Mobile Communications Systems

Coursework

Prepared by

Othmane Belarbi

Department of Electrical and Electronic Engineering

University of Bristol

15th January 2021

Contents

List of Figures

List of Tables

Part 1: Mobile Communications Systems

Q1.1 Binary Double Erasure Channel

This is a symmetric channel, we expect the capacity to be maximised for a uniform distribution.

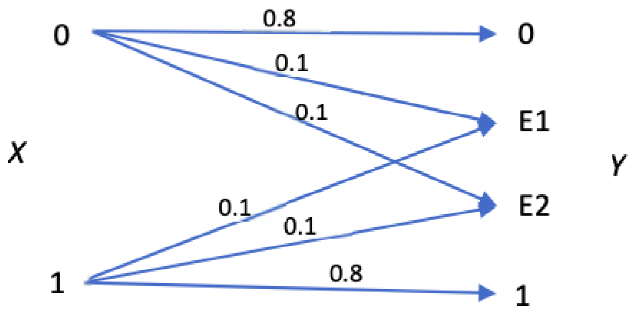

Figure 1: Binary Double Erasure Channel

Let $\mathcal{X} = \{0, 1\}$, $\mathcal{Y} = \{0, E_1, E_2, 1\}$,

$$p(y|x) = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}$$

We can note that $H(X|Y = 0) = H(X|Y = 1) = 0$ as this is deterministic.

Let's compute the marginal distribution of Y by using the fact that X needs to be uniformly distributed (symmetric channel - capacity) and this formula:

$$p(y = y_i) = \sum_{x_i \in \mathcal{X}} p(y = y_i|x = x_i)p(x = x_i)$$

Hence:

$$\begin{cases} p(y=0) & = 0.4 \\ p(y=E_1) & = 0.1 \\ p(y=E_2) & = 0.1 \\ p(y=1) & = 0.4 \end{cases}$$

First,

$$H(X|Y) = \sum_{y_i \in \mathcal{Y}} p(y=y_i) H(X|Y=y_i)$$
$$H(X|Y) = p(y=E_1)H(X|Y=E_1) + p(y=E_2)H(X|Y=E_2)$$

Hence,

$$H(X|Y) = 0.1 \times H(X|Y=E_1) + 0.1 \times H(X|Y=E_2)$$
$$H(X|Y) = 0.2H(X)$$

Therefore, (because $H(X|Y=E_1) = H(X|Y=E_2) = H(X)$)

$$I(X;Y) = H(X) - H(X|Y)$$
$$I(X;Y) = 0.8H(X)$$

The maximum value for H(X) is obtained when X is equiprobable. In this case:

$$C = max_{p(x)} I(X;Y) = 0.8 max H(X) = 0.8$$

The channel will erase 0.2n bits on average (out of total n transmitted bits). Therefore, the number of remaining bits available is 0.8n per channel use.

Q1.2 Binary non-symmetric Channel

This is a non-symmetric channel, we don't expect the capacity to be maximised for a uniform distribution. Let $P(Y=1|X=0)=a$ and $P(Y=0|X=1)=b$

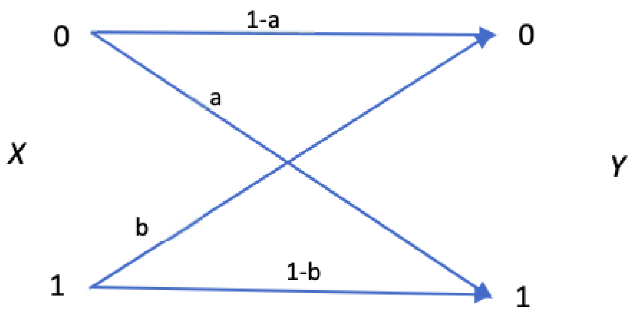

Figure 2: Binary Non-Symmetric Channel

I denote $p_X(0)=p$ and $p_X(1)=1-p$.

