Sparticle Spectrum Constraints

STEPHEN P. MARTIN

Department of Physics
Northeastern University
Boston MA 02115
and

PIERRE RAMOND[†]

Physics Research Division
Superconducting Super Collider Laboratory
Dallas TX 75237

ABSTRACT: The supersymmetric standard model with supergravity-inspired soft breaking terms predicts a rich pectrum of sparticles to be discovered at the SSC, LHC and NLC. Because there are more supersymmetric particles than unknown parameters, one can write down sum rules relating their masses. We discuss the pectrum of sparticles from this point of view. Some of the sum rules do not depend on the input parameters and can be used to test the consistency of the model, while others are useful in determining the input parameters of the theory. If supersymmetry is discovered but the sum rules turn out to be violated, it will be evidence of new physics beyond the minimal supersymmetric standard model with universal soft supersymmetry-breaking terms.

[†] Permanent address: Institute for Fundamental Theory, Physics Department, University of Florida, Gainesville FL 32611

1. INTRODUCTION

The extension of the Standard Model to $N = 1$ supersymmetry[1] is totally straightforward outside of the Higgs sector: to every chiral fermion (quark or lepton) one associates a complex spinless boson (squark or slepton), and to each gauge field (gluon, W-boson, Z-boson, photon), one adds a spin 1/2 Majorana gaugino (gluino, wino, zino, photino). The extension of the Higgs sector to supersymmetry is less straightforward, since by extending the single Higgs doublet of the Standard Model to a chiral Higgsino, one induces local (ABJ) and global (Witten) anomalies. This is easily solved by adding its vector-like completion; the result is a theory with two Higgs doublets of opposite weak hypercharge. This is fortuitous since the nature of supersymmetric couplings itself requires two Higgs doublets if both up and down type quarks and leptons are to be massive. However, theories with two Higgs doublets show an additional chiral global symmetry, of the type introduced by Peccei and Quinn. This symmetry is broken explicitly by QCD instanton effects and spontaneously by the electroweak symmetry breaking. This results in a pseudo-Nambu-Goldstone boson, the axion, of the type ruled out by experiment.

To be in accord with experiment, the PQ symmetry must be broken. Fortunately, this can be done without introducing new fields in the theory by adding a term which preserves supersymmetry; it has dimension 3, and is parametrized by a coupling with dimension of mass called μ. Its numerical value is to be considered one of the parameters of the $N = 1$ Standard Model. The model also comes with a potential but it is not capable of breaking the electroweak symmetry, since the Higgs scalar bosons have the same positive mass squared, μ^2.

In order to bring this model closer to reality, one must break supersymmetry. In the absence of any concrete theory of supersymmetry breaking, the effect is mocked up in the low energy Lagrangian by including terms which break the supersymmetry softly while preserving the gauge symmetries.

The generalization of the model to include supergravity allows for such a mechanism of supersymmetry breaking, with a particular set of soft terms specified at a given input scale[2]. They are:

- masses for the three gauginos [M_1 for weak hypercharge, M_2 for $SU(2)_L$ and M_3 for $SU(3)_c$];

- a common mass m_0 for all the spinless particles in the theory (squark, slepton, Higgs);

- cubic interactions among the squarks, sleptons and Higgs as allowed by R-parity and the gauge interactions of the theory, each equal at the input scale to the corresponding Yukawa coupling multiplied by a universal parameter A; and

- a scalar $(\text{mass})^2$ term in the Higgs sector which breaks both the Peccei-Quinn symmetry and supersymmetry.

In this scheme, the supersymmetry-breaking sector is parametrized by six masses; three gaugino masses M_i, the common scalar mass m_0, the trilinear scalar coupling parameter A, and the PQ-breaking and supersymmetry-breaking parameter B. Since the gauginos have not been observed to date, the masses M_i must certainly be non-zero. In fact, since they are strictly multiplicatively renormalized, they must not vanish at any scale for which the renormalization group equations are valid. It is possible that each of the other supersymmetry-breaking parameters are zero at the input scale, although this will not be maintained under renormalization group evolution. The low energy values of all the soft breaking parameters are constrained by the fact that no sparticles have been found yet.

A remarkable feature of the theory is that, with the supersymmetry breaking parameters specified at some high scale M_X, it is possible to trigger electroweak symmetry breaking[3]. It is even more amazing that the present bounds on the top-quark mass, which is constrained (in the context of the Standard Model) to be between 120 and 200 GeV by experiment, yields the correct value of M_Z for values of the supersymmetry breaking parameters not far above their experimental lower limits. One of the consequences of such a picture is that the superpartners of the elementary particles would have masses in the hundreds of GeV, quite accessible to the next generation of colliders: SSC, LHC and NLC[4]. Another remarkable consequence of the mechanism is that it suggests that the three gauge couplings of the Standard Model have a common origin[5] around $10^{15} - 10^{16}$ GeV, providing a strong hint in favor of a Grand Unified Theory (GUT)[6]. This in turn dovetails nicely with the requirement of R-parity conservation, which is necessary in order to avoid superfast proton decay, and which arises most naturally in supersymmetric GUTs with gauged $B - L$[7,8].

The scale at which the supersymmetry-breaking parameters are specified is in principle undetermined as long as it is below the Planck mass. However, if it is too far below, the

magnitude of the electroweak breaking comes out too low, given the lower bound on the top-quark mass. Thus a remarkably consistent picture emerges with the supersymmetry breaking parameters specified at $10^{15} - 10^{16}$ GeV, the scale at which the gauge couplings unify. Another coincidence is that around that scale, the relation $m_b = m_\tau$ is favored[9] by the data. One of the mysteries is that the PQ-breaking parameter μ is constrained to be of the same order of magnitude as the supersymmetry-breaking parameters. Since μ is logically uncorrelated with supersymmetry-breaking from the low-energy point of view, this hints at a deeper mechanism which would link PQ and supersymmetry breakings.

