Preventative measure: A mathematical model for forecasting the impact of vaccination programs of COVID-19 for the United Kingdom.

Introduction to Mathematics

Group Project Report

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Year of Study: 2022/23

Abstract

In this mathematical modeling project, we develop a mathematical model of the COVID-19 epidemic that can predict and evaluate the impact of vaccination programs in the United Kingdom (UK). To forecast the transmission rate, we used an ordinary differential equation-based dynamic SIR model. We introduced the unreported symptomatic infectious population as an addition to this model. To identify the number of unreported cases we utilize the parameterized model1. Our finds emphasize the importance and found evidence that the mass covid vaccination initiative helped the UK to reduce this virus transmission rate.

Objectives:

1) Develop a mathematical model to show preventative measures of vaccination programmes
2) The main features of this model
   a. Incorporation of asymptomatic (which is longer in COVID-19) and symptomatic infectiousness.
3) Reported daily case data will be used to parameterise the models
4) This model will forecast the epidemic with vaccination programs, social distancing measures and public health policies.

Model and Methodology

Ordinary differential equation-based dynamic model of an infectious disease is applied. This model is well suited to explain and understand basic concepts and dynamics for ideal cases (uniform homogeneous populations with homogeneous interaction dynamics)[1]. This type of models, for instance, does not consider that recovered individuals might infect others or could get infected again. In certain type of epidemics, dead individuals could infect living ones too.

In our model we have divided total population into four components.

1. *S(t)*- susceptible population at time *t*, who could potentially catch the disease
2. *I(t)*- infectious population at time *t* (asymptomatic) this would be people who currently have the disease and infecting others.
3. *R(t)*- symptomatic infectious population at time *t* (reported)
4. *U(t)*- symptomatic infectious population at time *t (*unreported*)*

Assumptions

Assumptions for this model: -

- To identify the transmissions rate[2]:
  - Susceptible become infected from asymptomatically infection and symptomatically infectious individuals such that $\tau(t)\, S(t)(I(t) + U(t))$
    - ($\tau(t)$ identify as a time dependent parameter)
    - *R(t)*- reported symptomatic symptomatic cases no longer contribute to transmitting the infection
- *I(t)* Asymptomatic individual's average period stay infectiousness of $1/v$. (People who are medically confirmed are typically isolated)
- Moreover, assuming that reported infections individuals are infected on average $1/\eta$; this assumption arises from the first order loss term in the equations (*U(t)* unreported symptomatic people)[3].
- *f* represents the fraction of population who become reported therefore reported symptomatic infectious at rate $v_1 = f\, v$ and the fraction who remain unreported 1 – *f* that fraction; therefore $v_2 = (1 - f)\, v$, where $v_1 + v_2 = v$; *v is new days*

Time units are days, and these assumptions arise from the first-order loss term in the equations. ✓

[1] Turkyilmazoglu, Mustafa Explicit formulae for the peak time of an epidemic from the SIR model. Physica D: Nonlinear Phenomena; Volume 422, August 2021, 132902

[2] Z. Liu, P. Magal, O. Seydi, G. Webb;Predicting the cumulative number of cases for the COVID-19 epidemic in China from early data; Mathematical Biosciences and Engineering doi: 10.3934/mbe.2020172

[3] Suli,Liua; Michael Y.Lib; Epidemic models with discrete state structures; Physica D: Nonlinear Phenomena Volume 422, August 2021, 132903

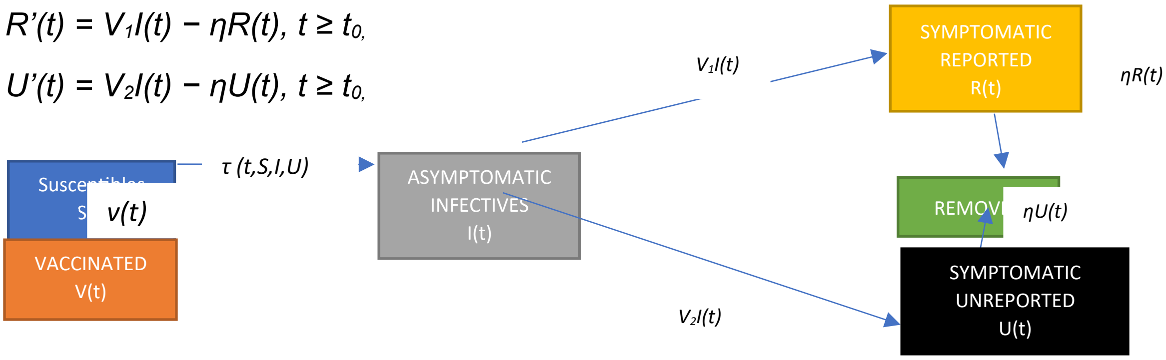
These are simplified assumptions which could be modified if better information is available.

$S'(t) = -\tau\,(t)S(t)(I(t) + U(t)) - v(t),\ t \geq t_0$[i]

$I'(t) = \tau\,(t)S(t)(I(t) + U(t)) - v(t),\ t \geq t_0$

$R'(t) = V_1 I(t) - \eta R(t),\ t \geq t_0,$

$U'(t) = V_2 I(t) - \eta U(t),\ t \geq t_0,$



Susceptible people become asymptomatically infectious at a certain rate and then they are lost at a rate certain fraction (*Greek New* v_1) $v_1 I(t)$ goes to the symptomatic reported class and remainder $v_2 = (1 - f)\ v$ *goes to* symptomatically unreported class. Assumption is both groups remain those classes for one before they move to the remove class. Furtherly, susceptible are removed as well by vaccination.