Let $\mathcal{X}=\{0,1\}$, $\mathcal{Y}=\{0,1\}$,

$$p(y|x)=\begin{bmatrix}1-a & a\\ b & 1-b\end{bmatrix}$$

First,

$$\begin{aligned}H(Y|X)&=p_X(0)\times H(Y|X=0)+p_X(1)\times H(Y|X=1)\\ H(Y|X)&=pH(\{1-a,a\})+(1-p)H(\{1-b,b\})\\ H(Y|X)&=pH_2(a)+(1-p)H_2(b)\end{aligned}$$

Moreover,

$$\begin{aligned}H(Y)&=H(\{p(1-a)+(1-p)b,pa+p(1-b)\})\\ H(Y)&=H_2(p(1-a)+(1-p)b)\\ H(Y)&=H_2(p(1-a-b)+b)\end{aligned}$$

Therefore,

$$\begin{aligned}I(X;Y)&=H(Y)-H(Y|X)\\ I(X;Y)&=H_2(p(1-a-b)+b)-pH_2(a)-(1-p)H_2(b)\end{aligned}\tag{1}$$

As mentioned before, we don't expect the capacity to be maximised for a uniform distribution. Hence, we need to take the derivative of (1) in order to find the value of p that maximizes the I(X; Y).

$$\frac{d}{dp}I(X;Y)=0$$

Hence,

$$\frac{d}{dp}\big[H_2(p(1-a-b)+b) - pH_2(a) - (1-p)H_2(b)\big] = 0$$
$$(1-a-b)\log\big(\frac{1}{p_{opt}(1-a-b)+b} - 1\big) - \big(H_2(a) - H_2(b)\big) = 0$$
$$\frac{1}{p_{opt}(1-a-b)+b} = 1 + 2^{\frac{H_2(a)-H_2(b)}{(1-a-b)}}$$

I denote $z = 2^{\frac{H_2(a)-H_2(b)}{(1-a-b)}}$. Then, we can simplify the last equation as follow:

$$\frac{1}{p_{opt}(1-a-b)+b} = 1+z$$
$$p_{opt}(1-a-b) = \frac{1}{1+z} - b$$
$$p_{opt} = \frac{1}{(1-a-b)}(\frac{1}{1+z} - b)$$
$$p_{opt} = \frac{1-b(1+z)}{(1-a-b)(1+z)}$$

Therefore, the capacity is maximised when $p_X(0) = p_{opt}$ and $p_X(1) = 1 - p_{opt}$. Hence, the **capacity of the binary non-symmetric channel** is:

$$C = H_2(p_{opt}(1-a-b)+b) - p_{opt}(H_2(a)+H_2(b)) - H_2(b)$$
$$C = H_2(\frac{1-b(1+z)}{(1+z)} + b) - \frac{1-b(1+z)}{(1-a-b)(1+z)}(H_2(a)+H_2(b)) - H_2(b)$$

Using the definition of z, we can simplify:

$$C = H_2(\frac{1}{(1+z)}) - \frac{\log z}{1+z} + b\log z - H_2(b)$$
$$C = \frac{1}{1+z}\log(1+z) + \frac{z}{1+z}\log\frac{1+z}{z} - \frac{\log z}{1+z} + b\log z - H_2(b)$$
$$C = \log(1+z) - \log z + b\log z - H_2(b)$$

Therefore,

$$C = \log(1+z) - \frac{1-b}{1-a-b}H_2(a) + \frac{a}{1-a-b}H_2(b)$$

We can check if our result makes sense by computing the capacity in case of a symmetrical channel, in other words a = b. So, z=1 by definition of z. Hence,

$$C_{BSC} = \log(1+1) - \frac{1-a}{1-2a}H_2(a) + \frac{a}{1-2a}H_2(a)$$
$$C_{BSC} = 1 + \frac{-(1-a)+a}{1-2a}H_2(a)$$
$$C_{BSC} = 1 - H_2(a)$$

The result makes sense as it's what we're expecting.

Q1.3 MIMO Wireless channel

The MIMO wireless channel is assumed to be square and unitary. Therefore:

$$\begin{cases} n_R & = n_T = n \\ H^\dagger & = H^{-1} \end{cases}$$

The mutual information can be expanded as:

$$\begin{aligned}
I(x;y) &= h(y) - h(y|x) \\
I(x;y) &= h(y) - h(Hx + n|x) \\
I(x;y) &= h(y) - h(n|x) \\
I(x;y) &= h(y) - h(n)
\end{aligned}$$

Assuming Gaussian signalling input X, the covariance matrix of the received complex vector y is given by:

$$\begin{aligned}
E\{yy^\dagger\} &= E\{(Hx + n)(Hx + n)^\dagger\} \\
E\{yy^\dagger\} &= E\{Hxx^\dagger H^\dagger\} + E\{nn^\dagger\} \\
E\{yy^\dagger\} &= H\phi H^\dagger + K^n \\
E\{yy^\dagger\} &= K^d + K^n
\end{aligned}$$

