Naturalness and the Status of Supersymmetry

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Abstract

For decades, the unnaturalness of the weak scale has been the dominant problem motivating new particle physics, and weak-scale supersymmetry has been the dominant proposed solution. This paradigm is now being challenged by a wealth of experimental data. In this review, we begin by recalling the theoretical motivations for weak-scale supersymmetry, including the gauge hierarchy problem, grand unification, and WIMP dark matter, and their implications for superpartner masses. These are set against the leading constraints on supersymmetry from collider searches, the Higgs boson mass, and low-energy constraints on flavor and CP violation. We then critically examine attempts to quantify naturalness in supersymmetry, stressing the many subjective choices that impact the results both quantitatively and qualitatively. Finally, we survey various proposals for natural supersymmetric models, including effective supersymmetry, focus point supersymmetry, compressed supersymmetry, and R-parity-violating supersymmetry, and summarize their key features, current status, and implications for future experiments.

Contents

I.   INTRODUCTION

Good physical theories are expected to provide *natural* explanations of experimental data and observations. Although physicists disagree about the definition of "natural," the idea that the criterion of naturalness exists and is a useful pointer to deeper levels of understanding has a long and storied history. In 1693, for example, when asked by clergyman Robert Bentley to explain how the law of universal gravitation was consistent with a static universe, Isaac Newton wrote [1]:

> That there should be a central particle, so accurately placed in the middle, as to be always equally attracted on all sides, and thereby continue without motion, seems to me a supposition fully as hard as to make the sharpest needle stand upright on its point upon a looking-glass.

Newton went on to conclude that this unnatural state of affairs could be taken as evidence for an infinite universe with initial conditions set by a divine power. Three hundred years later, the assumption of a static universe appears quaint, but we are no closer to a natural explanation of our accelerating universe than Newton was to his static one. More generally, the image of a needle balanced upright on a mirror remains the classic illustration of a possible, but unnatural, scenario that cries out for a more satisfactory explanation, and the notion of naturalness continues to play an important role in many areas of physics.

In particle physics today, the role of naturalness is nowhere more central than in the statement of the gauge hierarchy problem, the question of why the weak scale $m_{\mathrm{weak}} \sim 0.1 - 1$ TeV is so far below the (reduced) Planck scale $M_{\mathrm{Pl}} = \sqrt{\hbar c/(8\pi G_N)} \simeq 2.4 \times 10^{18}$ GeV. For many years, this has been the dominant problem motivating proposals for new particles and interactions. Chief among these is supersymmetry, which solves the gauge hierarchy problem if there are supersymmetric partners of the known particles with masses not far above the weak scale. This has motivated searches for superpartners at colliders, in low-energy experiments, and through astrophysical observations. So far, however, no compelling evidence for weak-scale supersymmetry has been found, and recent null results from searches at the Large Hadron Collider (LHC) have disappointed those who find supersymmetry too beautiful to be wrong and led its critics to declare supersymmetry dead.

In this article, we review the status of weak-scale supersymmetry at this brief moment in time when a Higgs-like particle has been discovered at the 8 TeV LHC, and the LHC has entered a two-year shutdown period before turning on again at its full center-of-mass energy. The field of supersymmetry phenomenology is vast, and we will necessarily review only a small subset of its many interesting aspects. As we will see, however, weak-scale supersymmetry is neither ravishingly beautiful (and hasn't been for decades), not is it excluded by any means; the truth lies somewhere in between. The goal of this review is to understand the extent to which naturalness and experimental data are currently in tension and explore models that resolve this tension and their implications for future searches.

We begin in Sec. II with a brief discussion of some of the longstanding theoretical motivations for weak-scale supersymmetry and their implications for superpartner masses. We then discuss some of the leading experimental constraints on weak-scale supersymmetry in Sec. III. In Sec. IV, we critically review attempts to quantify naturalness. Naturalness is a highly contentious subject with many different approaches leading to disparate conclusions. We will highlight some of the assumptions that are often implicit in discussions of naturalness and discuss the various subjective choices that impact the conclusions, both qualitatively and quantitatively.

	Effective SUSY	Focus Point SUSY	Compressed SUSY	R_p-Violating SUSY
Naturalness	√	√	√	√
Grand Unification	√	√	√	√
WIMP Dark Matter	√	√	√	
LHC Null Results	√	√	√	√
Higgs Mass		√		
Flavor/CP Constraints	√	√		

TABLE I: Some of the supersymmetric models discussed in this review, the virtues they are intended to preserve, and the constraints they are designed to satisfy, with varying degrees of success. For the rationale behind the check marks, see Secs. II, III, and V for discussions of the virtues, constraints, and models, respectively.

With all of these considerations in hand, we then turn in Sec. V to an overview of model frameworks that have been proposed to reconcile naturalness with current experimental constraints, summarizing their key features, current status, and implications for future searches. As a rough guide to the discussion, these models and the problems they attempt to address are shown in Table I. We conclude in Sec. VI.

II. THEORETICAL MOTIVATIONS

To review the status of supersymmetry, we should begin by recalling the problems it was meant to address. Supersymmetry [2–4] has beautiful mathematical features that are independent of the scale of supersymmetry breaking. In addition, however, there are three phenomenological considerations that have traditionally been taken as motivations for weak-scale supersymmetry: the gauge hierarchy problem, grand unification, and WIMP dark matter. In this section, we review these and their implications for superpartner masses.

A. The Gauge Hierarchy Problem

1. The Basic Idea

The gauge hierarchy problem of the standard model [5–7] and its possible resolution through weak-scale supersymmetry [8–11] are well-known. (For reviews and discussion, see, e.g., Refs. [12–16].) The standard model includes a fundamental, weakly-coupled, spin-0 particle, the Higgs boson. Its bare mass receives large quantum corrections. For example,

given a Dirac fermion f that receives its mass from the Higgs boson, the Higgs mass is

$$m_h^2 \approx m_{h\,0}^2 - \frac{\lambda_f^2}{8\pi^2} N_c^f \int^\Lambda \frac{d^4p}{p^2} \approx m_{h\,0}^2 + \frac{\lambda_f^2}{8\pi^2} N_c^f \Lambda^2 \;, \tag{1}$$

where $m_h \approx 125$ GeV is the physical Higgs boson mass [17, 18], $m_{h\,0}$ is the bare Higgs mass, and the remaining term is $m_{h\,\text{1-loop}}^2$, the 1-loop correction. The parameters λ_f and N_c^f are the Yukawa coupling and number of colors of fermion f, Λ is the largest energy scale for which the standard model is valid, and subleading terms have been neglected. For large Λ, the bare mass and the 1-loop correction must cancel to a large degree to yield the physical Higgs mass. Attempts to define naturalness quantitatively will be discussed in detail in Sec. IV, but at this stage, a simple measure of naturalness may be taken to be

$$\mathcal{N}^0 \equiv \frac{m_{h\,\text{1-loop}}^2}{m_h^2} \;. \tag{2}$$

For $\Lambda \sim M_{\text{Pl}}$ and the top quark with $\lambda_t \simeq 1$, Eq. (1) implies $\mathcal{N}^0 \sim 10^{30}$, *i.e.*, a fine-tuning of 1 part in 10^{30}.

Supersymmetry moderates this fine-tuning. If supersymmetry is exact, the Higgs mass receives no perturbative corrections. With supersymmetry breaking, the Higgs mass becomes

$$m_h^2 \approx m_{h\,0}^2 + \frac{\lambda_f^2}{8\pi^2} N_c^f \left(m_{\tilde f}^2 - m_f^2 \right) \ln \left(\Lambda^2 / m_{\tilde f}^2 \right) \;, \tag{3}$$

where $\tilde f$ is the superpartner of fermion f. The quadratic dependence on Λ is reduced to a logarithmic one, and even for $\Lambda \sim M_{\text{Pl}}$, the large logarithm is canceled by the loop suppression factor $1/(8\pi^2)$, and the Higgs mass is natural, provided $m_{\tilde f}$ is not too far above m_h. Requiring a maximal fine-tuning $\mathcal{N}_{\text{max}}^0$, the upper bound on sfermion masses is

$$m_{\tilde f} \lesssim 800 \text{ GeV} \, \frac{1.0}{\lambda_f} \left[\frac{3}{N_c^f} \right]^{\frac{1}{2}} \left[\frac{70}{\ln(\Lambda^2/m_{\tilde f}^2)} \right]^{\frac{1}{2}} \left[\frac{\mathcal{N}_{\text{max}}^0}{100} \right]^{\frac{1}{2}} \;, \tag{4}$$

where λ_f and N_c^f have been normalized to their top quark values, the logarithm has been normalized to its value for $\Lambda \sim M_{\text{Pl}}$, and $\mathcal{N}_{\text{max}}^0$ has been normalized to 100, or 1% fine-tuning.

2. First Implications

Even given this quick and simple analysis, Eq. (4) already has interesting implications:

- Naturalness constraints vary greatly for different superpartners. As noted as early as 1985 [19], the 1-loop contributions of first and second generation particles to the Higgs mass are suppressed by small Yukawa couplings. For the first generation sfermions, naturalness requires only that they be below 10^4 TeV! In fact, this upper bound is strengthened to ~ 4 TeV $-$ 10 TeV by considerations of D-term and 2-loop effects, as discussed in Sec. IV C. Nevertheless, it remains true that *without additional theoretical assumptions, there is no naturalness reason to expect first and second generation squarks and sleptons to be within reach of the LHC.*

- Naturalness bounds on superpartner masses are only challenged by LHC constraints for large Λ, such as $\Lambda \sim M_{\rm Pl}$ or $\Lambda \sim m_{\rm GUT} \simeq 2 \times 10^{16}$ GeV, the grand unified theory (GUT) scale. For low Λ, the loop suppression factor is not compensated by a large logarithm, and naturalness constraints are weakened by as much as an order of magnitude. For example, even for top squarks, for low Λ such that $\ln(\Lambda^2/m_{\tilde{f}}^2) \sim 1$, the naturalness bound for $\mathcal{N}_{\max}^0 = 1$ becomes $m_{\tilde{t}} \lesssim 700$ GeV, beyond current LHC bounds, and for $\mathcal{N}_{\max}^0 = 100$, the bound is $m_{\tilde{t}} \lesssim 7$ TeV, far above even the reach of the 14 TeV LHC. This is as expected for a 1-loop effect. The heuristic expectation that $\mathcal{O}(1)$ fine-tuning requires $m_{\tilde{t}} \sim m_h$ assumes implicitly that the 1-loop suppression factor is compensated by a large logarithm.

  Of course, supersymmetry makes it possible to contemplate a perturbative theory all the way up to $m_{\rm GUT}$ or $M_{\rm Pl}$, and grand unification, radiative electroweak symmetry breaking, and other key virtues of supersymmetry make this highly motivated. There are therefore strong reasons to consider $\Lambda \sim m_{\rm GUT}, M_{\rm Pl}$. This observation, however, suggests that if the enhancement from large logarithms may somehow be removed, supersymmetric theories with multi-TeV superpartners may nevertheless be considered natural; this is the strategy of models that will be discussed in Sec. V B.

- Last, all naturalness bounds depend on what level of fine-tuning is deemed acceptable, with mass bounds scaling as $\sqrt{\mathcal{N}_{\max}^0}$. This is an irreducible subjectivity that must be acknowledged in all discussions of naturalness.

  There is a sociological observation perhaps worth making here, however. In the past, in the absence of data, it was customary for some theorists to ask "What regions of parameter space are most natural?" and demand fine-tuning of, say, less than 10% ($\mathcal{N}_{\max}^0 = 10$). This requirement led to the promotion of models with very light superpartners, and heightened expectations that supersymmetry would be discovered as soon as the LHC began operation.

  In retrospect, however, this history has over-emphasized light supersymmetry models and has little bearing on the question of whether weak-scale supersymmetry is still tenable or not. As with all questions of this sort, physicists vote with their feet. Rather than asking "What regions of parameter space are most natural?", a more telling question is, "If superpartners were discovered, what level of fine-tuning would be sufficient to convince you that the gauge hierarchy problem is solved by supersymmetry and you should move on to researching other problems?" An informal survey of responses to this question suggests that values of $\mathcal{N}_{\max}^0 = 100, 1000$, or even higher would be acceptable. From this perspective, the normalization of $\mathcal{N}^0$ in Eq. (4) is reasonable, and current bounds from the LHC do not yet preclude a supersymmetric solution to the hierarchy problem, especially given the many caveats associated with attempts to quantify naturalness, which will be discussed in Sec. IV.

