall i \in S' i \neq 0, \forall r \in R$ $$I_i^r \geq 0 \quad (16)$$

State of charging are allowed to be below the safe rate in charging station

$\forall r \in R$ $$I_0^r = (1 - y^r)I_{bat} \quad (17)$$

$$\forall i \in S \cup S', \forall r \in R \qquad I_i^r \leq I_{bat} \qquad (18)$$

The constraints in the mathematical model for the VRP with stationary charging stations are as follows:

- Constraint (1) ensures every stop is visited on time, excluding charging stations.
- Constraint (2) guarantees the flow of vehicles, ensuring that if a vehicle arrives at a stop, it must also leave from there.
- Constraint (3) relates to the utilization of vehicles, ensuring that each route starts from the depot.
- Constraints (4) and (5) are vehicle capacity constraints, ensuring that the accumulated weight at each stop does not exceed the vehicle's capacity.
- Constraint (6) deals with the change in vehicle weight at each stop due to loading and unloading.
- Constraints (7) and (8) manage time continuity at stops and charging stations, respectively.
- Constraints (9) and (10) are time window constraints, ensuring service times fall within specified limits.
- Constraints (11) and (12) relate to the total time spent on a route, ensuring it does not exceed the depot's closing time.
- Constraints (13) and (14) manage the charging rate, taking into account travel distance, vehicle weight, and whether wireless charging is available.
- Constraint (15) maintains the state of charge at safe levels for customers, while Constraint (16) allows lower states of charge at charging stations.
- Constraint (17) ensures the vehicle starts with a full battery, and Constraint (18) ensures the battery state does not exceed its capacity.

The objective function in the mathematical model for the Vehicle Routing Problem (VRP) with stationary charging stations is designed to minimize the total cost. This cost includes several key components:

- **Driver's Wage**: This component represents the labor cost, calculated based on the time drivers spend on their routes.
- **Battery Charging Cost**: This part addresses the expenses related to recharging the electric vehicle batteries at the charging stations.