The maximum mutual information of a random MIMO channel where the channel is assumed to be square and unitary is given by:

$$\begin{aligned}
I(x;y) &= h(y) - h(n) \\
I(x;y) &= \log_2 \Big[\det(\pi e(K^d + K^n))\Big] - \log_2 \Big[\det(\pi e K^n)\Big] \\
I(x;y) &= \log_2 \Big[\det(K^d + K^n)(K^n)^{-1}\Big] \\
I(x;y) &= \log_2 \Big[\det(K^d (K^n)^{-1} + I_n)\Big] \\
I(x;y) &= \log_2 \Big[\det(H\phi H^\dagger (K^n)^{-1} + I_n)\Big]
\end{aligned}$$

When the transmitter has no knowledge about the channel, a uniform power distribution is used. The transmit covariance matrix is then $\phi = \frac{P_T}{n_T} I_n$.

Moreover, the uncorrelated noise as seen at each receiver is described by the covariance matrix $K^n = \sigma^2 I_n$.

The capacity for fixed H is then:

$$C = \log_2 \Big[\det(I_n + \frac{P_T}{\sigma^2 n} HH^\dagger)\Big]$$

Where the average signal-to-noise (SNR) is $\rho = \frac{P_T}{\sigma^2}$. Then, the ergodic capacity of the channel becomes:

$$C = E_H\Big\{\log_2\Big[\det(I_n + \frac{P_T}{\sigma^2 n} HH^\dagger)\Big]\Big\}$$
$$C = \log_2\Big[\det(I_n + \frac{\rho}{n} I_n)\Big]$$
$$C = \log_2\Big[\det((1 + \frac{\rho}{n}) I_n)\Big]$$
$$C = \log_2\Big[(1 + \frac{\rho}{n})^n\Big]$$
$$C = n \log_2\Big[1 + \frac{\rho}{n}\Big]$$

The capacity grows linearly with the number of antennas and there is no fading. In this case, the MIMO channel is equivalent to a set of n parallel, non-interfering identical standard gaussian channel.

We can compare the capacity with the case where the individual channels in the wireless MIMO system are all independent and iid complex circular symmetric Gaussian after scaling the channels in the unitary case.

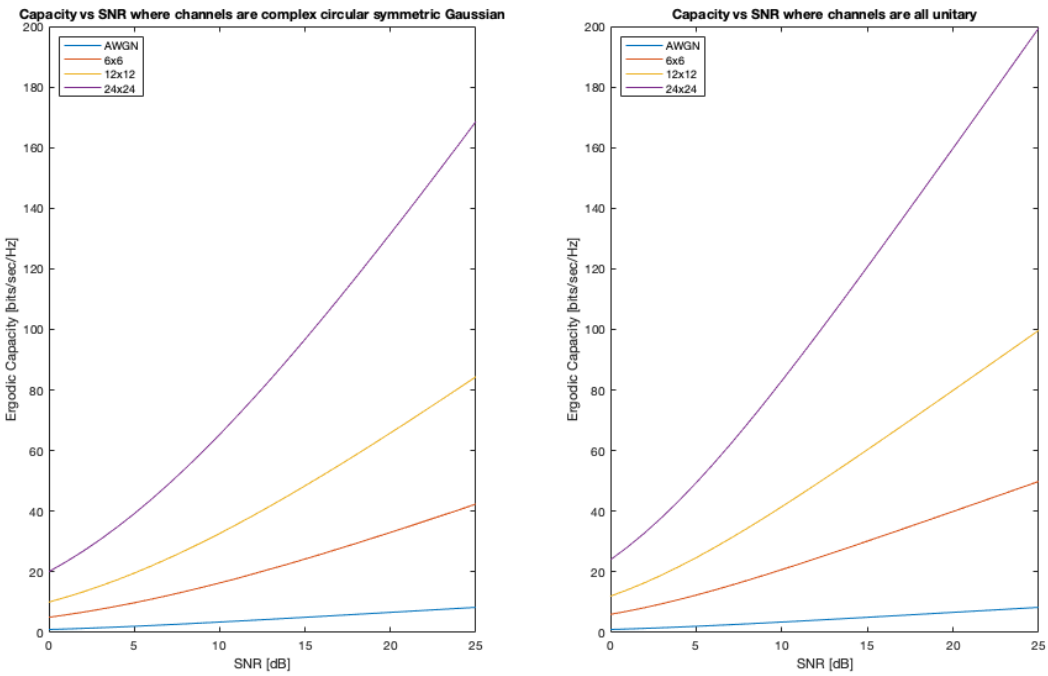

Figure 3: Comparison of the capacity in unitary and normal case

First, the ergodic capacity increases when the number of antennas increases. As expected the capacity where the channels are all unitary is better than the other case. (The difference is obvious for high values of SNR)