Collecting the data:

A major problem we found in working with models of Covid 19 is to use of data and the data we have is daily reported cases. Typically, we assume there are more unreported cases daily than reported cases. In the model, we simplified the data by calling a function *DR(t).* The daily transmission rate in the model can be obtained from the daily reported cases data[4]. Hence, we connect the daily reported cases in the model to the daily reported cases data[ii]:

Let,

The number of daily reported cases is *DR(t)*[5]

$$DR'(t) = V_1 I(t) - 1\ DR(t) \rightarrow I(t) = \frac{DR'(t) + DR(t)}{V1}$$

Since, the transmission rate in the equation:

$$I'(t) = \tau\,(t)S(t)(I(t) + U(t)) - vI(t)\ ;\ (\tau\,(t)S(t)(I(t) + U(t) = \textit{transmission rate})$$

So,

$$\tau\,(t)S(t)(I(t) + U(t)) = I'(t) + vI(t),$$

$$= \frac{DR''(t) + DR'(t)}{V1} + v\left(\frac{DR'(t) + DR(t)}{V1}\right) \text{ (since, } I(t) = \frac{DR'(t) + DR(t)}{V1}\text{))}$$

Connecting the daily reported cases in the Model to the daily reported cases in the data:

[4] Tang et al., 2020; B. Tang, N.L. Bragazzi, Q. Li, S. Tang, Y. Xiao, J. Wu; An updated estimation of the risk of transmission of the novel coronavirus (2019-nCov) Infectious Disease Modelling, 5 (2020), pp. 248-255

[5] Z. Liu, P. Magal, O. Seydi, G. Webb; Predicting the cumulative number of cases for the COVID-19 epidemic in China from early data; Mathematical Biosciences and Engineering doi: 10.3934/mbe.2020172;

Since the daily reported cases of COVID-19 epidemics are heavily fluctuated and typically very erratic, it varies with locations. It can be doubled from one day to the next in some cases. Since, the difficulty of using the date, we use rolling weekly moving average daily reported cases of that data to smooth the daily reported cases i.e our data to be interpolated by a smooth continuum B-spline curve *BS(t)*[iii]

Therefore, DR(t) can be equated in the model to BS(t) in the data and the derivatives DR'(t) = BS'(t) and DR''(t)= BS''(t) can be obtained.

$\tau\,(t)S(t)(I(t) + U(t)) = \frac{BS''(t)+BS'(t)}{V1}+v\left(\frac{BS'(t)+BS(t)}{V1}\right)$ (formula for the transmission in our model)

Therefore, we can solve for this function $\tau\,(t)$

$$\tau\,(t) = \frac{BS''(t)+BS'(t)}{V1}+v\left(\frac{BS'(t)+BS(t)}{V1}\right)/\left(S(t)\left(I(t)+U(t)\right)\right)$$

Since we know the transmission up to the last day of reported data, that we can incorporate into our model.

Background of the Covid-19 Model to the United Kingdom[6]:

February: First cases reported

March: Beginning of the first lock down. The UK Government introduces different lockdown measures, along with rising covid infection and death cases[7]

Late April, May, June: The British Government started to ease the restrictions

July and August: Cases remained low during the summertime

September and October: Winter approaching, the Government reintroduce the Covid measures

November: (End of Nov.) the UK reported a high number of deaths

December: The British government. imposed stricter lockdown measures

Parameters for Covid-19 Model[8]

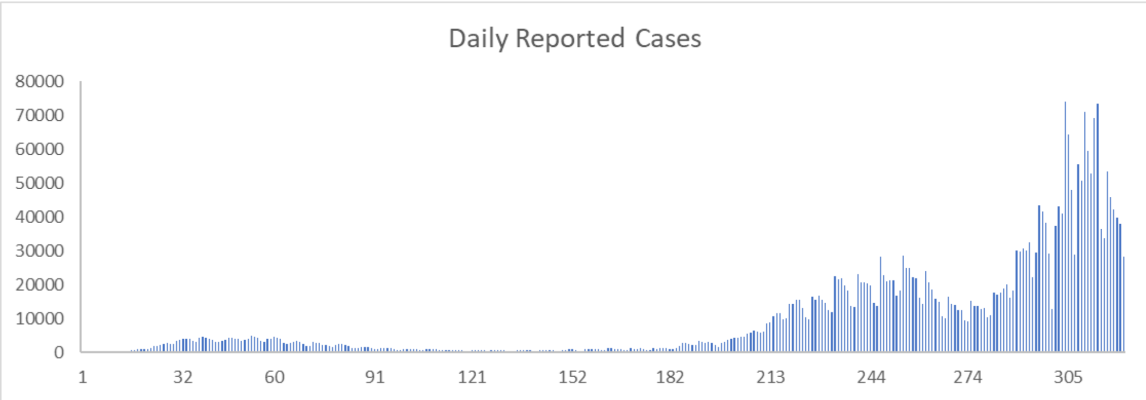
[6] Uk health security agency, 'COVID-19: epidemiology, virology and clinical features' (GovUK, 17 May 2022) <https://www.gov.uk/government/publications/wuhan-novel-coronavirus-background-information> accessed 1 November 2022

[7] Institutie for Government, 'Timeline of UK government coronavirus lockdowns and restrictions'(LeadingThinkTank, 21 Jan 2022) < https://www.instituteforgovernment.org.uk/charts/uk-government-coronavirus-lockdowns >

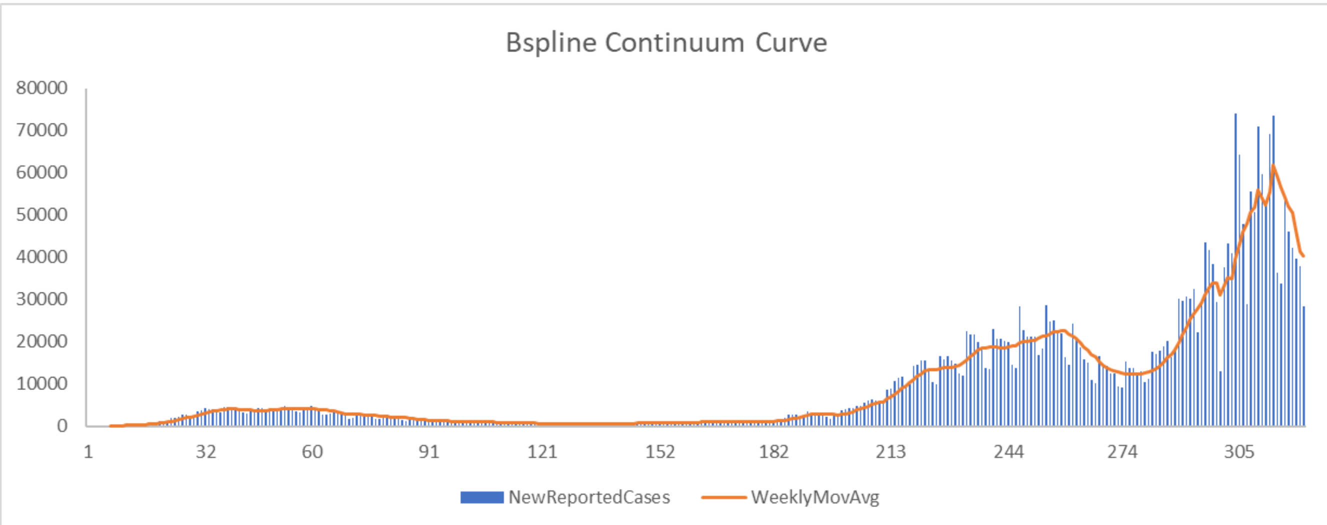
[8] Liu, Z.; Magal, P.; Seydi, O.; Webb, G. Understanding Unreported Cases in the COVID-19 Epidemic Outbreak in Wuhan, China, and the Importance of Major Public Health Interventions. Biology 2020, 9, 50.