The masses of the superpartners are determined in terms of the soft breaking parameters[10,11]. Various authors[12,13,14,15,16,17] have presented numerical results based on computer analysis for the sparticle masses, using different sets of input parameters. Since there are more superpartners than breaking parameters, there are many sum rules among superpartner masses. These sum rules will test the validity of this picture, and will be of importance for the SSC, LHC, and NLC. It is the purpose of this paper to present these sum rules in a simple, unified format, without using computers. Some are new; some have already appeared in the literature, but not all in one place. Their study will enable us to offer some specific scenarios in conducting the experimental search for superpartners.

Since the superpartner masses are in the several hundred GeV range, we neglect the masses and Yukawa couplings of the leptons and quarks of the first two families. It follows that we need only consider the Yukawa couplings y_t, y_b and y_τ, and the trilinear scalar couplings of $H_u \tilde{t}_L \tilde{t}_R^*$, $H_d \tilde{b}_L \tilde{b}_R^*$ and $H_d \tilde{\tau}_L \tilde{\tau}_R^*$, which we denote $y_t A_t$, $y_b A_b$ and $y_\tau A_\tau$, respectively. At the input scale, $A_t = A_b = A_\tau$. We denote the squarks and sleptons of the first two families by their first-family names, that is $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{u}_R$, $\tilde{d}_R$, $\tilde{e}_L$, $\tilde{e}_R$, $\tilde{\nu}_e$. Thus $\tilde{u}_L$ can be taken to be either the left-handed up or charmed squark. The squarks and sleptons associated with the third family will be denoted by $(\tilde{t}_{L,R}, \tilde{b}_{L,R}, \tilde{\tau}_{L,R}, \tilde{\nu}_\tau)$.

2. RENORMALIZATION GROUP EQUATIONS

In this section we remind the reader of the one-loop renormalization group equations which are relevant to this work. They are

$$16\pi^2 \frac{d}{dt} g_i = -b_i g_i^3 \qquad i = 1, 2, 3; \qquad (2.1)$$

$$16\pi^2 \frac{d}{dt} M_i = -2b_i M_i g_i^2 \qquad\qquad i = 1, 2, 3; \qquad\qquad (2.2)$$

for the three gauge couplings and three gaugino masses, respectively, and above the super-partner mass thresholds,

$$b_i = \begin{cases} -\frac{3}{5} - 2n_f & i = 1 \\ 5 - 2n_f & i = 2 \\ 9 - 2n_f & i = 3 \end{cases}$$

with $i = 1$ for weak hypercharge in a GUT normalization, $i = 2$ for $SU(2)_L$ and $i = 3$ for $SU(3)_c$. Together, (2.1) and (2.2) imply that the three quantities M_i/α_i do not run with scale:

$$\frac{M_i(t)}{\alpha_i(t)} = \frac{M_i(t_0)}{\alpha_i(t_0)} . \qquad\qquad (2.3)$$

The light squark and slepton masses obey the RG equations

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m_{\tilde{Q}_L}^2 &= -\frac{2}{15} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{1}{5} g_1^2 \mathrm{Tr}(Ym^2) , \\
16\pi^2 \frac{d}{dt} m_{\tilde{u}_R}^2 &= -\frac{32}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 - \frac{4}{5} g_1^2 \mathrm{Tr}(Ym^2) , \\
16\pi^2 \frac{d}{dt} m_{\tilde{d}_R}^2 &= -\frac{8}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{2}{5} g_1^2 \mathrm{Tr}(Ym^2) , \\
16\pi^2 \frac{d}{dt} m_{\tilde{L}_L}^2 &= -\frac{6}{5} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{3}{5} g_1^2 \mathrm{Tr}(Ym^2) , \\
16\pi^2 \frac{d}{dt} m_{\tilde{e}_R}^2 &= -\frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 \mathrm{Tr}(Ym^2) ,
\end{aligned}
\qquad (2.4)
$$

where

$$\mathrm{Tr}(Ym^2) = m_{H_u}^2 - m_{H_d}^2 + \sum_1^{n_f} \left(m_{\tilde{Q}_L}^2 - 2m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 - m_{\tilde{L}_L}^2 + m_{\tilde{e}_R}^2 \right) .$$

The renormalization group equations for the sparticles of the third family are different because they involve the Yukawa couplings; for the squarks they read

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m_{\tilde{t}_L, \tilde{b}_L}^2 &= 2y_t^2 \Sigma_t^2 + 2y_b^2 \Sigma_b^2 - \frac{2}{15} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{1}{5} g_1^2 \mathrm{Tr}(Ym^2) , \\
16\pi^2 \frac{d}{dt} m_{\tilde{t}_R}^2 &= 4y_t^2 \Sigma_t^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 - \frac{4}{5} g_1^2 \mathrm{Tr}(Ym^2) , \\
16\pi^2 \frac{d}{dt} m_{\tilde{b}_R}^2 &= 4y_b^2 \Sigma_b^2 - \frac{8}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{2}{5} g_1^2 \mathrm{Tr}(Ym^2) ,
\end{aligned}
\qquad (2.5)
$$

and for the sleptons,

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m_{\tilde{\tau}_L, \tilde{\nu}_\tau}^2 &= 2y_\tau^2 \Sigma_\tau^2 - \frac{6}{5} g_1^2 M_1^2 - 6 g_2^2 M_2^2 - \frac{3}{5} g_1^2 \mathrm{Tr}(Ym^2) , \\
16\pi^2 \frac{d}{dt} m_{\tilde{\tau}_R}^2 &= 4y_\tau^2 \Sigma_\tau^2 - \frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 \mathrm{Tr}(Ym^2) ,
\end{aligned}
\qquad (2.6)
$$

where

$$\Sigma_t^2 = (m_{H_u}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2),$$
$$\Sigma_b^2 = (m_{H_d}^2 + m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2 + A_b^2),$$
$$\Sigma_\tau^2 = (m_{H_d}^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 + A_\tau^2).$$