Part 2: Mobile Communications Systems

Q2.1 Coursework assignment using Matlab

a) As shown in the **Figure 4**, the noise immunity of the 16QAM is worse than QPSK for the same signal powers. That's why the QPSK performes better than the 16QAM in gaussian noise.

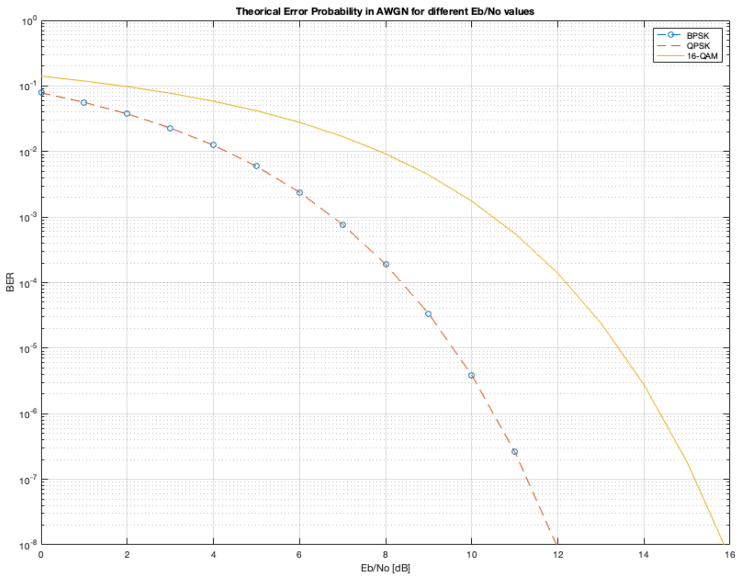

Figure 4: Theoretical Error Probability in AWGN

However, the 16QAM provides a better data rate and spectral efficiency than the QPSK.

b) Here is a diagram that gives a brief insight on how I performed the baseband simulation of a QPSK communication system in AWGN channel. (You can find more details about the implementation in the MATLAB code)

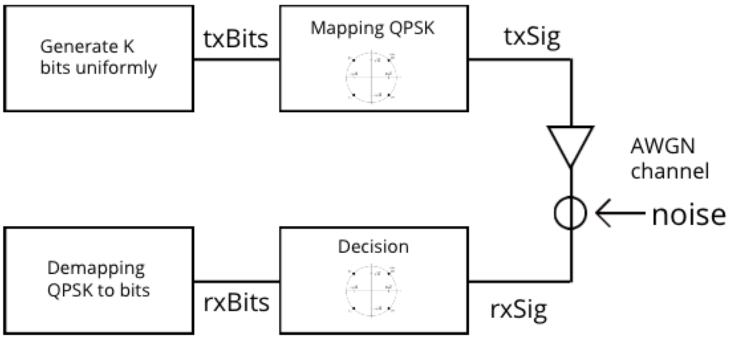

Figure 5: Baseband Simulation

First, I generated K bits uniformly using the **randi** method. Then, I modulated these K bits using the quadrature phase shift keying method (more details in the **modulationQPSK function**). So, that's for the transmission part.

Then, these symbols go through an AWGN channel (AWGN channel created using the **randn** method). I demodulated in order to retrieve the symbols transmitted and reconstitute the initial K bits (more details in the **demodulationQPSK function**).

Finally, I implemented all this process in the **simulate** function and I did a **Monte Carlo Simulation** in order to get the best value of BER.