- V = 1/7 the period of asymptomatic infectiousness to one week
- *f* = .4 fraction of reported asymptomatic cases
- *1 – f* = .6 fraction of the unreported symptomatic infectious
- $S(t_0) = 67{,}900{,}000$ initial population of the UK

We initially collected the data from the beginning of March to January 15 (322 days); later we extended our forecast till the 1st of September to show the vaccination impact on the transmission rate. Furthermore, in our project, we compared our model data with the data published by the British Government.



The daily reported cases without Bspline interpolation *BS(t)*, which tell the data heavily fluctuated.



The daily reported cases with Bspline interpolation *BS(t)*, the line chart went over the bar chart to show the effectiveness of the moving average data rather than daily data entry point.

Incorporation of Vaccination into the Model:

On Dec the 8 United Kingdom (UK) began their vaccination program with a 2-dose Pfizer vaccine. Then, on Dec 30th, the NHS delayed the second dose vaccine for over 500k people after they had received the first dose. This endeavour aimed at providing the first dose vaccine to as many people as possible[9]. AstraZeneca was approved on the 30th of Dec as a vaccine, and it followed

[9] Institutie for Government, 'Timeline of UK government coronavirus lockdowns and restrictions'(LeadingThinkTank, 21 Jan 2022) < https://www.instituteforgovernment.org.uk/charts/uk-government-coronavirus-lockdowns >

the same policy to delay vaccination for the second dose. Both doses followed similar policy for delaying second dose approximately 12 weeks delay from the first dose. According to The BBC, during the beginning of January, the UK was vaccinating around 2 million people per week[10].

Let $0 < f \leq 1$, the model equations include the loss term f x 2,000,000/7 in the susceptible population equation, since the vaccination begun from January 1= day 307, with effectiveness at 100% x f (approximately f x 285,000 people per day)[11].

The equations down below explain the vaccination removes susceptible from becoming infected[iv].

Since[12]*,*

$S'(t) = -\tau(t)S(t)(I(t) + U(t)) - v(t),\ t \geq t_0$

Therefore,

$S'(t) = -\tau(t)S(t)(I(t) + U(t)) - f \times 285000\ t \geq t_0$

$I'(t) = \tau(t)S(t)(I(t) + U(t)) - v(t),\ t \geq t_0$

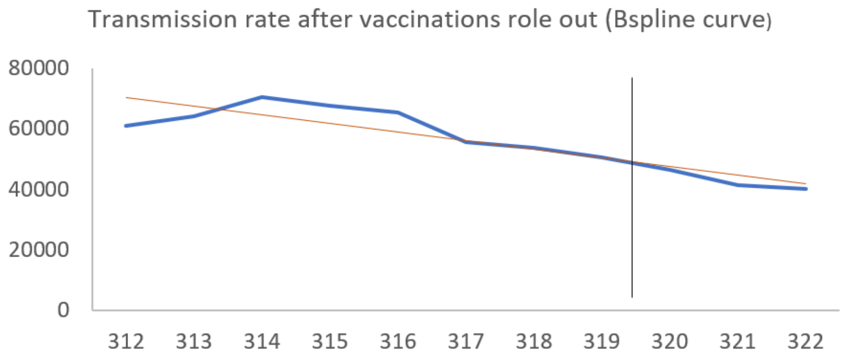
$R'(t) = V_1 I(t) - \eta R(t),\ t \geq t_0,$

$U'(t) = V_2 I(t) - \eta U(t),\ t \geq t_0,$

It is necessary to find the effectiveness of that vaccination rate, as a successful immunisation. Since not every person was vaccinated. In addition, after the jab susceptible people are not instantaneously immune to infection. In fact, it can take up to three of more weeks before a susceptible individual becomes immune to infection.

Going forward for a month designing the transmission rate after January 15 to find out the effectiveness of the vaccination

<u>Model to predict effectiveness of vaccination in UK:</u>(Designing the transmission rate after the vaccination role out)



Since the vaccination program began on day 307 = January 1, we can take that transmission rate which we have in our model up to January 15, and put it in a graph. After setting a linear

[10] Bbc, 'Covid vaccine: How many people are vaccinated in the UK?' (*Coronavirus pandemic*, 4 March 2022) <https://www.bbc.co.uk/news/health-55274833> accessed 3 November 2022

[11] Thompson,R. N.; Novel coronavirus outbreak in Wuhan, China, 2020: Intense surveillance Is vital for preventing sustained transmission in new locations. J. Clin. Med. 9(2), (2020), 498

[12] Nishiura,H; Jung,SM; Linto,Natalie; Kinoshita,Ryo; The Extent of Transmission of Novel Coronavirus in Wuhan, China, 2020; J. Clin. Med. 2020, 9(2), 330; https://doi.org/10.3390/jcm9020330

regression line to visualize the transmission rate relationship after the vaccination, it is clear that t_x is the going forward value.

We identified a time $t_x \leq t_1$ in which is the time that we will change from reported cases data information daily to a new form in the model going forward from that date. To clarify, we will take the last value where the red line intersects. Therefore, the transmission rate in the week before t_1 such that the transmission rate in the week before t_1 is represented by $\tau(t_x)$. To find the transmission rate we fit a linear regression line and take the fourth intersection value, Therefore, $t_x = 319.3$ for the value of linear regression graph with $\tau(t)S(t)(I(t) + U(t))$.