When all of the squark, slepton and Higgs masses are the same at the initial scale, we have

$$\text{Tr}(Y m^2) = m_0^2 \text{Tr}(Y) = 0, \tag{2.7}$$

as required by the absence of the gravitational mixed anomaly. Furthermore the condition (2.7) is maintained by the RG evolution, and so holds at all scales. Hence we neglect it in the following, which greatly simplifies these equations. Fortunately, we will not need the renormalization group equations for $m_{H_u}^2, m_{H_d}^2, A_t, A_b, A_\tau$ or μ in this analysis.

3. FIRST AND SECOND FAMILY SQUARKS AND SLEPTONS

The sum rules involving masses of the squarks and sleptons associated with the first two families are particularly simple. Besides the universal m_0^2, there are four other contributions to the squared masses of the squarks and sleptons, as follows.

First, there are contributions from the renormalization group running of the scalar masses down to experimental scales, as given by (2.4).

Second, the D^2 term in the scalar potential contributes to the scalar masses after the Higgs scalar bosons get vacuum expectation values. For each squark or slepton ϕ with third component of weak isospin I_3^L and weak hypercharge Y, this contribution is given by

$$\Delta_\phi = M_Z^2 \cos(2\beta) \left[\cos^2 \theta_W I_3^L - \sin^2 \theta_W Y \right], \tag{3.1}$$

where

$$\tan \beta = v_u / v_d$$

is the ratio between the two expectation values of the Higgs.

Third, there is a supersymmetric contribution which is just equal to the (mass)^2 of the corresponding quark or lepton. This contribution is utterly negligible for all but the scalar partners of the top quark.

Finally, there are contributions to the scalar $(\text{mass})^2$ matrix which mix the scalar partners of the left and right handed squarks and the left and right handed charged sleptons. These contributions are again quite negligible for the first two families.

The physical squared masses are obtained from all of the above contributions. We will use $M_{\tilde{q}}, M_{\tilde{l}}$ to denote the physical masses of squarks and sleptons. Since we can neglect Yukawa couplings the spectrum is arranged in seven distinct groups of degenerate scalar states $(\tilde{u}_L, \tilde{c}_L)$; $(\tilde{d}_L, \tilde{s}_L)$; $(\tilde{u}_R, \tilde{c}_R)$; $(\tilde{d}_R, \tilde{s}_R)$; $(\tilde{e}_L, \tilde{\mu}_L)$; $(\tilde{e}_R, \tilde{\mu}_R)$; $(\tilde{\nu}_e, \tilde{\nu}_\mu)$. The members of each group transform in the same way under $SU(3) \times SU(2) \times U(1)$.

Now, experimental constraints on flavor-changing neutral currents are most easily evaded if the scalar partners of the down and strange squarks are nearly degenerate, and likewise for the up and charm squarks and the selectron and smuon, so there is already indirect experimental evidence in favor of the hypothesis of a universal mass m_0.

With the assumption of a common m_0^2, the RG equations for the squarks and sleptons can be integrated down to experimental scales to yield

$$
\begin{aligned}
M_{\tilde{u}_L}^2 &= m_0^2 + C_3 + C_2 + \frac{1}{36}C_1 + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)M_Z^2\cos(2\beta) \\
M_{\tilde{d}_L}^2 &= m_0^2 + C_3 + C_2 + \frac{1}{36}C_1 + (-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W)M_Z^2\cos(2\beta) \\
M_{\tilde{u}_R}^2 &= m_0^2 + C_3 + \frac{4}{9}C_1 + \frac{2}{3}\sin^2\theta_W M_Z^2\cos(2\beta) \\
M_{\tilde{d}_R}^2 &= m_0^2 + C_3 + \frac{1}{9}C_1 - \frac{1}{3}\sin^2\theta_W M_Z^2\cos(2\beta) \\
M_{\tilde{e}_L}^2 &= m_0^2 + C_2 + \frac{1}{4}C_1 + (-\frac{1}{2} + \sin^2\theta_W)M_Z^2\cos(2\beta) \\
M_{\tilde{\nu}_e}^2 &= m_0^2 + C_2 + \frac{1}{4}C_1 + \frac{1}{2}M_Z^2\cos(2\beta) \\
M_{\tilde{e}_R}^2 &= m_0^2 + C_1 - \sin^2\theta_W M_Z^2\cos(2\beta)
\end{aligned} \tag{3.2}
$$

where we have added the contributions of the D^2-term (3.1). The C_i factors are given by

$$
C_i(t) = \left\{ \begin{array}{c} 3/5 \\ 3/4 \\ 4/3 \end{array} \right\} \frac{1}{2\pi^2} \int_t^{t_X} dt\, g_i(t)^2 M_i(t)^2
$$

which, after performing the integration can be written as

$$
C_i(t) = \left\{ \begin{array}{c} 3/5 \\ 3/4 \\ 4/3 \end{array} \right\} \frac{2M_i^2(t)}{b_i} \left[1 - \frac{\alpha_i^2(t_X)}{\alpha_i^2(t)} \right]. \tag{3.3}
$$

In (3.2), the functions $C_i(t)$ should be evaluated at the corresponding squark and sleptons mass poles.