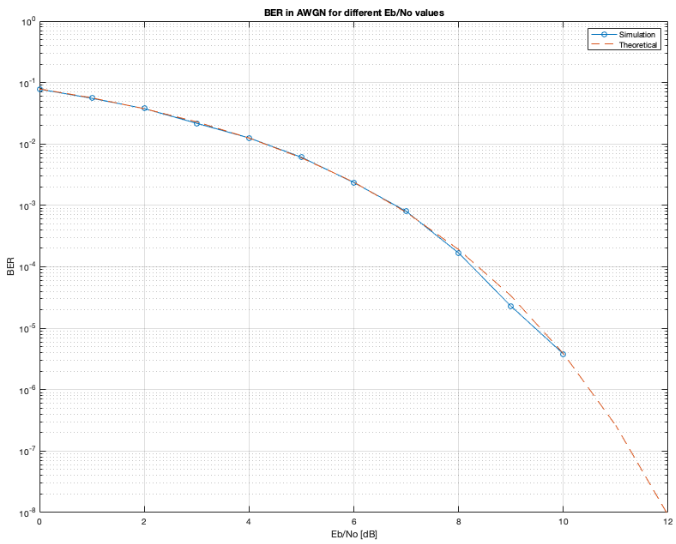

Figure 6: Practical BER in AWGN

c) Rayleigh fading occurs when there are multiple indirect paths between the transmitter and receiver and no distinct dominant path. Fast fading can be attributed to the phasor addition of the various multi-path signals. In this case, the amplitude and phase fluctuate significantly as the user moves.

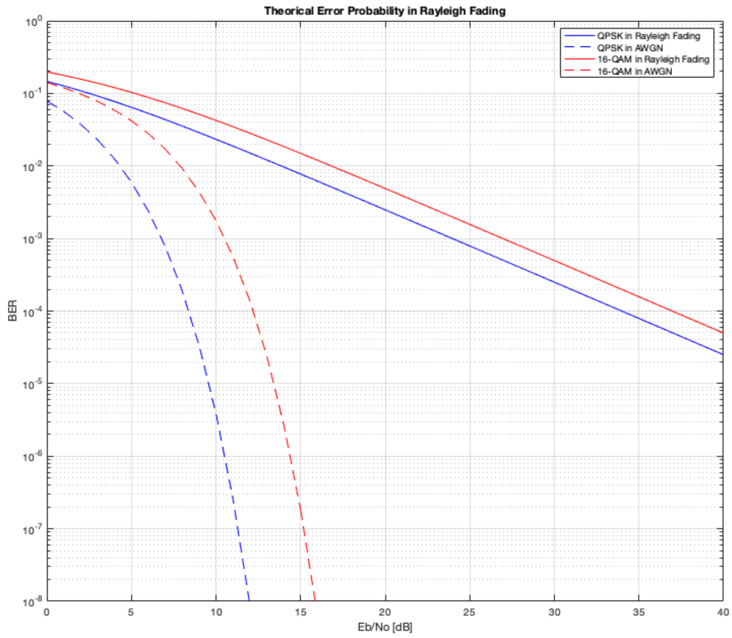

Figure 7: BER in Rayleigh fading

We can see that the theoretical error probability in a Rayleigh channel, requires higher Eb/No values to reach error-free region. For instance, Eb/No should be greater than 34dB to maintain an error probability of 1 in 10,000 in Rayleigh Fading for the QPSK. (Whereas Eb/No should be greater than 9dB to maintain an error probability of 1 in 10,000 in AWGN)

Rician fading occurs when one of the paths arrives with more power, a path with greater amplitude compared to the remaining paths. Whereas all multi-paths have approximately the same amplitude in Rayleigh Fading. Rayleigh fading can be considered like a special case of Rician fading for when there is no line of sight signal.

d) It's possible to improve the performance in Rayleigh Fading using diversity. The **Figure 8** shows that these fluctuations diminish into a more acceptable range when the order increases, in other words the effects of fading reduces.

The more the order of the diversity is, The better the theoretical error probability in Rayleigh Fading is.