B. Grand Unification

The fact that the standard model particle content fits neatly into multiplets of larger gauge groups, such as SU(5), SO(10), or $\rm E_6$, is striking evidence for GUTs [20–23]. In the standard model, the strong, weak, and electromagnetic gauge couplings do not unify at any scale. However, in the minimal supersymmetric standard model (MSSM), the supersymmetric extension of the standard model with minimal field content, the gauge coupling renor-

malization group equations (RGEs) are modified above the superpartner mass scale. With this modification, if the superpartners are roughly at the weak scale, the gauge couplings meet at $m_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, further motivating both grand unification and weak-scale supersymmetry [24–28].

Gauge coupling unification is sensitive to the superpartner mass scale, since this governs when the RGEs switch from non-supersymmetric to supersymmetric. However, the sensitivity to the superpartner mass scale is only logarithmic. Furthermore, full SU(5) multiplets of superpartners, such as complete generations of squarks and sleptons, may be heavy without ruining gauge coupling unification. Note, however, that the MSSM particles do not completely fill SU(5) multiplets; in particular, the Higgs bosons must be supplemented with Higgs triplets. One might therefore hope that the masses of SU(2) doublet Higgsinos might be stringently constrained by gauge coupling unification, but even this is not the case. A full justification requires a complete discussion of GUTs and proton decay [29, 30], but roughly speaking, heavy sfermions suppress the leading contributions to proton decay, and there is sufficient freedom in threshold corrections from the GUT-scale spectrum to allow unification even for relatively heavy Higgsinos; see, *e.g.*, Refs. [31, 32].

In summary, grand unification is a significant motivation for supersymmetry, but gauge coupling unification is a blunt tool when it comes to constraining the superpartner mass scale. Note, however, that the relations imposed by grand unification may have a strong impact on naturalness bounds, either weakening or strengthening them significantly; see Sec. IV C.

C.   Dark Matter

Supersymmetry provides an excellent WIMP dark matter candidate when the neutralino is the lightest supersymmetric particle (LSP) [33, 34]. Neutralinos naturally freeze out with approximately the correct thermal relic density. This density is inversely proportional to the thermally-averaged annihilation cross section, which, on dimensional grounds, is inversely proportional to the superpartner mass scale squared:

$$\Omega_\chi \propto \frac{1}{\langle \sigma v \rangle} \propto \tilde{m}^2 \; . \tag{5}$$

The requirement $\Omega_\chi \leq 0.23$ therefore places an upper bound on the superpartner mass scale $\tilde{m}$.

Unfortunately, when the constants of proportionality are included, the upper bounds for some types of neutralinos are far above current LHC sensitivities. For example, for mixed Bino-Higgsino [35–37] and pure Wino-like [38] neutralino dark matter, the upper bounds are

$$\begin{aligned} m_{\tilde{B}-\tilde{H}} &< 1.0 \text{ TeV} \\ m_{\tilde{W}} &< 2.7 - 3.0 \text{ TeV} \; . \end{aligned} \tag{6}$$

Such neutralinos may be produced in the cascade decays of squarks and gluinos, but this is model-dependent. The model-independent search strategy is to consider Drell-Yan production of neutralino pairs with a radiated jet or photon, which contributes to mono-jet and mono-photon searches [39–43]. The limits in Eq. (6) are far above current LHC sensitivities [44, 45]. The spin-independent and spin-dependent scattering cross sections of such neutralinos are also consistent with current bounds from direct search experiments [46].

Of course, dark matter may be composed of other particles, such as axions, sterile neutrinos, hidden sector dark matter, or gravitinos [47]. There is no requirement that superpartners be light in these dark matter scenarios. In fact, some scenarios in which gravitinos are the dark matter provide motivation for extremely heavy superpartners, which freeze out with $\Omega \gg 0.23$, but then decay to gravitinos with $\Omega_{\tilde{G}} \simeq 0.23$ [48].

In summary, the requirement of WIMP dark matter provides upper bounds on superpartner masses, but these upper bounds are high and far beyond the reach of current colliders. In addition, the dark matter doesn't have to be made of supersymmetric WIMPs. As with the case of grand unification, the possibility of WIMP dark matter is a significant virtue of weak-scale supersymmetry, but it does not provide stringent upper bounds on superpartner masses.

III. EXPERIMENTAL CONSTRAINTS

A. Superpartner Searches at Colliders

The search for weak-scale supersymmetry has been ongoing for decades at many colliders. Before the 2013-14 shutdown, however, the LHC experiments ATLAS and CMS each collected luminosities of more than 5 fb^{-1} at $\sqrt{s} = 7$ TeV and 20 fb^{-1} at $\sqrt{s} = 8$ TeV, and the resulting LHC limits supersede previous collider constraints in almost all scenarios. We will therefore confine the discussion to LHC results and focus on a small subset that is particularly relevant for the following discussion. For a summary of pre-LHC constraints, see Ref. [49], and for the full list of LHC analyses, see Refs. [50, 51].

1. Gluinos and Squarks

The greatest mass reach at the LHC is for strongly-interacting particles, such as gluinos and squarks, which are produced through $pp \to \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q}$. The limits depend, of course, on the decays. Limits in the $(m_{\tilde{g}}, m_{\tilde{q}})$ plane, assuming the decays $\tilde{g} \to q\bar{q}\chi$ and $\tilde{q} \to q\chi$, leading to jets $+ \not{E}_T$, are shown in Fig. 1. The results imply $m_{\tilde{q}} \gtrsim 1.3$ TeV for decoupled gluinos, $m_{\tilde{g}} \gtrsim 1.2$ TeV for decoupled squarks, and $m_{\tilde{g}} = m_{\tilde{q}} \gtrsim 1.5$ TeV in the degenerate case. Note, however, that the squarks appearing in this analysis are squarks of the first two generations. The lightest neutralino is assumed massless, but top and bottom squarks, as well as all other superpartners, are assumed heavy and decoupled.

The resulting bounds may also be applied to the framework of minimal supergravity (mSUGRA), also know as the constrained MSSM. This framework has 4 continuous parameters defined at the GUT scale (a unified scalar mass m_0, a unified gaugino mass $M_{1/2}$, a unified tri-linear scalar coupling A_0, the ratio of Higgs boson vacuum expectation values $\tan \beta \equiv \langle H_0^u \rangle / \langle H_0^d \rangle$), and one discrete choice, the sign of the Higgsino mass parameter μ. Constraints in this model parameter space are shown in Fig. 1. In the limit of heavy sfermions (large m_0), the jets $+ \not{E}_T$ search implies $m_{\tilde{g}} \gtrsim 1.0$ TeV.
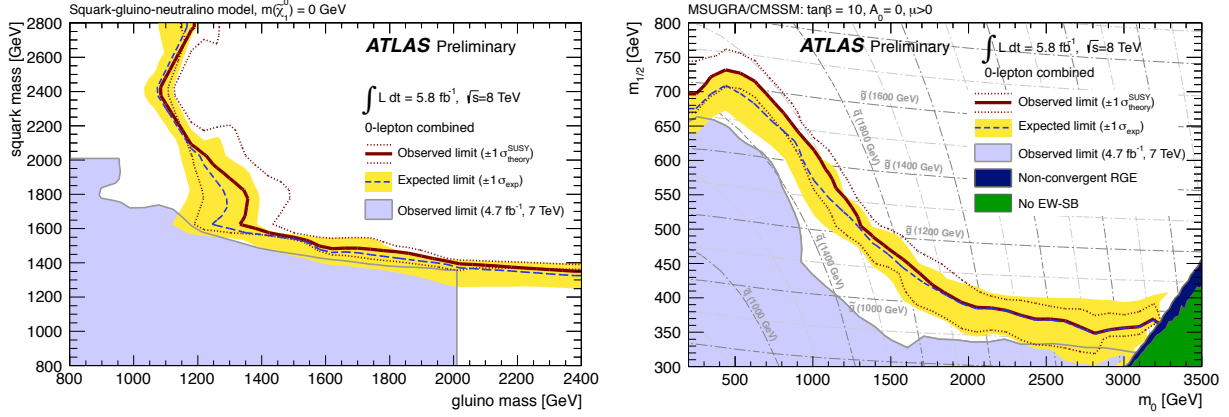
FIG. 1: Constraints on gluinos and first and second generation squarks from ATLAS at the LHC with $L = 5.8$ fb^{-1} and $\sqrt{s} = 8$ TeV [52]. Left: Limits in the $(m_{\tilde{g}}, m_{\tilde{q}})$ plane from $pp \to \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q}$ followed by $\tilde{g} \to q\bar{q}\chi$ and $\tilde{q} \to q\chi$, leading to jets$+\not{E}_T$. The analysis assumes $m_{\tilde{q}} \equiv m_{\tilde{u}_{L,R}} = m_{\tilde{d}_{L,R}} = m_{\tilde{s}_{L,R}} = m_{\tilde{c}_{L,R}}$, $m_\chi = 0$, and that all other superpartners, including the top and bottom squarks, are very heavy. The shaded region boundaries at $m_{\tilde{g}}, m_{\tilde{q}} = 2$ TeV are artifacts of the previous 7 TeV analysis. Right: Limits from the jets $+\not{E}_T$ search in the $(m_0, M_{1/2})$ plane of mSUGRA, with $\tan\beta = 10$, $A_0 = 0$, and $\mu > 0$.

2. Top and Bottom Squarks

The constraints of Fig. 1 might appear to require all superpartners to be above the TeV scale. As noted in Sec. II A, however, naturalness most stringently constrains the top and bottom squarks, but allows effectively decoupled first and second generation squarks, exactly the opposite of the assumptions made in deriving these bounds. It is therefore important to consider other analyses, including searches for light top and bottom squarks. Results from such searches are shown in Fig. 2. Limits from direct stop pair production followed by $\tilde{t} \to t\chi$ are shown, as are limits from gluino pair production followed by $\tilde{g} \to \tilde{t}^*\bar{t} \to t\bar{t}\chi$, which is the dominant decay mode if stops are significantly lighter than all other squarks. In the case of stop pair production, we see that stops as light as 500 GeV are allowed for massless neutralinos, and much lighter stops are allowed if one approaches the kinematic boundary $m_{\tilde{t}} - m_\chi = m_t$. In the case of gluino pair production, the bound is $m_{\tilde{g}} \gtrsim 1.1$ TeV for $m_\chi = 0$, but again, there are allowed regions with much lighter gluinos near the kinematic boundary $m_{\tilde{g}} - m_\chi = 2m_t$. Searches for light stops in other channels, as well as searches for bottom squarks, yield roughly similar constraints [50, 51].

3. R-Parity Violation

The search results presented so far require missing transverse energy. Although WIMP dark matter motivates this possibility, large $\not{E}_T$ is far from a requirement of supersymmetry, and $\not{E}_T$ signals may be degraded in a number of ways, for example, by compressed superpartner spectra, a possibility discussed in Sec. V C.