Let us suppose for the moment that β is known. Then we have seven physical masses $M_{\tilde{u}_L}$, $M_{\tilde{d}_L}$, $M_{\tilde{u}_R}$, $M_{\tilde{d}_R}$, $M_{\tilde{e}_L}$, $M_{\tilde{\nu}_e}$, $M_{\tilde{e}_R}$ which essentially depend on just four unknown parameters, namely m_0^2 and C_1, C_2, and C_3. Therefore, there should be three independent sum rules which do not contain the unknown input parameters.

We can immediately use the equations to relate the masses of the squarks and sleptons which live in the same $SU(2)_L$ doublet:

$$M_{\tilde{d}_L}^2 - M_{\tilde{u}_L}^2 = -\cos(2\beta)M_W^2 \tag{3.4}$$

$$M_{\tilde{e}_L}^2 - M_{\tilde{\nu}_e}^2 = -\cos(2\beta)M_W^2 \tag{3.5}$$

For the choice $\tan\beta > 1$, $\cos(2\beta)$ is negative, so that $M_{\tilde{d}_L} > M_{\tilde{u}_L}$ and $M_{\tilde{e}_L} > M_{\tilde{\nu}_e}$. Note that these two sum rules do not rely on the assumption of universal m_0 or on the equality of the gaugino masses M_i at any initial scale. This is simply because e.g. the left-handed squarks live in the same irreducible gauge multiplet before electroweak symmetry breaking. Thus they must have the same m_0, and must be renormalized in the same way down to the electroweak scale. So the only difference in the masses of $M_{\tilde{d}_L}$ and $M_{\tilde{u}_L}$ comes from the electroweak D-term, yielding (3.4). The same argument for the left-handed sleptons yields (3.5).

We also obtain a third sum rule by taking linear combinations of (3.2):

$$2(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + (M_{\tilde{d}_R}^2 - M_{\tilde{d}_L}^2) + (M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) = \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta . \tag{3.6}$$

This relation *does* depend on the assumption of universal m_0, but again does not depend on any particular assumptions about the gaugino mass parameters. The functions C_i cancel out. The sum rule (3.6) is thus a test of the universality of m_0, without making assumptions about the other input parameters.

The remaining four independent equations can be inverted to yield expressions for the input parameters in terms of the squark and slepton masses:

$$m_0^2 = M_{\tilde{e}_R}^2 - 3(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + 4\sin^2\theta_W M_Z^2 \cos 2\beta, \tag{3.7}$$

$$C_3 = (M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2) + \frac{8}{3}(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta \tag{3.8}$$

$$C_2 = (M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) + \frac{9}{4}(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) + \left(\frac{1}{2} - \frac{17}{4}\sin^2\theta_W\right) M_Z^2 \cos 2\beta \tag{3.9}$$

$$C_1 = 3(M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2) - 3\sin^2\theta_W M_Z^2 \cos 2\beta \tag{3.10}$$

The function $C_3(t)$ varies significantly as a function of scale even over the range from M_Z to a TeV; here it should be evaluated at a typical squark mass.

In terms of the gluino mass $M_{\tilde{g}} = M_3(t_{\tilde{g}})$, we have from (3.3)

$$C_3(t) = \frac{8}{9}\frac{M_{\tilde{g}}^2}{\alpha_3^2(t_{\tilde{g}})} \left[\alpha_3^2(t) - \alpha_3^2(t_X)\right], \tag{3.11}$$

where we have used (2.3). With the assumption of a GUT, one can further require that all three gaugino masses be the same at M_X. While there are theories, derived from superstrings, where the unification of the gauge couplings does not imply the equality of the gaugino masses at that scale, in the following we may choose to assume the following GUT relation

$$M_i(t_X) \equiv m_{1/2}. \tag{3.12}$$

It follows from (2.3) that

$$M_i(t) = \alpha_i(t)\frac{M_{\tilde{g}}}{\alpha_3(t_{\tilde{g}})}.$$

Then the seven equations for the squarks and sleptons of a light family are expressed in terms of two mass parameters m_0 and $M_{\tilde{g}}$, and the angle β. We can now test the assumption of equal gaugino masses at t_X, since it implies that

$$C_1(t) = \frac{2}{11}\frac{M_{\tilde{g}}^2}{\alpha_3^2(t_{\tilde{g}})} \left[\alpha_1^2(t_X) - \alpha_1^2(t)\right], \tag{3.13}$$

$$C_2(t) = \frac{3}{2}\frac{M_{\tilde{g}}^2}{\alpha_3^2(t_{\tilde{g}})} \left[\alpha_2^2(t_X) - \alpha_2^2(t)\right]. \tag{3.14}$$

We can estimate the values for C_1, C_2 and C_3 in terms of the gluino mass, by assuming that at the unification scale, $\alpha_1(t_X) = \alpha_2(t_X) = \alpha_3(t_X) = .04$. These estimates depend strongly on the value of the QCD coupling constant at low energies, which we take to be $.08 < \alpha_3(t_{\tilde{g}}) < .11$, with the lower (upper) bound corresponding to a heavy (light) gluino. [We take $\alpha_3(M_Z) = .115$.] Then for C_1 and C_2 (in the hundreds of GeV range) we estimate

$$.020\, M_{\tilde{g}}^2 < C_1 < .037\, M_{\tilde{g}}^2, \tag{3.15}$$

$$.058\, M_{\tilde{g}}^2 < C_2 < .115\, M_{\tilde{g}}^2. \tag{3.16}$$

The other parameter C_3 should be evaluated at a typical squark mass scale $t_{\tilde{q}}$. If m_0 is small, then $t_{\tilde{q}}$ is slightly less than $t_{\tilde{g}}$, and we find

$$.67\, M_{\tilde{g}}^2 < C_3 < .80\, M_{\tilde{g}}^2 \,. \tag{3.17}$$

For larger values of m_0, the squarks will be heavier than the gluino, so from (3.11) we find that the estimate for C_3 decreases. However, a reasonable general range is

$$.35\, M_{\tilde{g}}^2 < C_3 < .80\, M_{\tilde{g}}^2 \,. \tag{3.18}$$

More precision in the expected values of the functions C_i must await a better determination of the gauge couplings as a function of scale in the sparticle mass range. The discovery of the sparticles will then allow the RG thresholds to be implemented, and the idea of gaugino mass unification tested.