Identify the time we call t_x (the time that we will change from reported cases data information daily to a new form in the model going forward from that date)
Let $t_0 = 1$ = March 1, 2020, the first day of reported cases data. The last date of daily reported cases date in day $t_1 = 322$ = *January 15.* The transmission rate before day t1 is
(Before January 15 we the value of $\tau(t)S(t)(I(t) + U(t))$ susceptible population losses through asymptomatic)

$\tau(t)S(t)(I(t) + U(t)) = I'(t) + vI(t)$[13],

$$= \frac{DR''(t)+DR'(t)}{V1} + v\left(\frac{DR'(t)+DR(t)}{V1}\right),\ t_0 \leq t \leq t_1$$

The transmission rate for the Model going forward (absence of any relaxation of social distancing)

Before time $t_x = 319.3$ from the t_0 = March 1st

$$\tau(t)S(t)(I(t) + U(t)) = \frac{DR''(t)+DR'(t)}{V1} + v\left(\frac{DR'(t)+DR(t)}{V1}\right),\ t_0 \leq t \leq t_x$$

Forward from t_x the formula, is the continuous function of time[14]

$\tau(t)S(t)(I(t) + U(t)) = \left[\frac{DR''(tx)+DR'(tx)}{Vx} + v\left(\frac{DR'(tx)+DR(tx)}{V}\right)\right] \times \frac{S(t)(I(t)+U(t))}{S(tx)(I(tx)+U(tx))}$, $t \geq tx$ (At time t_x this $\frac{S(t)(I(t)+U(t))}{S(tx)(I(tx)+U(tx))} = 1$; Therefore, both above equations match at time t_x)

After time tx, transmission is dependent on S(t), I(t), U(t), but not on R(t). Vaccination is implemented from day 307 = January 1 to a time $t > t_1$.

This equation brings in the dynamics of the susceptible loss and the number of infected people,i.e., both asymptomatic and symptomatic. The transmission rate is continuous of function of time.

Going forward in time t_x we define $\tau(t, S(t), (I(t), U(t))$. *(Considering relaxation of pandemic measures due to mass vaccination)*

[13] Lau, E.H.Y.;Wu, P.; Hao, X.; Wong, J.Y.; Wu, J.T.; Leung, K.S.M.; Leung, G.M; Cowling, B.J.; Real-time tentative assessment of the epidemiological characteristics of novel coronavirus infections in Wuhan, China, as at 22 January 2020.
[14] Z. Liu, P. Magal, O. Seydi, G. Webb; Predicting the cumulative number of cases for the COVID-19 epidemic in China from early data; Mathematical Biosciences and Engineering doi: 10.3934/mbe.2020172;

Forward from t_x = 319.3 value (before the last day of daily reported cases to the next day of daily reported cases) to t_2 = March 1,

For $t_x < t \le t_2$ = March 1,

$$\tau\,(t,\ S(t)(I(t) + U(t)) = \tau\,((t)S(t),I(t),U(t))\ \frac{S(t)(I(t)+U(t))}{S(tx)(I(tx)+U(tx))}$$

<u>Changes of social behaviour</u>

Undoubtedly, the relaxation of social distancing rules happened because of the mass vaccination program. Consequently, there is a chance of increasing the transmission rate:

We estimated three intervals of time:

For $t_2 \le t < t_3$ = May 1, increase in transmission due to this relaxation
$\tau\,(t)S(t)(I(t) + U(t)) =$

$$(1+.03(t - t_2))\ \tau\,((t_x)S(t_x),I(t_x),U(t_x))\ \frac{S(t)(I(t)+U(t))}{S(tx)(I(tx)+U(tx))}$$

(if t = t_0 which is 1 and it matches the previous equations; but t > t_0 then it will be increased transmission .03(t – t_2) will be added);

<u>Further relaxation of social behaviour:</u>

for t_3 = *May 1 <* t_4 = *July 1,*

$$\tau\,(t)S(t)(I(t) + U(t)) =$$

$$(1+.03(t - t_2)+.02(t - t_3))\ \tau\,((t_x)S(t_x),I(t_x),U(t_x))\ \frac{S(t)(I(t)+U(t))}{S(tx)(I(tx)+U(tx))}$$

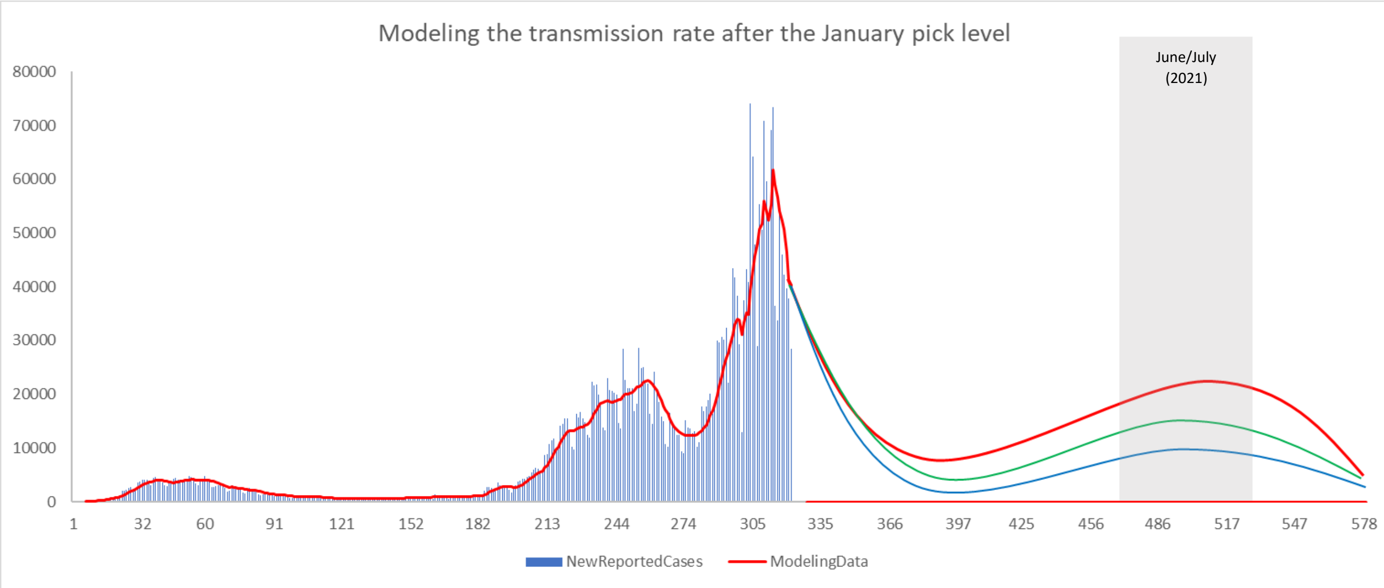
<u>Furthermore relaxation:</u>

for t_4 = *July 1 < t = June 1,*

$$\tau\,(t)S(t)(I(t) + U(t)) =$$

$$(1+.03(t - t_2) +.02(t - t_3)+.01(t - t_4))\ \tau\,((t_x)S(t_x),I(t_x),U(t_x))\ \frac{S(t)(I(t)+U(t))}{S(tx)(I(tx)+U(tx))}$$