Perhaps the most dramatic way is with R-parity (R_p) violation. When the standard model is extended to include supersymmetry, there are many new gauge-invariant, renormalizable interactions. If any one of these is present, all superpartners decay, effectively eliminating
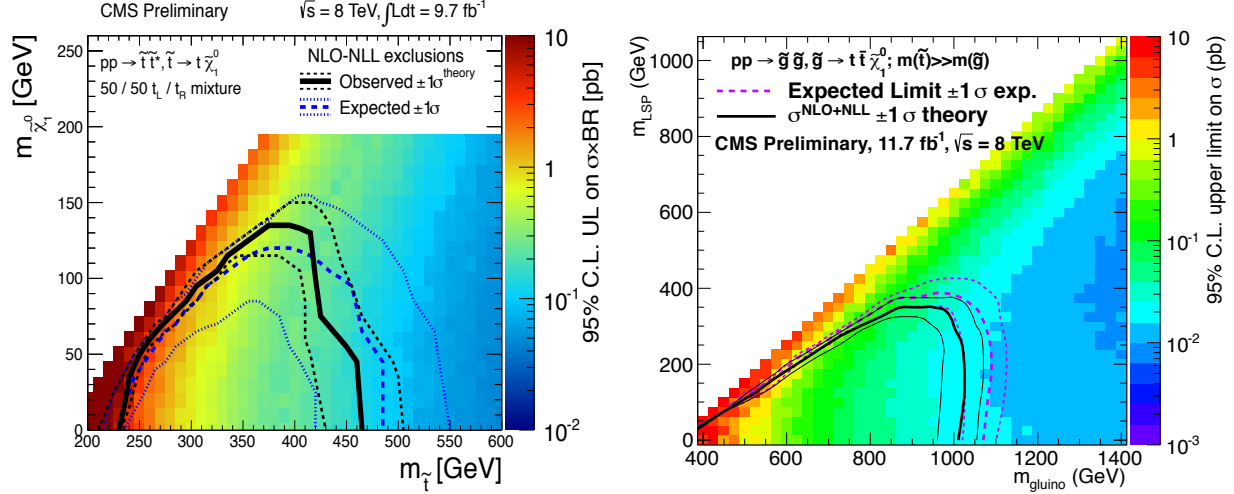
FIG. 2: Constraints on gluinos and top squarks from CMS at the LHC. Left: Limits from $L = 9.7$ fb^{-1} at $\sqrt{s} = 8$ TeV in the $(m_{\tilde{t}}, m_\chi)$ plane from $pp \to \tilde{t}\tilde{t}^*$, followed by $\tilde{t} \to t\chi$, leading to the signature of $l + b$-jet $+ \not{E}_T$ [53]. Right: Limits from $L = 11.7$ fb^{-1} at $\sqrt{s} = 8$ TeV in the $(m_{\tilde{g}}, m_\chi)$ plane from $pp \to \tilde{g}\tilde{g}$, followed by $\tilde{g} \to t\bar{t}\chi$, leading to signatures of N_j jets $+ N_b$ b-jets $+ \not{E}_T$, where $2 \leq N_j \leq 3$ or $N_j \geq 4$, and $N_b = 0, 1, 2, 3$, or ≥ 4 [54].

the $\not{E}_T$ signature. These R_p-violating (RPV) terms arise from superpotentials of the form

$$W_{\not{R}_p} = \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \lambda''_{ijk} U_i D_j D_k + \mu_i L_i H_u \ , \tag{7}$$

where the first three types of terms are categorized as leptonic, semi-leptonic, and hadronic, and $i, j, k = 1, 2, 3$ are generation indices. If any of these couplings is non-zero, all gauginos may decay to three standard model fermions.

The most difficult case for the LHC is hadronic RPV. In Fig. 3, we show results from $pp \to \tilde{g}\tilde{g}$ followed by the RPV decay $\tilde{g} \to qqq$ through a λ'' operator [55]. The resulting bound on the gluino mass is 670 GeV, far weaker than in cases where the gluino cascade decay includes significant $\not{E}_T$.

4. Sleptons, Charginos, and Neutralinos

Finally, the mass reach for superpartner searches is, of course, greatly reduced for un-colored superpartners. In Fig. 4, we show constraints from CMS on Drell-Yan slepton pair production and associated chargino-neutralino pair production [56]. The limits are impressive, as they extend LEP bounds of $\tilde{m} \gtrsim 100$ GeV to much higher masses, requiring $m_{\tilde{e}}, m_{\tilde{\mu}} \gtrsim 275$ GeV and $m_{\chi_1^\pm} = m_{\chi_2^0} \gtrsim 330$ GeV for $m_\chi = 0$. Note, however, that these limits do not apply to staus, they degrade significantly for larger m_χ and more degenerate spectra, and they bound superpartner masses that are not highly constrained by naturalness in the absence of additional theoretical assumptions.
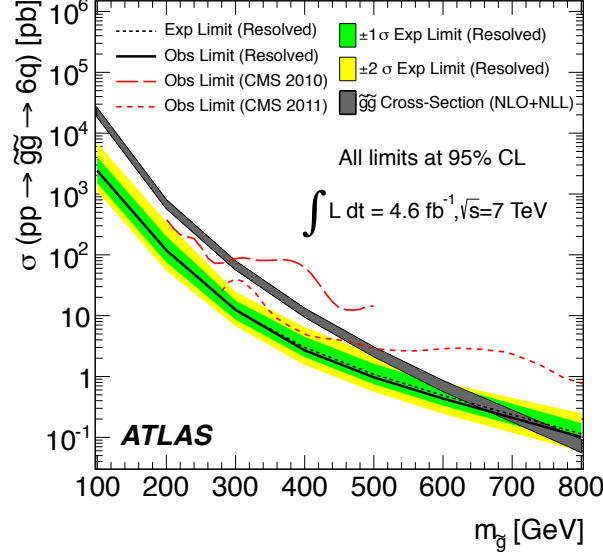
FIG. 3: Constraint on gluinos in supersymmetry with R_p violation from ATLAS at the LHC with $L = 4.6$ fb^{-1} and $\sqrt{s} = 7$ TeV [55]. The constraint is on gluino pair production $pp \to \tilde{g}\tilde{g}$ followed by the hadronic RPV decay $\tilde{g} \to qqq$, leading to 6 jets with no $\not{E}_T$, and implies $m_{\tilde{g}} > 670$ GeV.

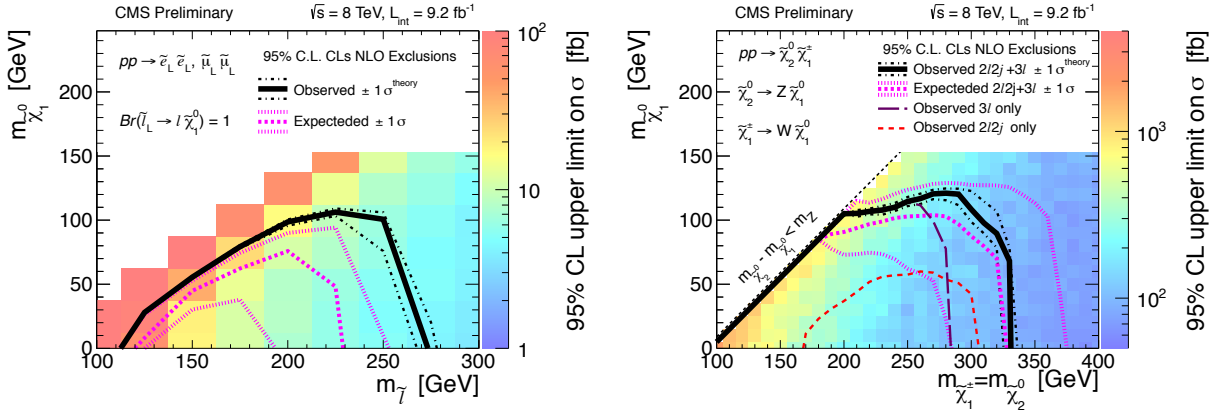

FIG. 4: Constraints on sleptons and electroweak gauginos from Drell-Yan production at CMS at the LHC with $L = 9.2$ fb^{-1} and $\sqrt{s} = 8$ TeV [56]. Left: Limits in the $(m_{\tilde{l}}, m_\chi)$ plane from $pp \to \tilde{l}_L \tilde{l}_L^*$, where $l = e, \mu$, followed by $\tilde{l}_L \to l\chi$, leading to $2l + \not{E}_T$ events. Right: Limits in the $(m_{\chi_1^\pm} = m_{\chi_2^0}, m_\chi)$ plane from $pp \to \chi_1^\pm \chi_2^0$, followed by $\chi_1^\pm \to W\chi$ and $\chi_2^0 \to Z\chi$, leading to $2j\, 2l + \not{E}_T$ and $3l + \not{E}_T$ events.

B. The Higgs Boson

The Higgs boson, or at least an eerily similar particle, has been discovered at the LHC [17, 18]. Constraints on the Higgs boson mass and the signal strength in the $h \to \gamma\gamma$ and $h \to ZZ^* \to 4l$ channels are shown in Fig. 5. Early hints of slight inconsistencies between the mass measurements and signal strengths in various channels have now largely disappeared. At ATLAS, the $\gamma\gamma$ mass is slightly larger than the ZZ^* mass, and both signal strengths are slightly above SM expectations. None of these discrepancies is significant, however, and the
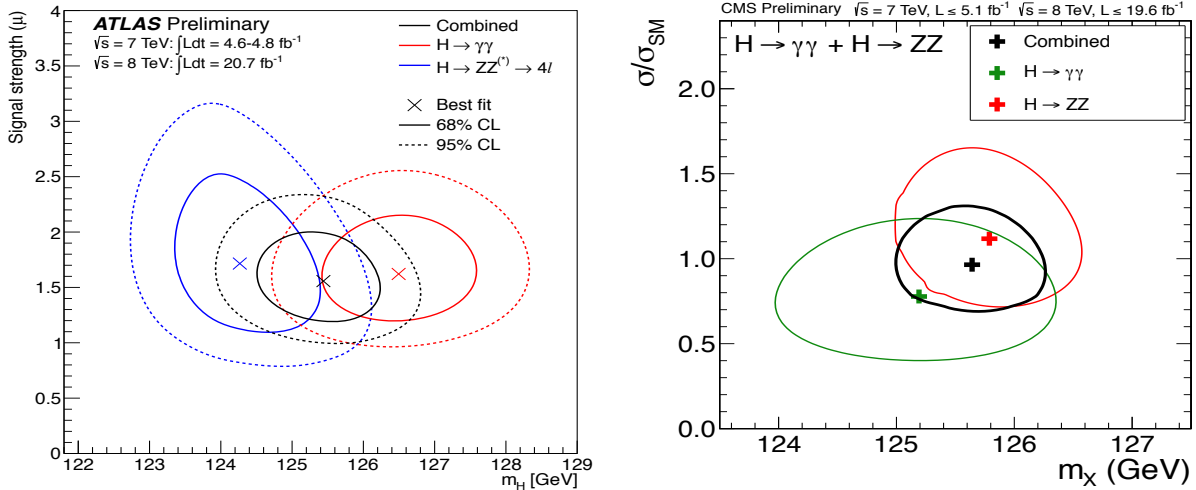
FIG. 5: Constraints in the $(m_h, \sigma/\sigma_{\rm SM})$ plane for the $h \to \gamma\gamma$ and $h \to ZZ^* \to 4l$ channels from ATLAS [57] (left) and CMS [58] (68% CL) (right).

results of the two experiments are also quite consistent, as evident in Fig. 5.

At present, the most pressing concern for supersymmetry is simply the Higgs boson mass. In the MSSM the Higgs boson is generically light, since the quartic coupling in the scalar potential is determined by the electroweak gauge couplings. Indeed, the tree level value $m_h(\text{tree}) = m_Z|\cos 2\beta|$ cannot exceed the Z boson mass. However, the Higgs mass may be raised significantly by radiative corrections [59–61]. For moderate to large $\tan\beta$, a 2-loop expression for the Higgs mass is [62, 63]

$$
\begin{aligned}
m_h^2 &\approx m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{2\pi^2 v^2} \left\{ \ln\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right) \right. \\
&\quad + \left. \frac{1}{16\pi^2}\left(\frac{3m_t^2}{v^2} - 32\pi\alpha_s\right)\left[\frac{2X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\ln\frac{M_S^2}{m_t^2} + \left(\ln\frac{M_S}{m_t^2}\right)^2\right] \right\},
\end{aligned}
\tag{8}
$$

where $v \simeq 246$ GeV, $M_S \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, $X_t \equiv A_t - \mu\cot\beta$ parameterizes the stop left-right mixing, and $\alpha_s \approx 0.12$. Several codes incorporate 2-loop [64–66], or even 3-loop [67, 68], corrections.

Equation (8) has several interesting features. First, increasing $\tan\beta$ increases the tree-level Higgs mass; this effect saturates for $\tan\beta \gtrsim 13$, where the tree-level mass is within 1 GeV of its maximum. Second, the Higgs boson mass may be greatly increased either by large stop mixing ($X_t \approx \pm\sqrt{6}M_S$) or by heavy stops ($M_S \gg m_t$). Numerical results are shown in Fig. 6. For negligible stop mixing, stop masses $M_S \gtrsim 4$ TeV are required to give a consistent Higgs boson mass. For near-maximal mixing, sub-TeV values of M_S are possible, but such large mixing is highly fine-tuned with respect to the A_t parameter [70]. The generic lesson to draw is that the measured Higgs mass favors stop masses well above a TeV.