From equations (3.8) and (3.9), we obtain

$$C_2 - \frac{3}{4}C_1 = M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2 + (\frac{1}{2} - 2\sin^2\theta_W)M_Z^2\cos(2\beta)$$

$$C_3 - \frac{8}{9}C_1 = M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2 - \frac{2}{3}\sin^2\theta_W M_Z^2\cos(2\beta) \,.$$

By taking their ratio we arrive at a sum rule which is independent of the gluino mass, namely (in GeV^2)

$$M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2 = \frac{C_2 - \frac{3}{4}C_1}{C_3 - \frac{8}{9}C_1}\left[M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2 - (36)^2\cos(2\beta)\right] + (20)^2\cos(2\beta) \tag{3.19}$$

or

$$M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2 = [.07 \text{ to } .31](M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2) + [(18)^2 \text{ to } (0)^2]\cos(2\beta) \,. \tag{3.20}$$

which tests the unification hypothesis for the gauge couplings and gaugino masses at M_X. The present large uncertainty in the numerical value of (3.20) is due partly to the uncertainty in $\alpha_3(M_Z)$ but more importantly to our lack of knowledge of the sparticle masses, which we need to tell us where to evaluate $C_{1,2,3}$ and where the thresholds are. At any rate, this formula shows that $M_{\tilde{d}_R} > M_{\tilde{e}_L}$.

4. THIRD FAMILY SQUARKS AND SLEPTONS

Sum rules involving the masses of third family squarks and sleptons are more complicated because of the Yukawa couplings. The presence of $y_t^2\Sigma_t^2$, $y_b^2\Sigma_b^2$ and $y_\tau^2\Sigma_\tau^2$ in the

RG equations (2.5) and (2.6), and the complicated RG evolution of the Higgs mass, makes these equations hard to integrate in a useful form. In addition, there are mixing terms in the mass matrices for the third family squarks and sleptons.

The mass-squared matrix for the top squarks is given by

$$\begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + \Delta_{\tilde{t}_L} & m_t(A_t + \mu \cot \beta) \\ m_t(A_t + \mu \cot \beta) & m_{\tilde{t}_R}^2 + m_t^2 + \Delta_{\tilde{t}_R} \end{pmatrix} \tag{4.1}$$

and that of the bottom squarks by

$$\begin{pmatrix} m_{\tilde{b}_L}^2 + m_b^2 + \Delta_{\tilde{b}_L} & m_b(A_b + \mu \tan \beta) \\ m_b(A_b + \mu \tan \beta) & m_{\tilde{b}_R}^2 + m_b^2 + \Delta_{\tilde{b}_R} \end{pmatrix} . \tag{4.2}$$

Despite these complications, with further assumptions concerning the relative magnitudes of the Yukawa couplings of the third family we can deduce some new sum rules for the third family squark masses.

The validity of the radiative electroweak symmetry breaking with a top quark mass much larger than the bottom quark mass puts restrictions on the relative magnitudes of y_b and y_t. For $y_t \ll y_b$, the radiative breaking scenario implies $m_b > m_t$. For $y_t \sim y_b$, the two Higgs develop similar vacuum expectation values, which in turn implies $m_b \sim m_t$ in the absence of fine-tuning. This leaves us with only one viable possibility, $y_t \gg y_b$. In this case, the radiative mechanism naturally favors a larger vacuum expectation value for H_u which couples to the top, yielding a consistent picture when $\tan \beta \ll m_t/m_b$. It is amusing to note that in $SO(10)$, this hierarchy of Yukawa couplings has a natural explanation provided that a **126** Higgs couples more strongly than the **10** to the top.

In the following, we therefore neglect y_b. (Numerical work[17] indicates that this is a reasonable approximation in realistic models for $\tan \beta$ less than about 10.) Then there is no mixing in the bottom squark mass matrix, and $\tilde{b}_L$ and $\tilde{b}_R$ are still the mass eigenstates. Thus, $\tilde{b}_R$ is degenerate with $\tilde{d}_R$ to a good approximation. In addition, from the RG equation for the running masses

$$16\pi^2 \frac{d}{dt}(m_{\tilde{b}_L}^2 - m_{\tilde{d}_L}^2) = 2y_t^2 \Sigma_t^2, \tag{4.3}$$

which has a positive RHS, we note the inequality

$$M_{\tilde{b}_L} < M_{\tilde{d}_L} \tag{4.4}$$

11

for the physical masses. It is also true that $\tilde{d}_R$ is lighter than $\tilde{d}_L$, but the relative placement of $\tilde{b}_L$ and $\tilde{d}_R$ cannot be determined without more detailed knowledge of the input parameters.