With this transmission rate, we show the model simulations of daily reported cases for three cases of the vaccination efficiency with vaccination beginning on January 1:

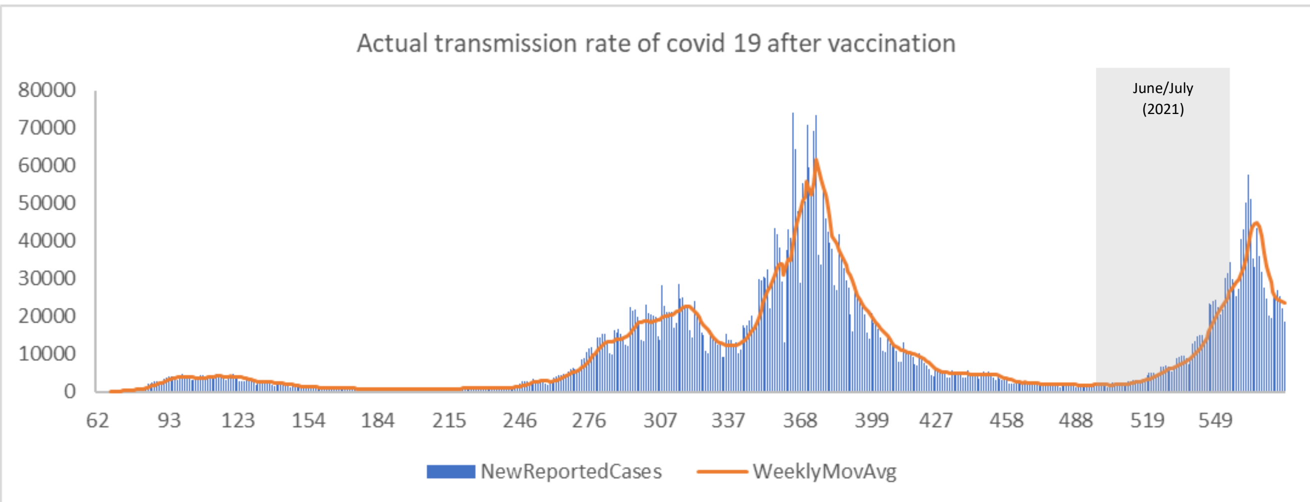


Red: efficiency 85%, 53,000,000 vaccinated by September 1. Daily reported cases data (bars); Green: efficiency 90%, 56,000,000 vaccinated by September. Blue: efficiency 95%, 59,000,000 vaccinated by September 1.

<u>Result:</u>

There are three different efficiency values are plotted in the graph. Up until January 15, the vertical bar represents the daily reported cases data. The red graph is the model simulation of the daily reported cases data, where the efficiency is 90%. Since the vaccination started on January 1st but after January 15th the vaccination starts to remove the transmission. Hence the number of cases started to slow down throughout January, and it continues to slow down forward from the previous day but then there is an upswing due to the relaxation of social distancing. This spick of transmission from June to the end of July is due to vaccination efficiency. The Red line represents the less efficient than the green line, however, the blue line represents the most efficiency and less transmission rate.

<u>Comparison between SIR model and published data:</u>



By making comparations between the two graphs, you can appreciate that our model correctly predicted the higher rate of transmission during the summertime due to the relaxation measures. However, in the data published by the UK government, the daily transmission rate continued to increase beyond the summer then it started to level out after September. According to our model, the incidence starts levelling out before September.

Furthermore, in our model, we introduced relaxation measures after March, and we predicted a slow increase in transmission rate. Therefore, you saw after January the 15th there was a big drop in the transmission rate. However, in the data published by the Government, the daily transmission rate suggests the transmission started to increase middle of February and it went at pick by mid-March before it started to level out after.

Nevertheless, our model did not consider the child vaccination factors, whereas during the beginning of March, the UK government reopened the schools without child vaccination[15]. Therefore, children and teenagers who were under 18 had a high rate of transmission rate, which helped to increase the total tally number. Another reason which could contribute to this higher transition rate during March was the increasing number of lateral flow tests, hence, the reported symptomatic population[16].

By contrast, when the UK government started to relax the pandemic measures, e.g., outdoor gatherings, parties (etc…) among the vaccinated group after March 2021, it is clear that the transmission rate did not increase amongst that group[17].

Conclusion

This forecast summarises, how the vaccination immunizes the population against covid-19. Vaccination programs lowered the transmission rate and the level of the epidemic for the UK. We can confirm that the vaccine has delayed transmission of the disease, but this was not an immediate effect, it happened over some time. In addition, a recent government reported evidence for lower transmission rates since the majority of the UK population had two doses of the vaccine[18].

[15] BBC/Health, Child Covid vaccinations: Your questions answered' (*Coronavirus pandemic*, 17th Feb 2022) < https://www.bbc.co.uk/news/health-60415846> accessed 3 November 2022

[16] BBC/Health, 'UK reports nearly 120,000 daily Covid cases' ((*Coronavirus pandemic*, 10th Dec 2021)) < https://www.bbc.co.uk/news/live/uk-59764750 >

[17] NHS/England; Covid-19 vaccinations/Data <https://www.england.nhs.uk/statistics/statistical-work-areas/covid-19-vaccinations> accessed 3 November 2022

[18] GOV.UK, Coronavirus (Covid-19) in the UK; (report published: 27th of Sep 2022) < https://coronavirus.data.gov.uk/details/cases?areaType=nation&areaName=England > accessed 3rd Nov 2022

[i] Susceptible populations are lost either by transmission or vaccination. Susceptible population has lost its transmission term - τ (t)S(t) because population is decreasing; this is the loss of individuals who become infected; **τ (t) is the function; we have to identify as a time dependent parameter;** (susceptible people are lost to the infected)
This mass action form *S(t)(I(t) + U(t))* depends on the number of susceptible * against the number of infected asymptomatic individuals + the unreported symptomatic infected individuals; in addition, v(t) is the loss term for the susceptible population, that's the number of people vaccinated each day or effectiveness of the vaccination for the people vaccinated each day (which is much less);
Moreover, both symptomatic reported and unreported cases stay in 1 week in their classes before they move to the remove class.