At present, the Higgs mass measurement is at least as significant a challenge to naturalness as the absence of superpartners at the LHC. First, the Higgs mass can only be raised to ~ 125 GeV by raising the masses of superpartners that couple strongly to the Higgs. But it is exactly these particles that, at least at first sight, must be light to preserve naturalness. Second, because the Higgs mass is only logarithmically sensitive to the top squark mass,
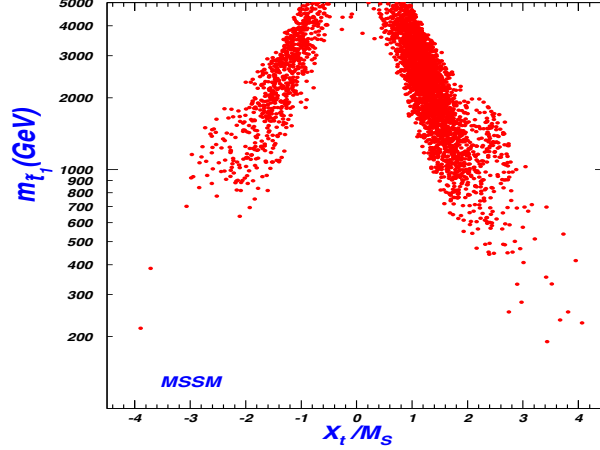
FIG. 6: Values of top squark parameters that give 123 GeV $< m_h <$ 127 GeV in viable MSSM models [69]. The parameters are $m_{\tilde{t}_1}$, the mass of the lighter stop, and X_t/M_S, where $X_t \equiv A_t - \mu \cot \beta$ parameterizes left-right stop mixing and $M_S \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ is the geometric mean of the physical stop masses.

it has tremendous reach, favoring, in the no-mixing case, stop masses that are far above current LHC bounds and even challenging for all proposed future colliders.

C. Flavor and CP Violation

Bounds on low-energy flavor and CP violation stringently constrain all proposals for new physics at the weak scale. For supersymmetry, these longstanding constraints are extremely stringent and are *a priori* a strong challenge to naturalness. The constraints on supersymmetry may be divided into two qualitatively different classes.

1. Flavor-Violating Constraints

The first are those that require flavor violation. Supersymmetry breaking generates sfermion masses that generically violate both flavor and CP. For example, for the left-handed down-type squarks, the mass matrix m_{ij}^2, where $i, j = \tilde{d}_L, \tilde{s}_L, \tilde{b}_L$, generically has off-diagonal entries that mediate flavor violation and complex entries that violate CP. Flavor and CP violation may also arise from all of the other mass matrices, as well as from the supersymmetry-breaking A-terms.

The constraints from low-energy flavor violation have been analyzed in many works. In Ref. [71], for example, constraints are derived by requiring that the supersymmetric box diagram contributions to meson mass splittings not exceed their observed values, and the supersymmetric penguin diagram contributions to radiative decays $l_i \rightarrow l_j \gamma$ not exceed current bounds. A small sample of these results include

$$\left[\frac{12 \text{ TeV}}{m_{\tilde{q}}}\right]^2 \left| \text{Re} \left(\frac{m_{\tilde{d}\tilde{s}}^2}{m_{\tilde{q}}^2}\right)^2 \right| \lesssim \frac{\Delta m_K}{3.49 \times 10^{-12} \text{ MeV}} \tag{9}$$

13

$$\left[\frac{16 \text{ TeV}}{m_{\tilde{q}}}\right]^2 \left|\text{Re}\left(\frac{m_{\tilde{u}\tilde{c}}^2}{m_{\tilde{q}}^2}\right)^2\right| \lesssim \frac{\Delta m_D}{1.26 \times 10^{-11} \text{ MeV}} \tag{10}$$

$$\left[\frac{5.4 \text{ TeV}}{m_{\tilde{q}}}\right]^2 \left|\text{Re}\left(\frac{m_{\tilde{d}\tilde{b}}^2}{m_{\tilde{q}}^2}\right)^2\right| \lesssim \frac{\Delta m_B}{3.38 \times 10^{-10} \text{ MeV}} \tag{11}$$

$$\left[\frac{160 \text{ TeV}}{m_{\tilde{q}}}\right]^2 \left|\text{Im}\left(\frac{m_{\tilde{d}\tilde{s}}^2}{m_{\tilde{q}}^2}\right)^2\right| \lesssim \frac{\epsilon_K}{2.24 \times 10^{-3}} \tag{12}$$

$$\left[\frac{2.4 \text{ TeV}}{m_{\tilde{l}}}\right]^4 \left|\frac{m_{\tilde{e}\tilde{\mu}}^2}{m_{\tilde{l}}^2}\right|^2 \lesssim \frac{B(\mu \to e\gamma)}{2.4 \times 10^{-12}} \tag{13}$$

$$\left[\frac{150 \text{ GeV}}{m_{\tilde{l}}}\right]^4 \left|\frac{m_{\tilde{e}\tilde{\tau}}^2}{m_{\tilde{l}}^2}\right|^2 \lesssim \frac{B(\tau \to e\gamma)}{3.3 \times 10^{-8}} \tag{14}$$

$$\left[\frac{140 \text{ GeV}}{m_{\tilde{l}}}\right]^4 \left|\frac{m_{\tilde{\mu}\tilde{\tau}}^2}{m_{\tilde{l}}^2}\right|^2 \lesssim \frac{B(\tau \to \mu\gamma)}{4.4 \times 10^{-8}} \ , \tag{15}$$

where the constraints apply to both left- and right-handed fermions and arise from the indicated observables, which have been normalized to current values [72]. Here $m_{\tilde{q}}$ and $m_{\tilde{l}}$ are average masses of the relevant squark and slepton generations, and we have set $m_{\tilde{g}} = m_{\tilde{q}}$ and $m_{\tilde{\gamma}} = m_{\tilde{l}}$. For $\mathcal{O}(1)$ flavor violation, low-energy constraints require that the first and second generation sfermions have masses $\gtrsim 10$ TeV, and if there are additionally $\mathcal{O}(1)$ phases, the down-type squarks must have masses $\gtrsim 100$ TeV. In contrast, constraints from processes involving third generation squarks and sleptons are generally much less severe, and are typically satisfied for sub-TeV masses.

2. Electric Dipole Moments

The second class of constraints arises from flavor-conserving, but CP-violating, observables, namely the electric dipole moments (EDMs) of the electron and neutron. There are well-known frameworks, *e.g.*, gauge-mediated supersymmetry breaking [73–78] and anomaly-mediated supersymmetry breaking [79, 80], in which the sfermion mass matrices are essentially diagonal, and all of the flavor-violating observables discussed above may be suppressed. Even in these frameworks, however, the gaugino masses M_i, A-terms, and the μ and B parameters may have CP-violating phases, and these generate potentially dangerous contributions to the EDMs.

The EDMs of the electron and neutron are generated by penguin diagrams with gauginos, Higgsinos and sfermions in the loop. The dominant diagram involves Wino-Higgsino mixing. The electron EDM is d_e and the neutron EDM is, assuming the naive quark model, $d_n = (4d_d - d_u)/3$. The electron and down quark EDMs are particularly dangerous in supersymmetry, as they are enhanced for large $\tan\beta$, and have the form [81]

$$d_f \sim e \frac{g_2^2}{64\pi^2} m_f \frac{|\mu M_2|}{m_{\tilde{f}}^4} \tan\beta \sin\theta_{\text{CP}} \ , \tag{16}$$

where $f = e, d$, $m_{\tilde{f}}$ is the mass scale of the heaviest superpartners in the loop, which we take to be $\tilde{f}$, and $\theta_{\text{CP}} \equiv \text{Arg}(\mu M_a B^*)$ is the CP-violating phase. Given the $\tan\beta$-enhanced

EDMs, and setting $m_d = 5$ MeV, the EDM constraints are

$$\left(\frac{2.5 \text{ TeV}}{m_{\tilde{l}}}\right)^2 \frac{|\mu M_2|}{m_{\tilde{l}}^2} \frac{\tan \beta}{10} \frac{\sin \theta_{\text{CP}}}{0.1} \lesssim \frac{d_e}{1.05 \times 10^{-27} \ e\,\text{cm}} \qquad (17)$$

$$\left(\frac{1.7 \text{ TeV}}{m_{\tilde{q}}}\right)^2 \frac{|\mu M_2|}{m_{\tilde{q}}^2} \frac{\tan \beta}{10} \frac{\sin \theta_{\text{CP}}}{0.1} \lesssim \frac{d_n}{2.9 \times 10^{-26} \ e\,\text{cm}} \ , \qquad (18)$$

where the electron and neutron EDMs are normalized to their current bounds [72].

The EDM constraints are extremely robust. The CP-violating phase can be suppressed only by a mechanism that correlates the phases of the supersymmetry-breaking gaugino masses, B, and the supersymmetry-preserving μ parameter. In many frameworks, such as gauge-mediated supersymmetry breaking, it is already challenging to generate μ and B parameters of the correct magnitude, much less to correlate their phases with the gaugino masses, and, of course, some CP-violation is desirable to generate the matter–anti-matter asymmetry of the universe. Although CP-conserving mechanisms have been proposed [82–84], they are typically far from the simple and elegant ideas proposed to eliminate flavor violation. In the absence of such mechanisms, the EDM constraints require multi-TeV first generation superpartners to be consistent with $\mathcal{O}(0.1)$ phases.

D. Hints of New Physics

In addition to constraints excluding large effects from new physics beyond the standard model, there are also experimental signals that may be taken as indications for new physics. Chief among these is the anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, where the final measurement from the Brookhaven E821 experiment [85] disagrees with standard model predictions by 2.6σ to 3.6σ [86, 87]. If supersymmetry is to resolve this discrepancy, the mass of the lightest observable superpartner, either a chargino or a smuon, must satisfy [88]

$$m_{\text{LOSP}} < 480 \text{ GeV} \left[\frac{\tan \beta}{50}\right]^{\frac{1}{2}} \left[\frac{287 \times 10^{-11}}{\Delta a_\mu}\right]^{\frac{1}{2}} \ , \qquad (19)$$

where Δa_μ has been normalized to the current discrepancy.

The anomalous magnetic moment of the muon is not the only potential signal for new physics, however. For example, A_{FB}^b, the forward-backward asymmetry in $Z \to b\bar{b}$ deviates from the standard model prediction by 2.8σ [89], the Higgs signal strength in $\gamma\gamma$ is 1σ to 2σ too large, and the various Higgs mass measurements discussed above disagree with each other at the 1σ to 3σ level.

In this review, as tempting as it is be optimistic, we do not consider these results to be compelling evidence for new physics. Of course, if well-motivated supersymmetric models elegantly explain a tantalizing anomaly, that should be noted, but here we will take a more conservative view and will not require supersymmetry to resolve these tentative disagreements between experiment and the standard model.

IV. QUANTIFYING NATURALNESS

We now return to naturalness and discuss attempts to quantify it in more detail. All such attempts are subject to quantitative ambiguities. However, this fact should not ob-

scure the many qualitative differences that exist in naturalness prescriptions proposed in the literature. In this section, we begin by describing a standard prescription for quantifying naturalness. We then critically review some of the many alternative prescriptions that have been proposed, stressing the qualitative differences and their implications. After this lengthy discussion has highlighted the many caveats in any attempt to quantify naturalness, we present some naturalness bounds on superpartner masses that may serve as a rough guide as we turn to models in Sec. V.

A. A Naturalness Prescription

We begin by describing a general five-step prescription for assigning a numerical measure of naturalness to a given supersymmetric model. So that clarity is not lost in abstraction, we also show how it is typically applied to mSUGRA, as implemented in software programs, such as SoftSUSY [65].