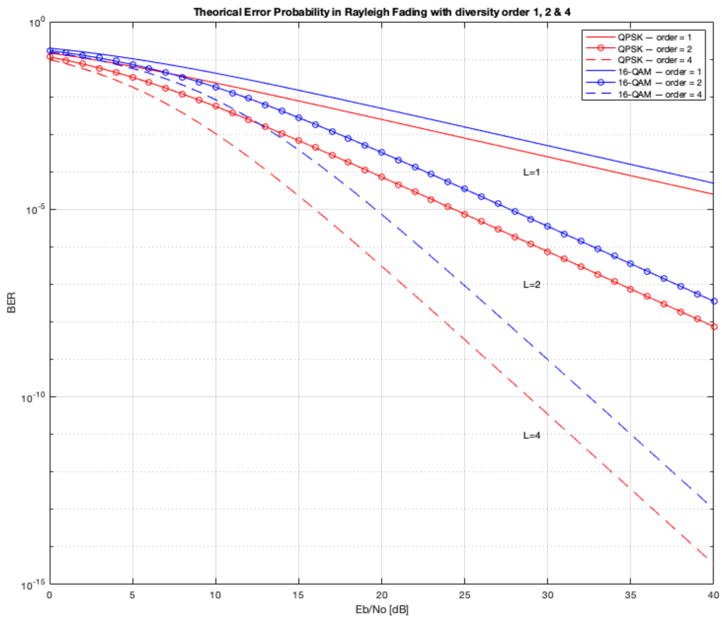

Figure 8: BER in Rayleigh fading with different diversity

Uncorrelated paths can be combined to improve the resulting signal to noise statistics. In ascending order of complexity, cost and performance, the strategies are:

-Switched (Scanning) Diversity
-Selection Diversity
-Equal Gain Combining
-Maximal Ratio Combining

Q2.2 Cellular planning

A digital micro-cellular system has 30MHz of available spectrum and operates with a cluster size of 7. Each channel requires 200 kHz of spectrum. The radio system uses TDMA with 8 calls per channel. Assume that each user represents a traffic load of 0.02 Erlangs. The cell radius is 160m, and the network comprises 40 base stations.

a) What is the channel bandwidth?

$$channel\ bandwidth = \frac{200}{8} = 25kHz$$

b) What is the maximum number of calls that can be supported in a cell?

$$calls\ per\ cell = \frac{30}{7 \times 0,200} \times 8 = 171.4$$

c) How many subscribers can we have per cell?

$$subscribers\ per\ cell = \frac{calls\ per\ cell}{traffic\ load} = 8,570$$

d) What is the maximum number of calls that can be supported in a cluster group?

Assuming a 200kHz call bandwidth, the 30MHz system supports a total of $\frac{30}{0,200} \times 8 = 1,200$ calls.

e) What is the cluster area?

These channels are used over a cluster area of $7 \times \pi \times 0,160^2 = 0.563km^2$

f) Determine the total capacity in terms of calls per MHz per kilometre square.

$$total\ capacity = \frac{1200}{30 \times cluster\ area} = 71.05calls/MHz/km^2$$

g) Determine the subscriber capacity in terms of subscribers per MHz per kilometre square.

$$subscriberscapacity = \frac{total\ capacity}{traffic\ load} = 3,552subscribers/MHz/km^2$$

h) What is the total subscriber capacity of the network?

$$total\ subscriber = 3552 \times 40 \times 0.0804 \times 30 = 342,697$$

i) What is the total coverage area?

$$subscribers\ per\ cell = number\ base\ stations \times area\ per\ cell = 1.608km^2$$

j) Why micro-cellular systems offer higher capacity?

Micro-cellular systems offer higher capacity because they offer a way to have a lot of subscriber in a small coverage area.

Q2.3 System design

a) With the aid of the graph given in the figure below, indicating the performance of various digital modulation schemes operating in an AWGN channel with an error rate of 10^{-5}, select suitable modulation schemes in order to meet the following criteria.

First, I computed the bandwidth efficiency required for each system using the definition of the bandwidth efficiency.

$$\eta_B = \frac{data\ rate}{bandwidth}$$

	System A	System B
Data Rate at BER $< 10^{-5}$	30Mbits/s	15Mbits/s
Channel Bandwidth	30MHz	7,5MHz
Eb/No	$< 10dB$	$< 16dB$
Bandwidth Efficiency	1 bit/s/Hz	2 bit/s/Hz

Table 1: Characteristics of the systems

The **BPSK** and **2-QAM** are both suitable modulation schemes of **the system A**. With the aid of the graph, We can see that they have both a bandwidth efficiency of 1 bit/s/Hz and a Eb/No lower than 10 dB.

The **4-PSK**, **4-DPSK** and **4-QAM** are suitable modulation schemes of **the system B**. With the aid of the graph, We can see that they have both a bandwidth efficiency of 2 bit/s/Hz and a Eb/No lower than 16 dB.

b) With the aid of the same graph explain why QPSK can offer a throughput enhancement compared to BPSK without either a bandwidth expansion or an increase in transmission power? In addition discuss why the noise performance of QPSK is superior to 8PSK and 16QAM is superior to 16PSK.