In the stop sector, the analysis is different, because $m_{\tilde{t}_L}^2$ and $m_{\tilde{t}_R}^2$, which are the result of running the universal value m_0^2 from Λ down to the electroweak scale, are not the actual mass eigenvalues. The mass eigenstates $\tilde{t}_1, \tilde{t}_2$ of the top squark system are found by diagonalizing the matrix (4.1), whose eigenvalues are the physical squared masses $M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2$. The sum of the (mass)^2 eigenvalues is just the sum of the diagonal entries in (4.1). So, by taking the trace, we find

$$M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2 = m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + 2m_t^2 + \frac{1}{2}M_Z^2\cos 2\beta .$$

We observe that there are two linear combinations of $m_{\tilde{t}_L}^2$, $m_{\tilde{t}_R}^2$, and $m_{\tilde{b}_L}^2$ for which the terms involving $y_t^2\Sigma_t^2$ in the RG equation (2.5) cancel out, and which therefore evolve like their counterparts from the first two families. One is the linear combination $m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 - 3m_{\tilde{b}_L}^2$ which runs exactly as the combination $m_{\tilde{u}_L}^2 + m_{\tilde{u}_R}^2 - 3m_{\tilde{d}_L}^2$ from the first two families. The D-term contributions to these two combinations are equal. We therefore obtain the interesting new sum rule

$$M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2 - 3M_{\tilde{b}_L}^2 - 2m_t^2 = M_{\tilde{u}_L}^2 + M_{\tilde{u}_R}^2 - 3M_{\tilde{d}_L}^2 \tag{4.5}$$

which relates masses of the squarks and quarks of the third family with the masses of the squarks of the first two families, without involving any input parameters.

If the top squark matrix is diagonalized by a rotation through an angle φ, we have

$$(M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2)\sin(2\varphi) = 2m_t(A_t + \mu\cot\beta),$$

$$(M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2)\cos(2\varphi) = m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 + (\frac{1}{2} - \frac{4}{3}\sin^2\theta_W)M_Z^2\cos 2\beta .$$

Eliminating the angle φ, we obtain

$$(M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2)^2 = 4m_t^2(A_t + \mu\cot\beta)^2 + \left[m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 + (\frac{1}{2} - \frac{4}{3}\sin^2\theta_W)M_Z^2\cos 2\beta\right]^2 .$$

Noting that the combination $m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2$ runs across the scales exactly like $-m_{\tilde{b}_L}^2 + m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 - m_{\tilde{u}_R}^2$, and taking into account the D-term contribution leads to our second sum rule for the third-family squarks:

$$(M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2)^2 = 4m_t^2(A_t + \mu\cot\beta)^2 + \left[M_{\tilde{b}_L}^2 - M_{\tilde{d}_L}^2 - M_{\tilde{u}_L}^2 + M_{\tilde{u}_R}^2\right]^2 \tag{4.6}$$

This equation provides a lower bound on the splitting between the top-squark masses, and illustrates how the parameters A_t and μ contribute to the splitting in the stop sector. We can also express the top-squark mixing angle in terms of the physical masses by

$$\cos 2\varphi = \frac{M_{\tilde{d}_L}^2 - M_{\tilde{b}_L}^2 + M_{\tilde{u}_L}^2 - M_{\tilde{u}_R}^2}{M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2} . \tag{4.7}$$

If the stop mixing angle φ can also be measured by other means, this may provide another interesting test.

From the form of the mixing matrix of the bottom squarks, it may be that neglecting y_b is inappropriate if μ and/or $\tan\beta$ is very large. In that case, $\tilde{b}_R$ and $\tilde{d}_R$ are no longer degenerate, and our sum rules may have to be modified.

By the same token, for large $\tan\beta$ and μ, one should also take into account the left-right mixing and the effect of the tau Yukawa coupling in the third family slepton sector. The stau (mass)^2 matrix is given by

$$\begin{pmatrix} m_{\tilde{\tau}_L}^2 + m_\tau^2 + \Delta_{\tilde{\tau}_L} & m_\tau(A_\tau + \mu\tan\beta) \\ m_\tau(A_\tau + \mu\tan\beta) & m_{\tilde{\tau}_R}^2 + m_\tau^2 + \Delta_{\tilde{\tau}_R} \end{pmatrix} . \tag{4.8}$$

For very large μ and $\tan\beta$, the splitting between $\tilde{\tau}_L$ and $\tilde{\tau}_R$ will be increased somewhat by the left-right mixing terms. Since the mixing angle is always small, we use the same names $\tilde{\tau}_L$ and $\tilde{\tau}_R$ for the mass eigenstates as for the gauge eigenstates. Also, $m_{\tilde{\nu}_\tau}^2$, $m_{\tilde{\tau}_L}^2$ and $m_{\tilde{\tau}_R}^2$ are pushed lower because of the terms proportional to $y_\tau^2 \Sigma_\tau^2$ in (2.6). Since

$$16\pi^2 \frac{d}{dt}(m_{\tilde{\nu}_\tau}^2 - m_{\tilde{\nu}_e}^2) = 2y_\tau^2 \Sigma_\tau^2 > 0$$

we know that

$$M_{\tilde{\nu}_\tau} < M_{\tilde{\nu}_e} . \tag{4.9}$$

By taking the traces of the stau (mass)^2 matrix and its selectron counterpart, and noting that the renormalization of the combination $m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 - 3m_{\tilde{\nu}_\tau}^2$ does not contain the τ Yukawa coupling, we derive the sum rule for the physical masses:

$$M_{\tilde{\tau}_L}^2 + M_{\tilde{\tau}_R}^2 - 3M_{\tilde{\nu}_\tau}^2 = M_{\tilde{e}_L}^2 + M_{\tilde{e}_R}^2 - 3M_{\tilde{\nu}_e}^2 . \tag{4.10}$$

We see from (4.9) and (4.10) that the center of mass-squared for the staus is less than that of the selectrons. Numerical work[17] shows that typically $\tilde{\tau}_L$ is slightly heavier than $\tilde{e}_L$

13

and $\tilde{\tau}_R$ is lighter than $\tilde{e}_R$ and $\tilde{\nu}_\tau$ is lighter than $\tilde{\nu}_e$ (by at most a few GeV in each case) when both $\tan\beta$ and μ are large.