[ii] DR'(t) change of daily reported cases (dependent variable) at time t is equal to the number of reported cases going into that variable and they will stay in that class for one day,

[iii] Idea is we just need to take the todays data of daily reported cases and we take the average daily reported cases data for the past 7 days; and we replace the value we get today by that average with the past 7days that's called the (rolling weekly averaged daily) data. Therefore, we get a value for that each day.
Furthermore, to incorporate this into the model; we take the daily averaged rolling weekly cases each day that a discreate set of points; one point each day; we want to continue that in our model; because it is a continues model and it based on averages over time. So, we want to replace that rolling weekly average data discrete with a continue interpolation of it involving cubic spline approximation.
We take that cubic spline approximation to that discrete data and transmission rate expression in our mode and replaces with this continuum cubic spline approximation to that discrete data

[iv] Where, *$-\tau$ (t)S(t)(I(t) + U(t)) is the transmission rate, v(t) = f x 285000 loss of susceptible via vaccination (we will take is value for this fraction and look at the outcome vaccination dependent on this fraction of successful vaccination)*

References

1. BBC/Health, Child Covid vaccinations: Your questions answered' (*Coronavirus pandemic*, 17th Feb 2022) < https://www.bbc.co.uk/news/health-60415846> accessed 3 November 2022

2. BBC/Health, 'UK reports nearly 120,000 daily Covid cases' ((*Coronavirus pandemic*, 10th Dec 2021)) < https://www.bbc.co.uk/news/live/uk-59764750 >

3. BBC, 'Covid vaccine: How many people are vaccinated in the UK?' (*Coronavirus pandemic*, 4 March 2022) <https://www.bbc.co.uk/news/health-55274833> accessed 3 November 2022

4. GOV.UK, Coronavirus (Covid-19) in the UK; (report published: 27th of Sep 2022) <https://coronavirus.data.gov.uk/details/cases?areaType=nation&areaName=England > accessed 3rd Nov 2022

5. Institutie for Government, 'Timeline of UK government coronavirus lockdowns and restrictions'(LeadingThinkTank, 21 Jan 2022) < https://www.instituteforgovernment.org.uk/charts/uk-government-coronavirus-lockdowns >

6. Institutie for Government, 'Timeline of UK government coronavirus lockdowns and restrictions'(LeadingThinkTank, 21 Jan 2022) < https://www.instituteforgovernment.org.uk/charts/uk-government-coronavirus-lockdowns >

7. Lau, E.H.Y. et al., 2020; Real-time tentative assessment of the epidemiological characteristics of novel coronavirus infections in Wuhan, China, as at 22 January.

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9. Liu, Z.; Magal, P.; Seydi, O.; Webb, G. Understanding Unreported Cases in the COVID-19 Epidemic Outbreak in Wuhan, China, and the Importance of Major Public Health Interventions. Biology 2020, 9, 50.

10. NHS/England; Covid-19 vaccinations/Data <https://www.england.nhs.uk/statistics/statistical-work-areas/covid-19-vaccinations> accessed 3 November 2022

11. Nishiura,H; Jung,SM; Linto,Natalie; Kinoshita,Ryo; The Extent of Transmission of Novel Coronavirus in Wuhan, China, 2020; J. Clin. Med. 2020, 9(2), 330; https://doi.org/10.3390/jcm9020330

12. Suli,Liua; Michael Y.Lib; Epidemic models with discrete state structures; Physica D: Nonlinear Phenomena, Volume 422, August 2021, 132903

13. Tang et al., 2020; An updated estimation of the risk of transmission of the novel coronavirus (2019-nCov) Infectious Disease Modelling, 5 (2020), pp. 248-255

14. Thompson,R. N.; Novel coronavirus outbreak in Wuhan, China, 2020: Intense surveillance Is vital for preventing sustained transmission in new locations. J. Clin. Med. 9(2), (2020), 498

15. Turkyilmazoglu, Mustafa Explicit formulae for the peak time of an epidemic from the SIR model. Physica D: Nonlinear Phenomena; Volume 422, August 2021, 132902

16. UK health security agency, 'COVID-19: epidemiology, virology and clinical features' (GovUK, 17 May 2022) <https://www.gov.uk/government/publications/wuhan-novel-coronavirus-background-information> accessed 1 November 2022

APPENDIX

In [172…

```python
import pandas as pd
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
```

We'll need to use **differential equations** to calculate the population change over time. Luckily we can just borrow this code and we'll be good!

> To understand this code, I recommend watching https://youtu.be/NKMHhm2Zbkw?t=119 - the timestamp I linked you to is where he starts walking through the equations below

In [173…

```python
# The SIR model differential equations.
def deriv(state, t, N, beta, gamma):
    S, I, R = state
    # Change in S population over time
    dSdt = -beta * S * I / N
    # Change in I population over time
    dIdt = beta * S * I / N - gamma * I
    # Change in R population over time
    dRdt = gamma * I
    return dSdt, dIdt, dRdt
```

We'll start by modeling our population. Having a thousand people and one infected person sounds reasonable.

Effective contact rate is `transmission rate * contact rate`, so:

- For example, 5% transmission rate and 5 contacts a day is `0.05 * 5 = 0.25`

Recovery rate is `1 / days`, so

- For example, 4 day recovery rate is `1 / 4 = 0.25`

In [268…

```python
effective_contact_rate = 0.5
recovery_rate = 1/4

# We'll compute this for fun
print("R0 is", effective_contact_rate / recovery_rate)

# What's our start population look like?
# Everyone not infected or recovered is susceptible
total_pop = 1000
recovered = 0
infected = 1
susceptible = total_pop - infected - recovered

# A list of days, 0-160
days = range(0, 160)

# Use differential equations magic with our population
ret = odeint(deriv,
```