Increasing M, the bit rate and spectral efficiency increase. Hence, $\eta_{QPSK} > \eta_{BPSK}$ for a given Eb/No. That's why QPSK can offer a throughput enhancement compared to BPSK.

In addition, we know that the **QPSK** and **BPSK** have the same error probability. Therefore, the error probability of the QPSK is identical to the BPSK but for a double spectral efficiency.

QPSK (resp. 16QAM) performes better than 8PSK (resp. 16PSK) in gaussian noise because the distance between symbols in QPSK (resp. 16QAM) is larger compared to 8PSK (resp. 16PSK) as shown in the **Figure 9**. Hence, it's less sensible to noise.

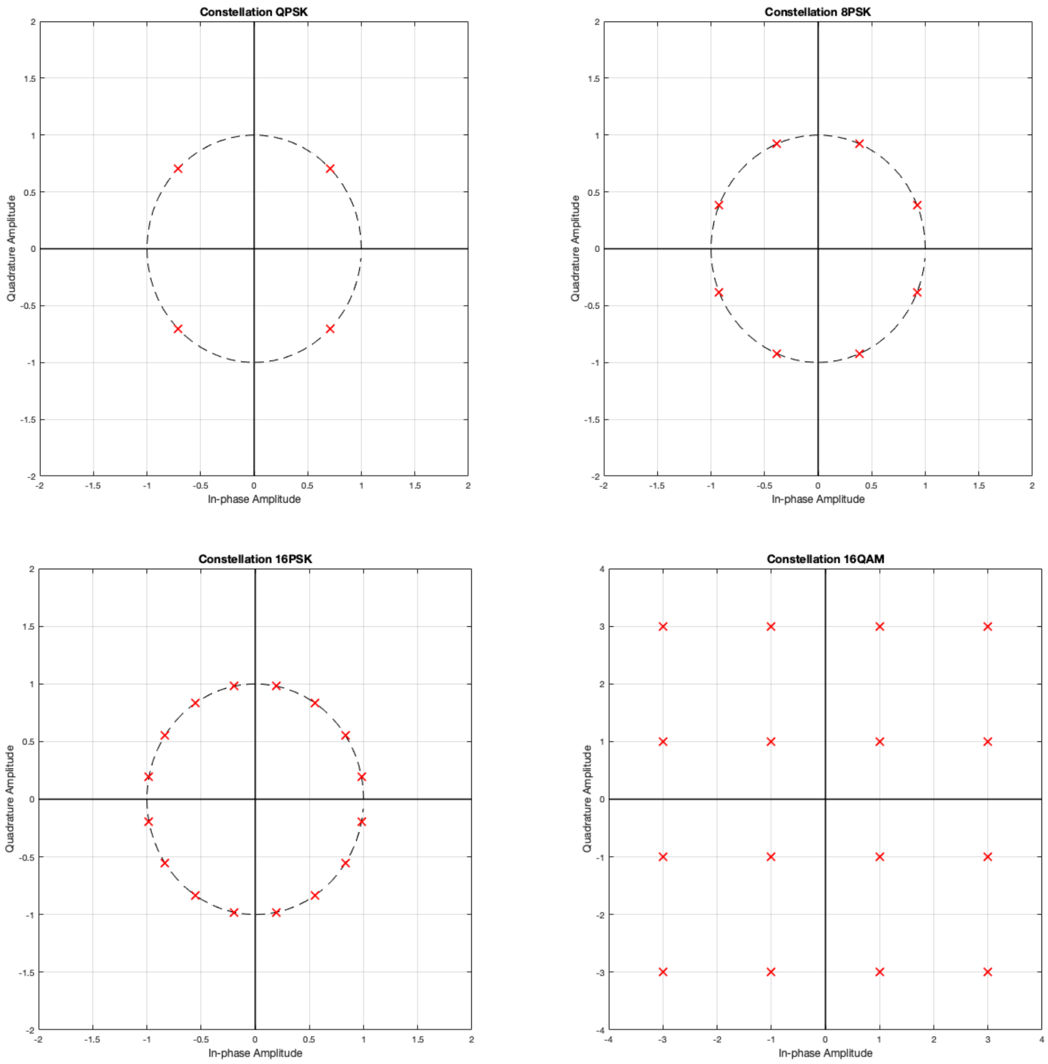

Figure 9: Constellations