5. CHARGINOS

The chargino sector consists of the fermionic partners of the charged electroweak gauge bosons and of the charged Higgs scalar bosons. The mass matrix is[18]

$$(\tilde{W}^+ \ \tilde{H}^+) \begin{pmatrix} M_2 & \sqrt{2}M_W \cos\beta \\ \sqrt{2}M_W \sin\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^- \\ -\tilde{H}^- \end{pmatrix} + \text{c.c.}$$

This mass matrix describes two charged Dirac fermion mass eigenstates $\tilde{C}_1$ and $\tilde{C}_2$ with masses

$$M_{\tilde{C}_{1,2}}^2 = \frac{1}{2}\left[(M_2^2 + \mu^2 + 2M_W^2) \pm \sqrt{(M_2^2 + \mu^2 + 2M_W^2)^2 - 4(\mu M_2 - M_W^2 \sin 2\beta)^2}\right].$$

If the gluino mass is known, then the gaugino mass parameter M_2 is $\alpha_2(M_{\tilde{g}}/\alpha_3)$, with α_3 taken at the gluino mass scale and α_2 evaluated self-consistently at M_2. Thus, measurement of the two chargino masses in principle determines the two unknown parameters μ and β. In fact, the sum of the squares of the charginos depends only on μ and not on β:

$$M_{\tilde{C}_1}^2 + M_{\tilde{C}_2}^2 = M_2^2 + 2M_W^2 + \mu^2 . \tag{5.1}$$

Also the product of the chargino eigenstates is given simply by

$$M_{\tilde{C}_1} M_{\tilde{C}_2} = \mu M_2 - M_W^2 \sin 2\beta . \tag{5.2}$$

Using these equations, and a measurement of the physical masses of $\tilde{g}$, $\tilde{C}_1$, $\tilde{C}_2$, and couplings α_2, α_3, one can solve for μ from (5.1) and then for $\sin 2\beta$ from (5.2). In a region $\tan\beta \gg 1$, this provides a more sensitive measure of the angle β than can be obtained in the squark and slepton sector via eqs. (3.4) or (3.5). The value of β determined by the chargino sector masses from (5.1) and (5.2) should therefore be used as an input for the squark and slepton sum rules.

In realistic models, it often happens that the chargino masses are close to being degenerate with two of the four neutralino masses. As we will see, this can be explained by considering the limit in which M_Z is small compared to $\mu \pm M_2$, so that electroweak

symmetry breaking can be treated as a perturbation in the chargino and neutralino mass matrices. From this point of view, the masses of the charginos are given to the lowest non-trivial order by

$$
\begin{aligned}
M_{\tilde{C}_1} &= M_2 - \frac{M_W^2(M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} \\
M_{\tilde{C}_2} &= \mu + \frac{M_W^2(\mu + M_2 \sin 2\beta)}{\mu^2 - M_2^2} \; .
\end{aligned}
\tag{5.3}
$$

The eigenstate $\tilde{C}_1$ is mostly wino and the eigenstate $\tilde{C}_2$ is mostly charged Higgsino in this limit.

6. NEUTRALINOS

The neutralino sector consists of the fermionic partners of the neutral electroweak gauge bosons and of the neutral Higgs scalar bosons. Electroweak symmetry breaking introduces mixing between these states. The mass spectrum and mixing angles are determined by the mass matrix

$$
\begin{pmatrix}
M_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\
0 & M_2 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\
-M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \cos\theta_W & 0 & -\mu \\
M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & -\mu & 0
\end{pmatrix}
\tag{6.1}
$$

in the basis $(\tilde{B}, \tilde{W}^0, -i\tilde{H}_u^0, -i\tilde{H}_d^0)$. The neutralino mass eigenvalues thus satisfy the characteristic equation

$$
\begin{aligned}
0 =&\lambda^4 - \lambda^3(M_1 + M_2) + \lambda^2(M_1 M_2 - \mu^2 - M_Z^2) \\
&+ \lambda(\mu^2[M_1 + M_2] + M_W^2[M_1 + M_2 \tan^2\theta_W] - \mu M_Z^2 \sin 2\beta) \\
&- \mu^2 M_1 M_2 + \mu M_W^2[M_1 + M_2 \tan^2\theta_W] \sin 2\beta \; .
\end{aligned}
\tag{6.2}
$$

The exact analytical expressions for the mass eigenvalues are quite complicated and not very illuminating. However, we can still make some relatively simple statements about the spectrum of neutralinos in the form of sum rules for the physical masses.

A simple relation governs the product of the neutralino masses, which is equal to the determinant of (6.1), and from (6.2) is given by

$$
M_{\tilde{N}_1} M_{\tilde{N}_2} M_{\tilde{N}_3} M_{\tilde{N}_4} = -\mu^2 M_1 M_2 + \mu M_W^2[M_1 + M_2 \tan^2\theta_W] \sin 2\beta \; .
\tag{6.3}
$$

This will provide an independent test of the values of μ and β obtained from the chargino spectrum via (5.1) and (5.2).