In [268…

```
effective_contact_rate = 0.5
recovery_rate = 1/4

# We'll compute this for fun
print("R0 is", effective_contact_rate / recovery_rate)

# What's our start population look like?
# Everyone not infected or recovered is susceptible
total_pop = 1000
recovered = 0
infected = 1
susceptible = total_pop - infected - recovered

# A list of days, 0-160
days = range(0, 160)

# Use differential equations magic with our population
ret = odeint(deriv,
             [susceptible, infected, recovered],
             days,
             args=(total_pop, effective_contact_rate, recovery_rate))
S, I, R = ret.T

# Build a dataframe because why not
df = pd.DataFrame({
    'suseptible': S,
    'infected': I,
    'recovered': R,
    'day': days
})

plt.style.use('ggplot')
df.plot(x='day',
        y=['infected', 'suseptible', 'recovered'],
        color=['#bb6424', '#aac6ca', '#cc8ac0'],
        kind='area',
        stacked=True)

# If you get the error:
#
#     When stacked is True, each column must be either all
#     positive or negative.infected contains both...
#
# just change stacked=True to stacked=False
```

```
R0 is 2.0
```

date	num	NewRepo	ModelingData	Cumulativ
01/03/2020	1	18		87
02/03/2020	2	40		127
03/03/2020	3	52		179
04/03/2020	4	49		228
05/03/2020	5	46		274
06/03/2020	6	74		348
07/03/2020	7	55	48	403
08/03/2020	8	50	52	453
09/03/2020	9	127	65	580
10/03/2020	10	229	90	809
11/03/2020	11	358	134	1167
12/03/2020	12	421	188	1588
13/03/2020	13	401	234	1989
14/03/2020	14	315	272	2304
15/03/2020	15	398	321	2702
16/03/2020	16	552	382	3254
17/03/2020	17	683	447	3937
18/03/2020	18	910	526	4847
19/03/2020	19	932	599	5779
20/03/2020	20	1096	698	6875
21/03/2020	21	1040	802	7915
22/03/2020	22	1212	918	9127
23/03/2020	23	2010	1126	11137
24/03/2020	24	2041	1320	13178
25/03/2020	25	2284	1516	15462
26/03/2020	26	2633	1759	18095
27/03/2020	27	2672	1985	20767
28/03/2020	28	2379	2176	23146
29/03/2020	29	2449	2353	25595
30/03/2020	30	3514	2567	29109
31/03/2020	31	3816	2821	32925
01/04/2020	32	4150	3088	37075
02/04/2020	33	4097	3297	41172
03/04/2020	34	4102	3501	45274
04/04/2020	35	3405	3648	48679
05/04/2020	36	3094	3740	51773
06/04/2020	37	4407	3867	56180
07/04/2020	38	4605	3980	60785
08/04/2020	39	4315	4004	65100
09/04/2020	40	4076	4001	69176
10/04/2020	41	3590	3927	72766
11/04/2020	42	3085	3882	75851
12/04/2020	43	2958	3862	78809
13/04/2020	44	3463	3727	82272
14/04/2020	45	3568	3579	85840
15/04/2020	46	4240	3569	90080
16/04/2020	47	4360	3609	94440
17/04/2020	48	4088	3680	98528
18/04/2020	49	4142	3831	102670
19/04/2020	50	3494	3908	106164
20/04/2020	51	3698	3941	109862
21/04/2020	52	3996	4003	113858
22/04/2020	53	4812	4084	118670
23/04/2020	54	4574	4115	123244
24/04/2020	55	4352	4153	127596
25/04/2020	56	3381	4044	130977
26/04/2020	57	3137	3993	134114
27/04/2020	58	4056	4044	138170
28/04/2020	59	4113	4061	142283
29/04/2020	60	4737	4050	147020

30/04/2020	Mar	4314	55	4013	151334
01/05/2020	Mar	4109	56	3978	155443
02/05/2020	Mar	2762	57	3890	158205
03/05/2020	Mar	2648	58	3820	160853
04/05/2020	Mar	2851	59	3648	163704
05/05/2020	Mar	3148	60	3510	166852
06/05/2020	Mar	3264	61	3299	170116
07/05/2020	Mar	3205	62	3141	173321
08/05/2020	Mar	2622	63	2929	175943
09/05/2020	Mar	1797	64	2791	177740
10/05/2020	Mar	2013	65	2700	179753
11/05/2020	Mar	3088	66	2734	182841
12/05/2020	Mar	2940	67	2704	185781
13/05/2020	Mar	2899	68	2652	188680
14/05/2020	Mar	2222	69	2512	190902
15/05/2020	Mar	2145	70	2443	193047
16/05/2020	Mar	1781	71	2441	194828
17/05/2020	Mar	1593	72	2381	196421
18/05/2020	Mar	2226	73	2258	198647
19/05/2020	Mar	2602	74	2210	201249
20/05/2020	Mar	2359	75	2133	203608
21/05/2020	Mar	2277	76	2140	205885
22/05/2020	Mar	1798	77	2091	207683
23/05/2020	Mar	1309	78	2023	208992
24/05/2020	Mar	1206	79	1968	210198
25/05/2020	Mar	1389	80	1849	211587
26/05/2020	Mar	1461	81	1686	213048
27/05/2020	Mar	1616	82	1579	214664
28/05/2020	Mar	1575	83	1479	216239
29/05/2020	Mar	1374	84	1419	217613
30/05/2020	Mar	1010	85	1376	218623
31/05/2020	Mar	984	86	1344	219607
01/06/2020	Mar	1319	87	1334	220926
02/06/2020	Mar	1365	88	1320	222291
03/06/2020	Mar	1251	89	1268	223542
04/06/2020	Mar	1147	90	1207	224689
05/06/2020	Mar	1020	91	1157	225709
06/06/2020	Mar	723	92	1116	226432
07/06/2020	Mar	668	93	1070	227100
08/06/2020	Mar	990	94	1023	228090
09/06/2020	Mar	1066	95	981	229156
10/06/2020	Mar	1086	96	957	230242
11/06/2020	Mar	915	97	924	231157
12/06/2020	Mar	938	98	912	232095
13/06/2020	Mar	785	99	921	232880
14/06/2020	Mar	752	100	933	233632
15/06/2020	Mar	944	101	927	234576
16/06/2020	Mar	996	102	917	235572
17/06/2020	Mar	914	103	892	236486
18/06/2020	Mar	938	104	895	237424
19/06/2020	Mar	818	105	878	238242
20/06/2020	Mar	626	106	855	238868
21/06/2020	Mar	550	107	827	239418
22/06/2020	Mar	821	108	809	240239
23/06/2020	Mar	727	109	771	240966
24/06/2020	Mar	732	110	745	241698
25/06/2020	Mar	643	111	702	242341
26/06/2020	Mar	640	112	677	242981
27/06/2020	Mar	506	113	660	243487
28/06/2020	Mar	413	114	640	243900
29/06/2020	Mar	666	115	618	244566
30/06/2020	Mar	578	116	597	245144
01/07/2020	Mar	607	117	579	245751

