



Figure: set of three routes for 24 customers and 3 charging station (including the depot). Green arcs are the wireless charging paths

Mathematical model

It is assumed that there is a single depot, and a set of n stops clustered in two subsets: customers S and charging stations S' . In this paper we look for- heuristically with a memetic algorithm - the best routes to visit the stops, each customer is mandatorily visited. But the charging stations are visited only if it is necessary when the battery is not enough charged to complete the assigned route.

Our problem calls for satisfying the following constraints:

- Each customer is visited one time.
- The vehicle capacity is not exceeded.
- The battery capacity stills in the safe range.
- Each customer has a time window to be visited at.
- All the route starts and end in the depot. This last has also an associated time window equals to the day working window.
- The energy consumption depends on the initial weight of the vehicle plus the loaded amount on it.

We consider the following inputs:

- R : Set of routes.
- C : Vehicle capacity.
- W : Vehicle weight.
- S : Set of costumers.
- S' : Set of charging stations including the depot with index 0.
- d_{ij} : The distance between the stop i and j ($i, j \in S \cup S'$).
- t_{ij} travelling time of arc (i, j) is calculated in term of d_{ij} and average moving speed.
- I_{bat} : Battery capacity.
- $I_{low} = \alpha I_{bat}$ min capacity should a battery reach. α mostly takes 20%
- $I_{high} = \beta I_{bat}$ max capacity should a battery reach. β mostly takes 80%
- CR_{ij} : Wireless charging rate in arc (i, j) which depend on charging segments installed on it. If there is no charging infrastructure installed then $CR_{ij} = 0$.
- CSR : Energy consumption rate in arc (i, j) depend on distance and the loaded and empty weight of the vehicle.
- a, b : are the starting/ending working time for all drivers.
- a_i, b_i : are time windows of the stop i . If the i is a charging station, then $a_i = a, b_i = b$.
- T_c : Waiting time in charging stations.
- c_i : Demand of stop i ($i \in S \cup S'$). A charging station has no demand, so $c_i = 0$
- A_i : Service time at stop i ($i \in S \cup S'$). A charging station has a service time equals to the waiting time to charge the battery.
- M : is a big enough number.

To model this problem, we need to define the following decision variables:

- D_i^r : is the accumulated weight of a vehicle at stop i by r ($r \in R$).
- w_i^r : is the timestamp at the stop i by r ($r \in R$).
- E^r : is the timestamp at the end of route r ($r \in R$).
- I_i^r : Charged amount of the battery at stop i .
- $x_{ij}^r = \begin{cases} 1 & \text{if the arc } (i, j) \text{ is traveled by route } r. \\ 0 & \text{otherwise.} \end{cases}$

$$\text{Minimize } \sum_{i,j \in S \cup S'} \sum_{r \in R} d_{ij} x_{ij}^r$$

$$\forall i \in S \cup \{0\} \quad \sum_{j \in S \cup S'} \sum_{r \in R} x_{ji}^r = 1 \quad (1)$$

$$\forall i \in S \cup S', \forall r \in R \quad \sum_{j \in S \cup S'} x_{ij}^r = \sum_{j \in S \cup S'} x_{ji}^r \quad (2)$$

$$\forall i \in S \cup S', \forall r \in R \quad D_i^r \leq C + W \quad (3)$$

$$\forall i \in S \cup S', \forall r \in R \quad D_i^r \geq W \quad (4)$$

$$\forall r \in R, \forall i, j \in S \cup S' \quad D_i^r - c_j \leq D_j^r + (1 - x_{ij}^r)M \quad (5)$$

$$\forall r \in R, \forall i \in S', j \in S \cup S' \quad w_i^r + c_i + t_{ij} \leq w_j^r + (1 - x_{ij}^r)M \quad (6)$$

$$\forall r \in R, \forall i \in S \cup S' \quad w_i^r \geq a_i \quad (7)$$

$$\forall r \in R, \forall i \in S \cup S' \quad w_i^r \leq b_i \quad (8)$$

$$\forall r \in R, \forall i \in S', j \in S \cup S' \quad I_{high} + (CR_{ij} - CSR D_i^r) d_{ij} \leq I_j^r + (1 - x_{ij}^r)M \quad (9)$$

$$\forall r \in R, \forall i \in S, j \in S \cup S' \quad I_i^r + (CR_{ij} - CSR D_i^r) d_{ij} \leq I_j^r + (1 - x_{ij}^r)M \quad (10)$$

$$\forall i \in S \cup S', \forall r \in R \quad I_i^r \leq I_{high} \quad (11)$$

$$\forall i \in S \cup S', \forall r \in R \quad I_i^r \geq I_{low} \quad (12)$$

The objective function is the summary of the travelled distance during the working day. The constraints of the problem are presented as follow: (1) every stop is visited on time (charging stations are excluded here), (2) each stop must be visited on time at most in every route. Vehicle capacity constraint is respected thanks to (3), (4), and (5). And time window constraint is respected thanks to (6), (7), and (8). Charging rate change according to the travelled distance, the vehicle weight, and either the arc (i, j) is travelled with wireless charging option or not (10). Also, if the stop i is a charging point it is assumed that the

battery is charged until the I_{high} value (9). (11) and (12) ensure that the charged quantity in battery must respect the safe rates I_{low} and I_{high} .

References

Cordeau J-F, Desaulniers G, Desrosiers J, Solomon MM, “ VRP with time windows”, In: Toth P, Vigo D, editors. The vehicle routing problem. Philadelphia, PA: SIAM; p. 157–93. 2002.

Solomon VRP instances <https://www.sintef.no/projectweb/top/vrptw/solomon-benchmark/25-customers/>