- *Step 1: Choose a framework with input parameters P_i.* In mSUGRA, the input parameters are $\{P_i\} = \{m_0, M_{1/2}, A_0, \tan\beta, \text{sign}(\mu)\}$.
- *Step 2: Specify a model.* A model is specified by choosing values for the input parameters and using experimental data and RGEs to determine all the remaining parameters. One key constraint on the weak-scale parameters is the relation

$$m_Z^2 = 2\frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - 2\mu^2 \; , \tag{20}$$

  suitably improved to include subleading corrections.
- *Step 3: Choose a set of fundamental parameters a_i.* These parameters are independent and continuously variable; they are not necessarily the input parameters. In mSUGRA, a common choice is the GUT-scale parameters $\{a_i\} = \{m_0, M_{1/2}, A_0, B_0, \mu_0\}$.
- *Step 4: Calculate the sensitivity parameters $\mathcal{N}_i$.* These parameters are [90, 91]

$$\mathcal{N}_i \equiv \left|\frac{\partial \ln m_Z^2}{\partial \ln a_i^2}\right| = \left|\frac{a_i^2}{m_Z^2}\frac{\partial m_Z^2}{\partial a_i^2}\right| \; . \tag{21}$$

  They measure the sensitivity of the weak scale, represented by the Z mass, to variations in the fundamental parameters.
- *Step 5: Determine the overall measure of naturalness $\mathcal{N} \equiv max\{\mathcal{N}_i\}$.* In mSUGRA, the overall measure of naturalness is, then, $\mathcal{N} \equiv \max\{\mathcal{N}_{m_0}, \mathcal{N}_{M_{1/2}}, \mathcal{N}_{A_0}, \mathcal{N}_{B_0}, \mathcal{N}_{\mu_0}\}$.

B. Subjective Choices

There are many subjective choices and caveats associated with each of the steps outlined in Sec. IV A. Here we highlight some of these for each step in turn.

1. Choosing a Framework

This initial step is absolutely crucial, as all naturalness studies are inescapably model-dependent. In any supersymmetry study, some fundamental framework must be adopted. In

studies of other topics, however, there exists, at least in principle, the possibility of a model-independent study, where no correlations among parameters are assumed. This model-independent study is the most general possible, in that all possible results from any other (model-dependent) study are a subset of the model-independent study's results. In studies of naturalness, however, the correlations determine the results, and there is no possibility, even in principle, of a model-independent study in the sense described above.

Given this caveat, there are two general approaches, each with their advantages and disadvantages. The first is a bottom-up approach, in which one relaxes as many theoretical assumptions as is reasonable in the hope that one might derive some generic insights. The drawback to this approach is that, since generic weak-scale supersymmetry is excluded by experimental constraints, we expect there to be structure in the supersymmetry-breaking parameters, which implies correlations, which impact naturalness. Ignoring these correlations is analogous to ignoring constraints from, say, the CPT theorem, allowing the electric charges of the electron and positron to be independent parameters, and concluding that the neutrality of positronium is incredibly fine-tuned. Of course, for supersymmetry, we do not know what the underlying correlations are, but we know there are some, and the only assumption that is guaranteed to be wrong is that the supersymmetry-breaking parameters are completely uncorrelated.

The second approach is a top-down approach, in which one takes various theoretical frameworks seriously and analyzes their naturalness properties, incorporating all the assumed correlations of the framework. The hope is that by examining various frameworks in sufficient detail and sampling enough of them, one can derive new insights to resolve known problems. The disadvantage here, of course, is that it is unlikely that any of the known frameworks correctly captures all the correlations realized in nature.

2. *Specifying a Model*

As noted above, it is important to include subleading corrections to the tree-level expression for m_Z^2. For example, it is important to use 2-loop RGEs and 1-loop threshold corrections, decouple superpartners at their masses, and minimize the electroweak potential at an appropriate scale (typically the geometric mean of the stop masses). The tree-level expression of Eq. (20) is very useful to obtain an intuitive understanding of many naturalness results, but it does not capture many dependencies, especially in the case of heavy superpartners.

3. *Choosing a Set of Fundamental Parameters*

Many naturalness studies differ at this step. As an example, let's consider mSUGRA. The choice given above follows from the view that GUT-scale parameters are more fundamental than weak-scale parameters and that sensitivity of the weak scale to variations in any of the parameters m_0, $M_{1/2}$, A_0, B_0, and μ_0 is an indication of fine-tuning.

Another choice is simply $\{a_i\} = \{\mu_0\}$. The advantage of this choice is that it is extremely simple to implement. The μ parameter is (barely) multiplicatively renormalized, and so $c_{\mu_0} \equiv \partial \ln m_Z^2 / \partial \ln \mu_0^2 = \partial \ln m_Z^2 / \partial \ln \mu^2 \simeq 2\mu^2/m_Z^2$. With this choice, naturalness is, therefore, deemed equivalent to low μ. Some string-inspired models in which all squarks masses are ~ 10 TeV are claimed to be natural based on this prescription [92].

Such claims are subject to caveats, however. Given our current understanding, the μ parameter is typically assumed to have an origin separate from the supersymmetry-breaking parameters. It is therefore reasonable to assume that it is not correlated with other parameters, and so low μ is a necessary condition for naturalness. (Note, however, that the discussion of EDMs in Sec. III C 2 provides a counterargument.) Much more problematic, however, is that low μ is certainly not a sufficient condition for naturalness. In the moderate to large $\tan\beta$ limit, Eq. (20) becomes $m_Z^2 \approx -2m_{H_u}^2 - 2\mu^2$. It is certainly possible for $m_{H_u}^2$ to be small as the result of large cancellations. In this case, μ will be small. But this does not imply there is no fine-tuning: the relation $a - b - c = 1$ with $a = 1,000,000$, $b = 999,998$, and $c = 1$ is fine-tuned, despite the fact that c is small. Claims that such theories are natural are implicitly assuming that some unspecified correlation explains the large cancellation that yields low $m_{H_u}^2$.

A third possible choice for the fundamental parameters is to include not only the dimensionful supersymmetry-breaking parameters, but all of the parameters of the standard model. Some naturalness studies include these [91, 93–96], while others do not [90, 97–101]. From a low-energy point of view, one should include all the parameters of the Lagrangian. However, by assuming some underlying high energy framework and defining our parameters at m_{GUT}, we have already abandoned a purely low-energy perspective. Once we consider the high-energy perspective, the case is not so clear. For example, the top Yukawa coupling y_t may be fixed to a specific value in a sector of the theory unrelated to supersymmetry breaking. An example of this is weakly-coupled string theory, where y_t may be determined by the correlator of three string vertex operators and would therefore be fixed to some discrete value determined by the compactification geometry. The fact that all of the Yukawa couplings are roughly 1 or 0 helps fuel such speculation.[1] In such a scenario, it is clearly inappropriate to vary y_t continuously to determine the sensitivity of the weak scale to variations in y_t. Dimensionless couplings may also be effectively fixed if they run to fixed points. Other such scenarios are discussed in Ref. [31]. In the end, it is probably reasonable to consider the fundamental parameters both with and without the dimensionless parameters and see if any interesting models emerge. Note that the question of which parameters to include in the $\{a_i\}$ is independent of which parameters have been measured; see Sec. IV B 4.

A final alternative approach is to choose the fundamental parameters to be weak-scale parameters. This is perhaps the ultimate bottom-up approach, and it has the advantage of being operationally simpler than having to extrapolate to the GUT or Planck scales. However, as noted above, many of the motivating virtues of supersymmetry are tied to high scales, and some structure must exist if weak-scale supersymmetry is to pass experimental constraints. Working at the weak-scale ignores such structure. It is possible, however, to view sensitivity to variations in electroweak parameters as a lower bound on sensitivities to variations in high-scale parameters, as they neglect large logarithm-enhanced terms; see, *e.g.*, Ref. [102].

---

[1] Of course, one might argue that some more fundamental theory will fix all parameters, including those that break supersymmetry. There are no known examples, however.

4. Calculating the Sensitivity Parameters

Alternative choices, sometimes found in the literature, are $\mathcal{N}_i \equiv |\partial \ln m_Z^2 / \partial \ln a_i|$ or $\mathcal{N}_i \equiv |\partial \ln m_Z / \partial \ln a_i^2|$. There is little reason to choose one over the others, except in the case of scalar masses, where m_0^2 is the fundamental parameter, not m_0 (m_0^2 may be negative, for example). In any case, these definitions differ by factors of only 2 or 4, which should be ignored. This is easier said than done: for example, one definition may yield $\mathcal{N}_i = 20$, or $\mathcal{O}(10)\%$ fine-tuning, while the other definition yields $\mathcal{N}_i = 80$ or $\mathcal{O}(1)\%$ fine-tuning, leading to a rather different impression. Such examples serve as useful reminders to avoid grand conclusions based on hard cutoffs in naturalness measures.

There are other caveats in defining the sensitivity parameters. The role of the sensitivity coefficients is to capture the possibility of large, canceling contributions to m_Z^2. In principle, it is possible to have a contribution to m_Z^2 that is small, but rapidly varying, or large, but slowly varying. It is also possible that m_Z^2 is insensitive to variations of any single parameter, but highly sensitive to variations in a linear combination of parameters. In all of these cases, the sensitivity coefficients are highly misleading, and these possibilities again serve as reminders of how crude naturalness analyses typically are.

Finally, some studies have advocated alternative definitions of sensitivity parameters that incorporate experimental uncertainties. For example, some authors have proposed that the definition of Eq. (21), be replaced by [97, 100]

$$\mathcal{N}_i^{\text{exp}} \equiv \left| \frac{\Delta a_i^2}{m_Z^2} \frac{\partial m_Z^2}{\partial a_i^2} \right| \ , \tag{22}$$

where Δa_i^2 is the experimentally allowed range of a_i^2. The intent of this definition is to encode the idea that naturalness is our attempt to determine which values of parameters are most likely to be realized in nature.

To contrast this definition with the conventional definition, consider, for example, the hypothetical scenario in which our theoretical understanding of supersymmetry has not improved, but the μ parameter is measured to be 10^9 GeV with very high accuracy. With the standard definition of Eq. (21), this model is fine-tuned, but with Eq. (22), it is not. In our view, Eq. (22) encodes an unconventional view of naturalness. Naturalness is not a measure of our experimental knowledge of the parameters of nature. Rather it is a measure of how well a given theoretical framework explains the parameters realized in nature. It is perfectly possible for values of parameters realized in nature to be unnatural — this is what the gauge hierarchy problem is! — and once a parameter's value is reasonably well-known, naturalness cannot be increased (or decreased) by more precise measurements.

5. Determining the Overall Measure of Naturalness

There are many possible ways to combine the $\mathcal{N}_i$ to form a single measure of naturalness. A simple variation, advocated by some authors, is to add the $\mathcal{N}_i$ in quadrature.

There are also reasons to consider normalizing the $\mathcal{N}_i$ either before or after combining them. The rationale for this is that in certain cases, all possible choices of a fundamental parameter may yield large sensitivities. A well-known example of this is the hierarchy between M_{Pl} and $\Lambda_{\text{QCD}} \sim M_{\text{Pl}} e^{-c/g^2}$, which is often considered the textbook example of how

to generate a hierarchy naturally. The related sensitivity parameter,

$$c_g \equiv \left| \frac{\partial \ln \Lambda_{\text{QCD}}}{\partial \ln g} \right| = \ln(M_{\text{Pl}}^2 / \Lambda_{\text{QCD}}^2) \sim 90 \; , \tag{23}$$

however, is large. The authors of Refs. [94, 95, 103] have argued that in such cases, only *relatively* large sensitivities should be considered fine-tuned, and conclusions based on sensitivity parameters consistently overestimate the degree of fine-tuning required. These authors propose replacing the sensitivity parameters $\mathcal{N}_i$ defined above, with fine-tuning parameters, defined as $\gamma_i \equiv \mathcal{N}_i / \bar{\mathcal{N}}_i$, where $\bar{\mathcal{N}}_i$ is an average sensitivity. These γ_i are then combined to form an overall measure of naturalness.

Unfortunately, the averaging procedure brings additional complications. If it is done only over a subspace of parameter space, it may mask important features [31], and so it should be carried out over the entire parameter space, which is computationally intensive. In addition, it requires defining a measure on the parameter space and defining its boundaries. These additional complications have dissuaded most authors from including an averaging procedure. Nevertheless, many would agree that the sensitivity parameters should, in principle, be normalized in some way, and the naturalness parameter derived from un-normalized sensitivity parameters exaggerates the fine-tuning required for a given model.

C. Naturalness Bounds

We now derive upper bounds on superpartner masses from naturalness considerations. Given all the caveats of Sec. IV B, it should go without saying that these should be considered at most as rough guidelines. The goal here is to give a concrete example of how naturalness bounds may be derived, compare these with the other theoretical and experimental constraints discussed in Secs. II and III, and provide a starting point for the discussion of models in Sec. V.