02/07/2020	Mar		561	118	567	246312
03/07/2020	Mar		536	119	552	246848
04/07/2020	Mar		385	120	535	247233
05/07/2020	Mar		544	121	554	247777
06/07/2020	Mar		664	122	554	248441
07/07/2020	Mar		565	123	552	249006
08/07/2020	Mar		682	124	562	249688
09/07/2020	Mar		688	125	581	250376
10/07/2020	Mar		524	126	579	250900
11/07/2020	Mar		433	127	586	251333
12/07/2020	Mar		351	128	558	251684
13/07/2020	Mar		694	129	562	252378
14/07/2020	Mar		661	130	576	253039
15/07/2020	Mar		726	131	582	253765
16/07/2020	Mar		644	132	576	254409
17/07/2020	Mar		548	133	580	254957
18/07/2020	Mar		473	134	585	255430
19/07/2020	Mar		419	135	595	255849
20/07/2020	Mar		769	136	606	256618
21/07/2020	Mar		697	137	611	257315
22/07/2020	Mar		764	138	616	258079
23/07/2020	Mar		741	139	630	258820
24/07/2020	Mar		720	140	655	259540
25/07/2020	Mar		499	141	658	260039
26/07/2020	Mar		518	142	673	260557
27/07/2020	Mar		824	143	680	261381
28/07/2020	Mar		796	144	695	262177
29/07/2020	Mar		994	145	727	263171
30/07/2020	Mar		879	146	747	264050
31/07/2020	Mar		644	147	736	264694
01/08/2020	Mar		508	148	738	265202
02/08/2020	Mar		518	149	738	265720
03/08/2020	Mar		963	150	757	266683
04/08/2020	Mar		965	151	782	267648
05/08/2020	Mar		974	152	779	268622
06/08/2020	Mar		1005	153	797	269627
07/08/2020	Mar		892	154	832	270519
08/08/2020	Mar		648	155	852	271167
09/08/2020	Mar		569	156	859	271736
10/08/2020	Mar		1375	157	918	273111
11/08/2020	Mar		1247	158	959	274358
12/08/2020	Mar		1126	159	980	275484
13/08/2020	Mar		1036	160	985	276520
14/08/2020	Mar		1059	161	1009	277579
15/08/2020	Mar		656	162	1010	278235
16/08/2020	Mar		534	163	1005	278769
17/08/2020	Mar		1159	164	974	279928
18/08/2020	Mar		955	165	932	280883
19/08/2020	Mar		1109	166	930	281992
20/08/2020	Mar		1254	167	961	283246
21/08/2020	Mar		1031	168	957	284277
22/08/2020	Mar		732	169	968	285009
23/08/2020	Mar		704	170	992	285713
24/08/2020	Mar		1157	171	992	286870
25/08/2020	Mar		1085	172	1010	287955
26/08/2020	Mar		1181	173	1021	289136
27/08/2020	Mar		1356	174	1035	290492
28/08/2020	Mar		1395	175	1087	291887
29/08/2020	Mar		1051	176	1133	292938
30/08/2020	Mar		987	177	1173	293925
31/08/2020	Mar		1279	178	1191	295204
01/09/2020	Mar		1962	179	1316	297166
02/09/2020	Mar		2696	180	1532	299862

03/09/2020	Mar	2716	181	1727	302578
04/09/2020	Mar	2637	182	1904	305215
05/09/2020	Mar	2200	183	2068	307415
06/09/2020	Mar	2079	184	2224	309494
07/09/2020	Mar	3530	185	2546	313024
08/09/2020	Mar	3071	186	2704	316095
09/09/2020	Mar	2893	187	2732	318988
10/09/2020	Mar	3143	188	2793	322131
11/09/2020	Mar	2893	189	2830	325024
12/09/2020	Mar	2224	190	2833	327248
13/09/2020	Mar	1722	191	2782	328970
14/09/2020	Mar	2941	192	2698	331911
15/09/2020	Mar	3057	193	2696	334968
16/09/2020	Mar	3778	194	2823	338746
17/09/2020	Mar	3979	195	2942	342725
18/09/2020	Mar	4290	196	3142	347015
19/09/2020	Mar	4215	197	3426	351230
20/09/2020	Mar	4652	198	3845	355882
21/09/2020	Mar	4727	199	4100	360609
22/09/2020	Mar	5400	200	4434	366009
23/09/2020	Mar	5933	201	4742	371942
24/09/2020	Mar	6361	202	5083	378303
25/09/2020	Mar	6105	203	5342	384408
26/09/2020	Mar	5704	204	5555	390112
27/09/2020	Mar	6084	205	5759	396196
28/09/2020	Mar	8641	206	6318	404837
29/09/2020	Mar	8832	207	6809	413669
30/09/2020	Mar	10663	208	7484	424332
01/10/2020	Mar	11444	209	8210	435776
02/10/2020	Mar	11719	210	9012	447495
03/10/2020	Mar	9820	211	9600	457315
04/10/2020	Mar	9966	212	10155	467281
05/10/2020	Mar	14218	213	10952	481499
06/10/2020	Mar	14458	214	11755	495957
07/10/2020	Mar	15418	215	12435	511375
08/10/2020	Mar	15452	216	13007	526827
09/10/2020	Mar	13159	217	13213	539986
10/10/2020	Mar	10301	218	13282	550287
11/10/2020	Mar	9781	219	13255	560068
12/10/2020	Mar	16420	220	13570	576488
13/10/2020	Mar	15660	221	13742	592148
14/10/2020	Mar	16646	222	13917	608794
15/10/2020	Mar	15429	223	13914	624223
16/10/2020	Mar	14761	224	14143	638984
17/10/2020	Mar	12487	225	14455	651471
18/10/2020	Mar	11982	226	14769	663453
19/10/2020	Mar	22369	227	15619	685822
20/10/2020	Mar	21638	228	16473	707460
21/10/2020	Mar	21762	229	17204	729222
22/10/2020	Mar	19866	230	17838	749088
23/10/2020	Mar	18359	231	18352	767447
24/10/2020	Mar	13713	232	18527	781160
25/10/2020	Mar	13354	233	18723	794514
26/10/2020	Mar	23033	234	18818	817547
27/10/2020	Mar	20761	235	18693	838308
28/10/2020	Mar	20547	236	18519	858855
29/10/2020	Mar	20257	237	18575	879112
30/10/2020	Mar	19899	238	18795	899011
31/10/2020	Mar	14565	239	18917	913576
01/11/2020	Mar	13611	240	18953	927187
02/11/2020	Mar	28394	241	19719	955581
03/11/2020	Mar	22690	242	19995	978271
04/11/2020	Mar	21053	243	20067	999324

Due to Microsoft word error we couldn’t upload our full appendix.