We will consider a bottom-up approach, following the general prescription of Sec. IV A. We consider a model defined at the GUT-scale with input parameters $P_i = M_1, M_2, M_3, m_{H_u}, m_{H_d}, m_{Q_3}, m_{U_3}, m_{D_3}, A_t, \ldots, \text{sign}(\mu)$. These include the gaugino masses M_i, the soft SUSY-breaking scalar masses, and the A-terms, all treated as independent. The weak-scale value of $|\mu|$ is determined by m_Z. The fundamental parameters are taken to be the GUT-scale values of the input parameters, with $\text{sign}(\mu)$ replaced by the GUT-scale value of μ. Sensitivity parameters are defined as in Eq. (21), and the overall naturalness parameter is defined as the largest one.

The weak-scale values of supersymmetry-breaking parameters may be determined analytically or numerically in terms of their GUT-scale values [104–108]. Recent analyses for $\tan \beta = 10$ and using 1- and 2-loop RGEs find [106, 107]

$$
\begin{align}
M_1(m_{\text{weak}}) &= 0.41 M_1 \tag{24} \\
M_2(m_{\text{weak}}) &= 0.82 M_2 \tag{25} \\
M_3(m_{\text{weak}}) &= 2.91 M_3 \tag{26} \\
-2\mu^2(m_{\text{weak}}) &= -2.18\mu^2 \tag{27} \\
-2m_{Hu}^2(m_{\text{weak}}) &= 3.84 M_3^2 + 0.32 M_3 M_2 + 0.047 M_1 M_3 - 0.42 M_2^2 \\
&\quad + 0.011 M_2 M_1 - 0.012 M_1^2 - 0.65 M_3 A_t - 0.15 M_2 A_t
\end{align}
$$

$$-0.025M_1 A_t + 0.22A_t^2 + 0.0040M_3 A_b$$
$$-1.27m_{H_u}^2 - 0.053m_{H_d}^2$$
$$+0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2$$
$$+0.051m_{Q_2}^2 - 0.110m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2$$
$$+0.051m_{Q_1}^2 - 0.110m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2 \ , \quad (28)$$

where all the parameters on the right-hand sides of these equations are GUT-scale parameters. The RGEs mix the parameters. Although H_u does not couple to gluinos directly, the gluino mass enters the squark mass RGEs and the squark masses enter the H_u RGE, and so $m_{H_u}^2(m_{\rm weak})$ depends on the gluino mass M_3. For the first and second generation sfermions, their Yukawa couplings are so small that their main impact on the Higgs potential is through hypercharge D-term contributions or, if GUT or other boundary conditions cause these terms to vanish, through 2-loop effects in the H_u RGE [109, 110].

The naturalness prescription of Sec. IV A is applicable to complete models, but we may derive rough bounds on individual superpartner masses by neglecting other parameters when deriving the bound on a given superpartner mass. As an example, keeping only the M_3^2 term in Eq. (28), we find

$$\mathcal{N}_{M_3} \equiv \left| \frac{\partial \ln m_Z^2}{\partial \ln M_3^2} \right| \approx \frac{M_3^2}{m_Z^2} \left| \frac{\partial [-2m_{H_u}^2(m_{\rm weak}) - 2\mu^2(m_{\rm weak})]}{\partial M_3^2} \right| = 3.84 \frac{M_3^2}{m_Z^2} \ . \quad (29)$$

Requiring $\mathcal{N}_{M_3} < \mathcal{N}_{\rm max}$ and using Eq. (26), we can derive a naturalness bound on the physical gluino mass $m_{\tilde{g}} \approx M_3(m_{\rm weak})$. Proceeding in a similar way for all the parameters, and using $m_{Q_3}^2(m_{\rm weak}) = 0.885m_{Q_3}^2 + \ldots$, $m_{U_3}^2(m_{\rm weak}) = 0.770m_{U_3}^2 + \ldots$, and $m_{\tilde{f}}^2(m_{\rm weak}) \approx m_{\tilde{f}}^2 + \ldots$ for all other sfermions [106], we find

$$m_{\tilde{H}} \lesssim 640 \text{ GeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (30)$$

$$m_{\tilde{B}} \lesssim 3.4 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (31)$$

$$m_{\tilde{W}} \lesssim 1.2 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (32)$$

$$m_{\tilde{g}} \lesssim 1.4 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (33)$$

$$m_{\tilde{t}_L, \tilde{b}_L} \lesssim 1.0 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (34)$$

$$m_{\tilde{t}_R} \lesssim 1.1 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (35)$$

$$m_{\tilde{b}_R} \lesssim 4.1 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (36)$$

$$m_{\tilde{\tau}_L, \tilde{\nu}_\tau} \lesssim 4.0 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (37)$$

$$m_{\tilde{\tau}_R} \lesssim 4.0 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (38)$$

$$m_{\tilde{c}_L, \tilde{s}_L, \tilde{u}_L, \tilde{d}_L} \lesssim 4.0 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (39)$$

$$m_{\tilde{c}_R, \tilde{u}_R} \lesssim 2.7 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (40)$$

$$m_{\tilde{s}_R, \tilde{d}_R} \lesssim 4.0 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (41)$$

$$m_{\tilde{\mu}_L, \tilde{\nu}_\mu, \tilde{e}_L, \tilde{\nu}_e} \lesssim 4.0 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \quad (42)$$

$$m_{\tilde{\mu}_R, \tilde{e}_R} \lesssim 4.0 \text{ TeV } (\mathcal{N}_{\rm max}/100)^{1/2} \ . \quad (43)$$

If the standard model particles are unified into GUT multiplets at the GUT scale, the correlations lead to significantly different conclusions. For example, assuming $M_{1/2} = M_3 =$

$M_2 = M_1$, $m_{\mathbf{10}_i} = m_{Q_i} = m_{U_i} = m_{E_i}$ and $m_{\mathbf{5}_i} = m_{D_i} = m_{L_i}$, where $i = 1, 2$, we find, within the accuracy of these numerical results,

$$- 2m_{H_u}^2(m_{\mathrm{weak}}) = 3.79M_{1/2}^2 + 0.0071m_{\mathbf{10}_2}^2 + 0.0013m_{\mathbf{5}_2}^2 + 0.0071m_{\mathbf{10}_1}^2 + 0.0013m_{\mathbf{5}_1}^2 + \ldots, \quad (44)$$

implying

$$m_{\tilde{B}} \lesssim 190 \text{ GeV} \, (\mathcal{N}_{\mathrm{max}}/100)^{1/2} \quad (45)$$

$$m_{\tilde{W}} \lesssim 380 \text{ GeV} \, (\mathcal{N}_{\mathrm{max}}/100)^{1/2} \quad (46)$$

$$m_{\tilde{g}} \lesssim 1.4 \text{ TeV} \, (\mathcal{N}_{\mathrm{max}}/100)^{1/2} \quad (47)$$

$$m_{\tilde{u}_L, \tilde{d}_L, \tilde{c}_L, \tilde{s}_L, \tilde{u}_R, \tilde{c}_R, \tilde{e}_R, \tilde{\mu}_R} \lesssim 11 \text{ TeV} \, (\mathcal{N}_{\mathrm{max}}/100)^{1/2} \quad (48)$$

$$m_{\tilde{d}_R, \tilde{s}_R, \tilde{\nu}_e, \tilde{e}_L, \tilde{\nu}_\mu, \tilde{\mu}_L} \lesssim 25 \text{ TeV} \, (\mathcal{N}_{\mathrm{max}}/100)^{1/2} \; . \quad (49)$$

These results may be understood as follows: The $\tilde{t}_L$, $\tilde{b}_L$, and $\tilde{t}_R$ masses enter the H_u RGE through top Yukawa couplings, and their bounds in Eqs. (34) and (35) are consistent with those of Eq. (4). For the other sfermions, the naturalness constraints are weaker. Generically, these masses enter the H_u RGE dominantly through hypercharge D-terms, and so one expects constraints on them to be weaker by a factor of $\sqrt{\alpha_{y_t}/\alpha_1} = y_t/g_1 \sim 3$, consistent with Eqs. (36)–(43).

In the case of GUTs, for the gauginos, the most stringent bound is from the gluino, with the Wino and Bino bounds following from the relations $M_{\tilde{B}} : M_{\tilde{W}} : M_{\tilde{g}} \approx 1 : 2 : 7$. For the scalars, in GUTs the masses enter only through two-loop terms, and so the constraints are weaker by a factor of $\sqrt{4\pi/\alpha_1} \sim 10$, as seen in Eqs. (48) and (49) [109, 110]. Note that the GUT correlations greatly strengthen the naturalness bounds on Binos and Winos, but greatly weaken the bounds on first and second generation scalars: the choice of underlying framework can have enormous qualitative implications for naturalness bounds.

V. MODEL FRAMEWORKS

We now discuss a few classes of models that have been proposed to relieve the tension between the various constraints discussed so far. To set the stage, we present all of the theoretical and experimental constraints discussed in this review in Fig. 7.

A. Effective Supersymmetry

As evident from Fig. 7, naturalness most stringently restricts the masses of scalars with large Yukawa couplings, since these are most strongly coupled to the Higgs sector. At the same time, experimental constraints are most stringent for scalars with small Yukawa couplings, since light fermions are most easily produced and studied. This suggests that light fermions have heavy superpartners and vice versa, which provides a promising way to reconcile naturalness with flavor and CP constraints [19, 109–111]. A representative spectrum for such models, known as "effective supersymmetry" [112] or, alternatively, "more minimal supersymmetry" or "inverted hierarchy models," is shown in Fig. 8.

The effective supersymmetry spectrum may be realized in many ways [112–126]. For example, if there is an extra anomalous or non-anomalous U(1) gauge group under which
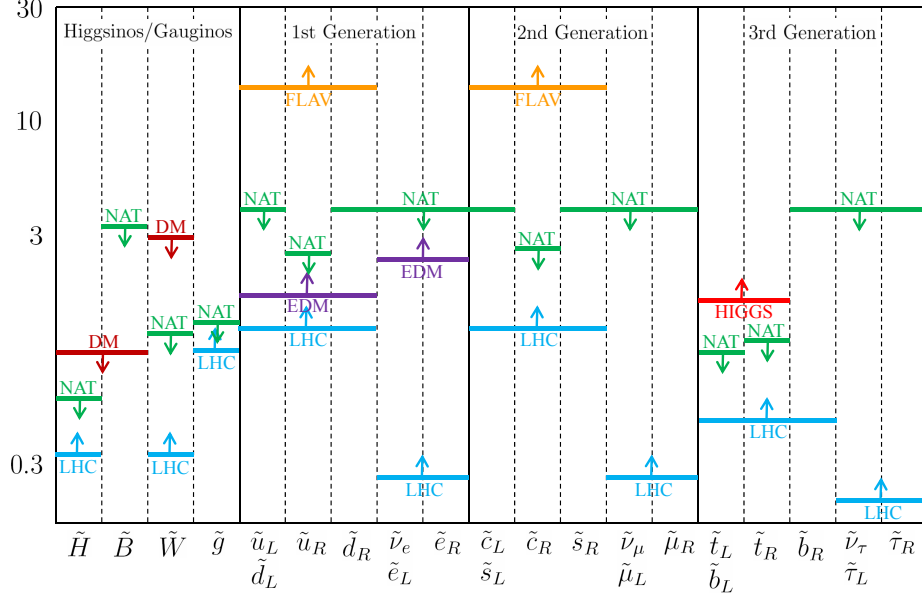
FIG. 7: A sample of constraints on the superpartner spectrum from naturalness (NAT), dark matter (DM), collider searches (LHC), the Higgs boson mass (HIGGS), flavor violation (FLAV), and EDM constraints (EDM). The constraints assume a moderate value of $\tan \beta = 10$. The naturalness constraints derive from a bottom-up analysis and scale as $(\mathcal{N}_{\max}/100)^{1/2}$, where $\mathcal{N}_{\max}$ is the maximally allowed naturalness parameter; see Sec. IV. All of the constraints shown are merely indicative and subject to significant loopholes and caveats; see the text for details.
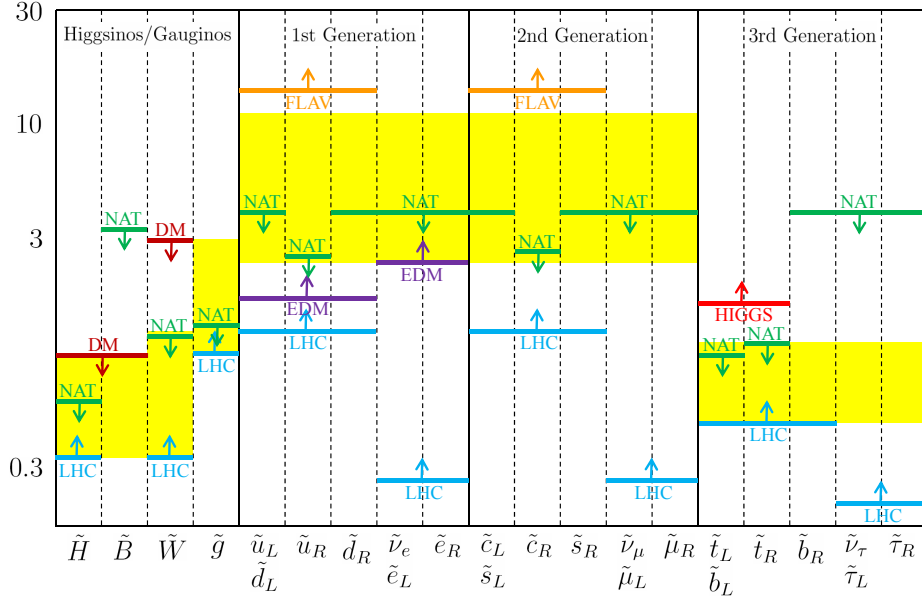

FIG. 8: Example superpartner mass ranges for effective supersymmetry (shaded) with constraints as given in Fig. 7. Heavy and degenerate first and second generation sfermions satisfy flavor and EDM constraints, and light third generation sfermions satisfy naturalness. The Higgs mass constraint requires either near-maximal stop mixing, or physics beyond the MSSM.

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the first two generations are charged, but the third generation is neutral, the first two generation sfermions may receive additional contributions to their mass. The U(1) symmetry may also allow fermion masses for the third generation, but forbid masses for the first and second, naturally explaining the inverted hierarchy structure. Alternatively, the split sparticle spectrum may be radiatively generated [122–124]. All sparticle masses may begin at ∼ 10 TeV at the GUT scale, but for particular GUT-scale boundary conditions, those with large Yukawa couplings may be driven to low values at the weak scale. In these scenarios, the large Yukawa coupling produces both heavy fermions and light sfermions, again naturally explaining the inverted hierarchy structure.

Effective supersymmetry predates not only the Higgs discovery, but even the most stringent LEP limits on the Higgs mass, and was not originally intended to explain the large Higgs boson mass. As discussed in Sec. III B, the Higgs boson mass may be consistent with sub-TeV stops, but only in the highly fine-tuned case when there is large left-right mixing; for a recent discussion of this in the context of effective supersymmetry, see Ref. [126]. Alternatively, more minimal supersymmetry may be made less minimal by adding extra fields to raise the Higgs mass; see, *e.g.*, Refs. [127, 128]. Effective supersymmetry with this extension has attracted renewed attention, sometimes under the confusingly generic moniker "natural supersymmetry," as a strategy to reconcile naturalness with LHC constraints [129].

In effective supersymmetry, the first and second generation sfermions are beyond the reach of the LHC, but gluinos, stops, and sbottoms may be within reach. The most promising collider signals are therefore direct stop and sbottom squark production, or gluinos with top- and bottom-rich cascade decays [130]. Effects in low-energy B physics may also be accessible [131]. Finally, new particles added to raise the Higgs mass may have associated signals.

B. Focus Point Supersymmetry

In focus point supersymmetry [31, 132, 133], correlations between parameters allow sparticle masses to be larger than their naive naturalness bounds. A representative spectrum with heavy scalars is given in Fig. 9. Heavy first and second generation scalars suppress flavor and CP violation, as in effective supersymmetry. In contrast to effective supersymmetry, however, the third generation is also heavy, naturally raising the Higgs mass to within current bounds. There are many theoretical reasons for expecting scalar superpartners to be heavier than the gauginos. For example, such a hierarchy follows from an approximate $U(1)_R$ symmetry, which suppresses the gaugino masses (and A-terms) but not the scalar masses. It also results if there are no singlet supersymmetry-breaking fields [79, 80]. Note that gaugino masses enter the scalar mass RGEs, but scalar masses do not enter the gaugino mass RGEs; the hierarchy $m_0 \gg M_{1/2}$ is therefore stable under RGE evolution, whereas $M_{1/2} \gg m_0$ is not.

The obvious difficulty is that heavy top squarks naively contradict naturalness. In focus point supersymmetry, correlations in GUT-scale parameters are invoked to alleviate this fine-tuning. A simple example is evident from Eq. (28). The weak-scale value of $m_{H_u}^2(m_{\text{weak}})$ is highly sensitive to the GUT-scale values of $m_{H_u}^2$, $m_{Q_3}^2$, and $m_{U_3}^2$, but if these have a unified value m_0^2 at the GUT scale, $-2m_{H_u}^2(m_{\text{weak}}) = -1.27m_{H_u}^2 + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + \ldots = 0.03m_0^2 + \ldots$, and the weak scale becomes highly insensitive to variations in these GUT-scale parameters, even if they are large. The reasoning here is similar to that leading to natural ∼ 10 TeV first and second generation sfermions with $\mathcal{N}_{\text{max}} \sim 100$ in the GUT case analyzed
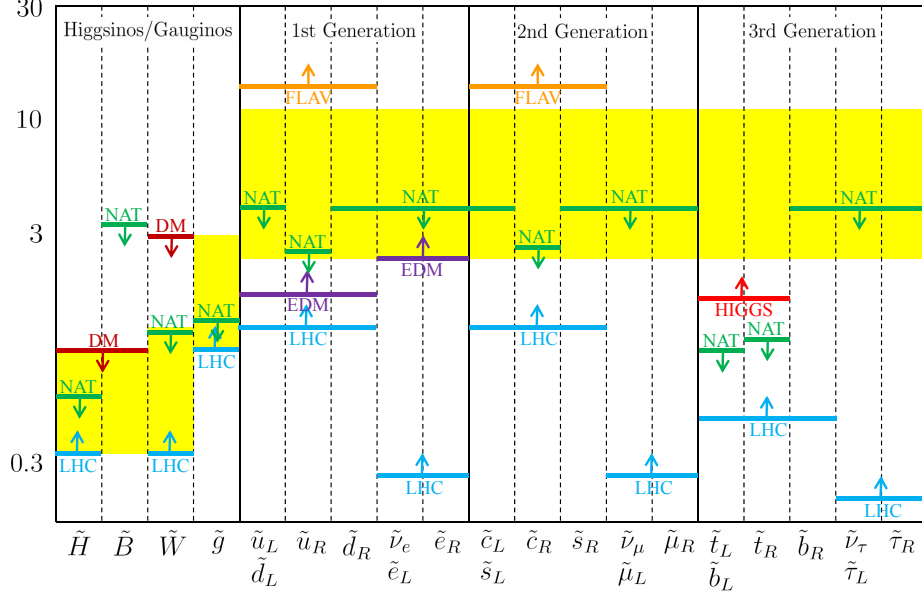
FIG. 9: Example superpartner mass ranges for focus point supersymmetry (shaded) with constraints as given in Fig. 7. Heavy and degenerate first and second generation sfermions satisfy flavor and EDM constraints and heavy third generation sfermions raise the Higgs mass, while naturalness is preserved despite heavy stops by correlations between GUT-scale parameters.

in Sec. IV C.

This behavior may be understood as a property of the RGEs. The $m_{H_u}^2$ RGEs in a focus point model are shown in Fig. 10. The RG trajectories have a focus point at the weak scale, and so the weak-scale value of $m_{H_u}^2$ is insensitive to variations in the GUT-scale parameters. The weak scale still receives quadratic contributions from heavy stops, but the large logarithm enhancement from RG evolution in Eq. (4) is absent, reducing the fine-tuning associated with multi-TeV stops by a factor of $\sim \ln(m_{\rm GUT}^2/m_{\rm weak}^2) \sim 60$. Such focusing occurs if the GUT-scale parameters satisfy [132]

$$(m_{H_u}^2, m_{\tilde{t}_R}^2, m_{\tilde{t}_L}^2) \propto (1, 1+x, 1-x) \tag{50}$$

for moderate values of $\tan\beta$, and

$$(m_{H_u}^2, m_{\tilde{t}_R}^2, m_{\tilde{t}_L}^2, m_{\tilde{b}_R}^2, m_{H_d}^2) \propto (1, 1+x, 1-x, 1+x-x', 1+x') \tag{51}$$

for large values of $\tan\beta$, where x and x' are arbitrary constants. Note that the scale at which focusing occurs is sensitive to dimensionless couplings, particularly the top Yukawa y_t. As discussed in Sec. IV B 3, one may include y_t as a fundamental parameter or not. If it is included, $\mathcal{N}_{y_t}$ is large for large superpartner masses, but it is large throughout parameter space [133]. If one adopts the averaging procedure described in Sec. IV B 5 to identify only relatively large sensitivities, the effect of including y_t as a fundamental parameter is greatly moderated.

A universal scalar mass obviously satisfies both Eqs. (50) and (51), and the large m_0 region of mSUGRA has become the canonical example of focus point supersymmetry [99, 132, 133]. Focus point supersymmetry is, however, a far more general phenomenon, as one may postulate many relations between the GUT-scale parameters to reduce the fine-tuning
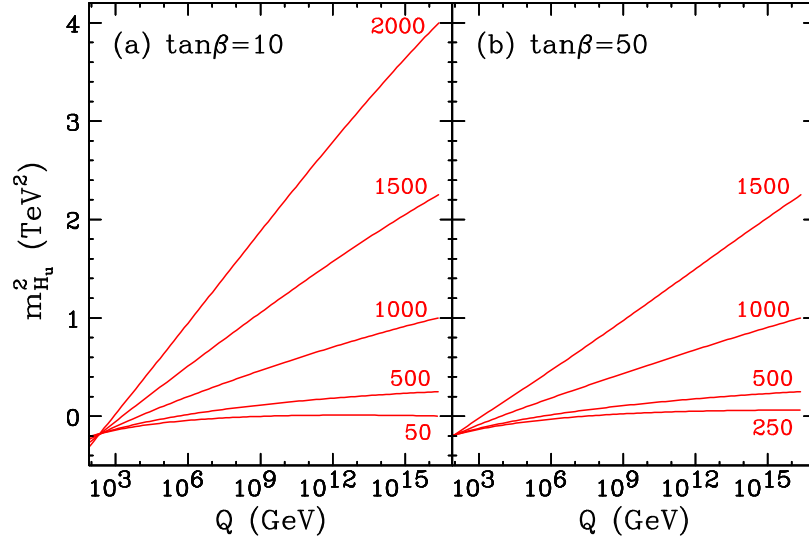
FIG. 10: The RG evolution of $m_{H_u}^2$ in the focus point region of mSUGRA for $\tan\beta = 10$ (left) and 50 (right), several values of m_0 (as shown in GeV), $M_{1/2} = 300$ GeV, and $A_0 = 0$. For both values of $\tan\beta$, $m_{H_u}^2$ exhibits an RG focus point near the weak scale, implying that the weak scale is insensitive to variations in the GUT-scale supersymmetry-breaking parameters [133].

in Eq. (28). For example, considering the M_2^2, $M_2 M_3$, and M_3^2 terms, one finds focusing for $M_3/M_2 \approx 0.3, -0.4$ at the GUT scale, allowing large, non-universal gaugino masses to be natural [106, 108, 134–136]. Focusing may also be found in models with right-handed neutrinos [137] and large A-terms [138], and may emerge from the boundary conditions of mirage mediation [139–141] or be enforced by a symmetry [136].

In the most studied focus point supersymmetry models, all scalars are heavy, but typically the stops are slightly lighter. They may be produced in future LHC runs, or may be beyond reach, but light enough to enhance the top content of gluino decays. The most promising LHC signals are therefore again direct stop production with cascade decays through charginos and neutralinos, or gluino production, followed by top- and bottom-rich cascade decays. As the first generation scalars are heavy but not extremely heavy, there may also be a signal in EDMs. Last, the prospects for WIMP dark matter detection are extremely promising in focus point models [37, 142]. In particular, focus point supersymmetry predicts a mixed Bino-Higgsino neutralino with a spin-independent proton cross sections typically above the zeptobarn level, which should be probed in the coming few years.

Last, note that the large logarithm enhancement may also be eliminated by adding additional particles. This is the approach of an entirely different class of models, typically called "supersoft supersymmetry" [143], where the MSSM is extended to include a gauge adjoint chiral superfield for each gauge group, providing an interesting alternative strategy for reconciling naturalness with experimental constraints [144, 145].

C. Compressed Supersymmetry

In many supersymmetric models, there is a large mass splitting between the gluino and squarks at the top of the spectrum and the lighter superparters at the bottom. This reduces the naturalness of these models in two ways. First, the large mass splittings imply that

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the gluino and squark cascade decays produce energetic particles and large $\not{E}_T$, leading to distinct signals and strong bounds on sparticle masses. Second, lower bounds on the masses of the lighter sparticles imply stringent lower bounds on gluino and squark masses, which decreases naturalness.

The superpartner spectrum may be much more degenerate, however. This has been explored in the context of "compressed supersymmetry" [107], in which there are small splittings between colored superpartners and an LSP neutralino. For the reasons given above, this leads to weaker bounds on sparticle masses and provides an interesting approach to developing viable and natural models [146, 147].

There are well-motivated reasons to expect large mass splittings. RG evolution drives up colored sparticles masses relative to uncolored ones. For example, assuming gaugino mass unification at the GUT scale, Eqs. (24)–(26) imply $|M_1| : |M_2| : |M_3| \approx 1 : 2 : 7$ at the weak scale. This is not a strict prediction of GUT models, however [148]. For example, if gaugino masses are generated not by gauge singlet F-terms, but by a **75** multiplet of SU(5), group theoretic factors imply $|M_1| : |M_2| : |M_3| \approx 5 : 3 : 1$ at the GUT scale, leading to $|M_1| : |M_2| : |M_3| \approx 5 : 6 : 6$ and a highly degenerate spectrum at the weak scale [149]. Note that the M_3^2 and M_2^2 terms enter with opposite signs in Eq. (28), and so when $|M_2|$ is a little larger than $|M_3|$ at the GUT scale, these terms partially cancel and naturalness is improved by essentially the same mechanism discussed in Sec. V B for focus point scenarios with non-universal gaugino masses.

A representative spectrum is shown in Fig. 11. The virtue of compressed supersymmetry is that it decreases the tension between naturalness and LHC superpartner search bounds. A shortcoming of these models is that the light spectrum exacerbates problems with flavor and CP violation. In particular, ~ 100 GeV superpartners generically require $\phi_{\rm CP} \lesssim 10^{-4} - 10^{-3}$ to satisfy EDM constraints, and so these models require some additional mechanism to suppress CP violation. In addition, the problem of obtaining a 125 GeV Higgs boson mass is present in compressed supersymmetry if the stops are light. As in the case of effective supersymmetry, physics beyond the MSSM [127, 128] is required to raise the Higgs mass to its measured value, bringing with it additional complications.

The collider signals of compressed supersymmetry have been explored in a number of studies [150–156]. The relevant signals depend on the degree of compression. For $m_{\tilde{t}} - m_\chi < m_t$, stops may dominantly decay via $\tilde{t} \to bW^+\chi$ or even $\tilde{t} \to c\chi$ or $\tilde{t} \to b f \bar{f}' \chi$, leading to softer leptons and a suppressed multi-lepton rate [150, 154]. For even greater degeneracies, the only possible signals are monophotons [155] and monojets [156]. At present these searches imply $m_{\tilde{g}} \gtrsim 500$ GeV. Implications for neutralino dark matter have been explored in Refs. [107, 150, 157].

Finally, there are many other models in which the $\not{E}_T$ signal is reduced. Interesting possibilities in which cascade decays go through hidden sectors include hidden valley models [158, 159] and stealth supersymmetry [160].

D. R-**Parity-Violating Supersymmetry**

As discussed in Sec. III A 3, the characteristic $\not{E}_T$ collider signal of supersymmetry may also be degraded in the presence of R-parity violation. If any of the superpotential terms of Eq. (7) is non-zero, all superpartners decay, and, provided the decay length is not too long, supersymmetric particles do not escape the detector. The phenomenology of RPV supersymmetry has been studied for a long time [161–163], but it has recently attracted
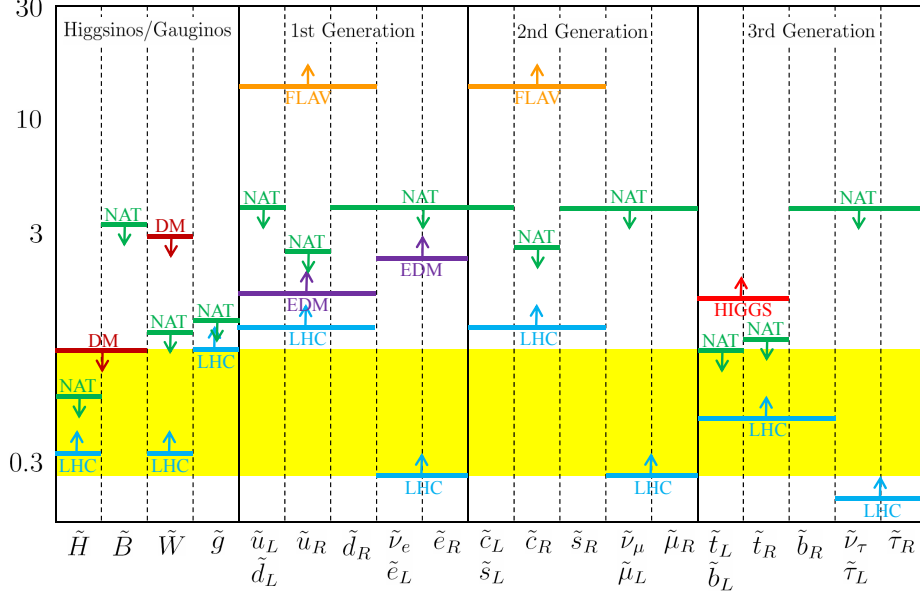
FIG. 11: Example superpartner mass ranges for compressed supersymmetry and RPV supersymmetry (shaded) with constraints as given in Fig. 7. Light sfermions preserve naturalness and evade LHC bounds because $\not{E}_T$ signals are degraded by superpartner degeneracies in compressed supersymmetry or by LSP decays in RPV supersymmetry. These models require mechanisms to eliminate flavor violation and reduce CP-violating phases to $\mathcal{O}(10^{-3}) - \mathcal{O}(10^{-4})$, and also require near-maximal stop mixing or physics beyond the MSSM to raise the Higgs mass. In addition, in RPV supersymmetry, there is no WIMP dark matter candidate.

renewed attention as a way to make light superpartners viable, and thereby reduce fine-tuning.

In general, once one allows R_p violation, one opens a Pandora's box of possibilities. There are few principled ways to violate R-parity conservation. The RPV couplings cannot all be sizable. In fact, there are stringent bounds on individual RPV couplings, and even more stringent bounds on products of pairs of couplings [164, 165]; in particular, if any lepton number-violating coupling and any baryon number-violating coupling are both non-zero, proton decay sets extremely stringent constraints.

If theory is any guide, one might expect that the RPV couplings follow the pattern of the R_p-conserving couplings, with those involving the third generation the biggest, the second generation smaller, and the first smaller still. Realizations of this hypothesis have been presented in Refs. [166–170], where models of R_p violation based on the principle of minimal flavor violation lead to scenarios in which only the hadronic RPV terms $\lambda''_{ijk} U_i D_j D_k$ are sizable, with λ''_{323} typically the largest. Such models somewhat moderate the naturalness motivation for R_p violation, as they imply that, say, the LSP neutralino decays dominantly to top and bottom quarks, leading to b-jets, leptons, and $\not{E}_T$ from neutrinos, all distinctive characteristics that one was hoping to avoid. Nevertheless, such RPV signals likely do reduce LHC limits somewhat, and, of course, one may always ignore theoretical bias and consider, say, λ''_{112} couplings that would lead to decays to light flavor and pure jet signals, such as those discussed in Sec. III A 3 and Fig. 3.

In summary, as with compressed supersymmetry, R_p violation provides another possibility for reducing the distinctiveness of supersymmetry signals at colliders and potentially

improving the naturalness of viable models. The shortcomings, however, are also similar: if all of the superpartners are light, the Higgs boson is generically too light, requiring physics beyond the MSSM, and the EDM constraints are generically not satisfied, requiring yet more structure to remove troubling CP-violating phases. In RPV supersymmetry, one also loses the motivation of WIMP dark matter, although the gravitino or other candidates may play this role.

VI. CONCLUSIONS

PARABLE. Some children notice that a soap bubble's width and height are remarkably similar. They get excited when they find that this can be explained by surface tension and rotational symmetry. Later, with amazing experiments, they find that the width and height are not identical, but differ by 1 part in 10^{28}. They remember, however, that wind can distort the shape of the bubble and calculate that, given typical winds, one would expect differences of 1 part in 10^{30}. Some of the children become despondent and wonder how such a beautiful solution could be so wrong; others consider alternative explanations; others postulate that bubbles can be any shape, but only nearly spherical ones are compatible with the presence of children; and others study the wind.

Supersymmetry has long been the leading candidate for new physics at the weak scale. In this review, we have evaluated its current status in light of many theoretical and experimental considerations.

The leading theoretical motivations for weak-scale supersymmetry are naturalness, grand unification, and WIMP dark matter. Each of these prefers supersymmetry breaking at the weak scale, but each argument is subject to caveats outlined in Sec. II. Of course, taken as a whole, these continue to strongly motivate supersymmetry.

Current experimental constraints are discussed in Sec. III and summarized in Fig. 7. For some varieties of supersymmetry models, the LHC now requires superpartner masses well above 1 TeV, but there are also well-motivated examples in which superpartners may be significantly lighter without violating known bounds. The 125 GeV Higgs boson mass prefers heavy top squarks in the MSSM, and longstanding flavor and CP constraints strongly suggest multi-TeV first and second generation sfermions. We have especially emphasized the robustness of the EDM constraints, which are present even in flavor-conserving theories. In the absence of a compelling mechanism for suppressing CP violation, the EDM constraints require first generation sfermions to be well above the TeV scale. Against the backdrop of these indirect constraints, LHC bounds on supersymmetry are significant because they are direct, but they are hardly game-changing. One may like supersymmetry or not, but to have thought it promising in 2008 and to think it much less promising now is surely the least defensible viewpoint.

In Sec. IV, we have critically examined attempts to quantify naturalness. There are many studies embodying philosophies that differ greatly from each other. We have expressed reservations about some, but for many, one can only acknowledge the subjective nature of naturalness and make explicit the underlying assumptions. Very roughly speaking, however, current bounds are beginning to probe naturalness parameters of $\mathcal{N} \sim 100$, corresponding to gluino masses of 1 TeV.

In Sec. V, we have described a few of the leading frameworks that attempt to preserve naturalness in viable models, giving their key features and implications for experimental

searches. Their primary motivations are summarized in Table I, and their rough implications for superpartner spectra are given in Figs. 8, 9, and 11. Although supersymmetry does not work "out of the box," these models provide longstanding (pre-LHC) and well-motivated frameworks that remain viable and preserve naturalness at the 1% level.

In summary, weak-scale supersymmetry is neither unscathed, nor is it dead. The true status is somewhere in between, and requires a nuanced view that incorporates at least some of the many caveats and subtleties reviewed here. Thankfully, the status quo will not last long, given expected experimental progress on many fronts. In particular, after the two-year shutdown from 2013-14, the LHC is currently expected to begin running again at ~ 13 TeV in 2015, with initial results available by Summer 2015, and 100 fb^{-1} of data analyzed by 2018. Such a jump in energy and luminosity will push the reach in gluino and squark masses from around 1 TeV to around 3-4 TeV, and probe models that are roughly an order of magnitude less natural. Given these exciting prospects for drastically improved sensitivity to supersymmetry or other new physics at the weak scale, patience is a virtue. In the grand scheme of things, we will soon know.

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