Knowledge of the sign of the determinant of the neutralino mass matrix is important in the derivation of neutralino mass sum rules. For $\mu < 0$, the determinant is obviously negative, and it is easy to show that one of its eigenvalues is negative and the other three positive. If $\mu > 0$, the determinant is still negative as long as $\mu M_2 > 1.6 M_W^2 \sin 2\beta$ where we have used the fact that M_1 is approximately $.5 M_2$. However, the present experimental bounds on the chargino masses ($M_{\tilde{C}_i} \geq M_Z/2$) and on the gluino mass ($M_{\tilde{g}} > 100$ GeV) still allow for the existence of a very restricted range of parameters for which the determinant is positive, namely

$$.45 < \tan \beta < 2.2 \,,$$

$$M_2^2 + \mu^2 < 1.5 M_W^2 \,,$$

$$\mu M_2 < .69 M_W^2 \,.$$

Note that LEPII can rule out the existence of this very small window by failing to detect any chargino lighter than the W. Also, the window shrinks rapidly as the lower limit on the gluino mass increases, disappearing entirely for $M_{\tilde{g}}$ greater than about 300 GeV.

The sum of the eigenvalues of the neutralino mass matrix is equal to its trace, which is $M_1 + M_2$, and thus does not depend on μ or β. In most of the allowed parameter space, where the determinant is negative, exactly one of the eigenvalues is negative. We call the neutralino eigenstate of (6.1) which corresponds to the negative eigenvalue the "flipped" neutralino. Then by relating M_1 and M_2 to the gluino mass, we arrive at the simple sum rule

$$|M_{\tilde{N}_1}| + |M_{\tilde{N}_2}| + |M_{\tilde{N}_3}| - |M_{\tilde{N}_4}| \;=\; (\alpha_1 + \alpha_2) \frac{M_{\tilde{g}}}{\alpha_3} \tag{6.4}$$

where $\tilde{N}_4$ is the flipped neutralino. In this expression, α_3 should be evaluated at the gluino mass scale, while α_1 and α_2 should be evaluated at the neutralino mass scale. Typically, one then finds very roughly that $(\alpha_1 + \alpha_2)/\alpha_3 \approx .5$ in eq. (6.4). We suggest that in future numerical work on the sparticle spectrum, it would be useful to specify not only the masses of the four neutralinos, but also which of them is the flipped neutralino in the sense discussed here.

In the very unlikely case discussed above of a positive determinant for (6.1), the term proportional to λ^2 in the characteristic equation, $M_1 M_2 - \mu^2 - M_Z^2$ is negative, which

implies that two of the eigenvalues are negative and two are positive. Then the trace sum rule (6.4) would be replaced by

$$|M_{\tilde{N}_1}| + |M_{\tilde{N}_2}| - |M_{\tilde{N}_3}| - |M_{\tilde{N}_4}| \;=\; (\alpha_1 + \alpha_2)\frac{M_{\tilde{g}}}{\alpha_3}\,. \tag{6.5}$$

The sum of the squares of the neutralino masses is given by the trace of the square of (6.1):

$$M_{\tilde{N}_1}^2 + M_{\tilde{N}_2}^2 + M_{\tilde{N}_3}^2 + M_{\tilde{N}_4}^2 = M_1^2 + M_2^2 + 2\mu^2 + 2M_Z^2\,. \tag{6.6}$$

Combining this with the chargino (mass)^2 relation (5.1), and writing M_1 and M_2 in terms of the gluino mass, we arrive at the sum rule

$$2(M_{\tilde{C}_1}^2 + M_{\tilde{C}_2}^2) - (M_{\tilde{N}_1}^2 + M_{\tilde{N}_2}^2 + M_{\tilde{N}_3}^2 + M_{\tilde{N}_4}^2) = (\alpha_2^2 - \alpha_1^2)\frac{M_{\tilde{g}}^2}{\alpha_3^2} + 4M_W^2 - 2M_Z^2\,. \tag{6.7}$$

In this formula α_3 should again be evaluated at the gluino mass scale. A corollary of (6.7) is that the average squared mass of the neutralinos is always less than the average squared mass of the charginos. The virtue of (6.4) and (6.7) is that all dependence on input parameters has been eliminated in favor of physical masses and coupling constants. They should hold in general as long as the GUT assumption relating the gaugino mass parameters M_1, M_2 and M_3 is true, notwithstanding the complicated dependence of the neutralino and chargino mixings on the unknown parameters μ and β.

The mass scale of the neutralino sector is set by μ, M_1, and M_2. In fact, with $M_Z = 0$, the neutralino mass eigenvalues of (6.1) are M_1, M_2, μ and $-\mu$, and there is no mixing between gauginos and Higgsinos. Now suppose that we turn on electroweak symmetry breaking. Then, expanding in M_Z, the neutralino mass eigenvalues are perturbed to

$$
\begin{aligned}
M_{\tilde{N}_1} &= M_1 - \frac{M_Z^2 \sin^2\theta_W (M_1 + \mu\sin 2\beta)}{\mu^2 - M_1^2} \\[2mm]
M_{\tilde{N}_2} &= M_2 - \frac{M_W^2 (M_2 + \mu\sin 2\beta)}{\mu^2 - M_2^2} \\[2mm]
M_{\tilde{N}_3} &= \mu + \frac{M_Z^2 (1 + \sin 2\beta)(\mu - M_1\cos^2\theta_W - M_2\sin^2\theta_W)}{2(\mu - M_1)(\mu - M_2)} \\[2mm]
M_{\tilde{N}_4} &= -\mu - \frac{M_Z^2 (1 - \sin 2\beta)(\mu + M_1\cos^2\theta_W + M_2\sin^2\theta_W)}{2(\mu + M_1)(\mu + M_2)}\,.
\end{aligned}
\tag{6.8}
$$

These expressions generalize the ones given in [14]. They are valid so long as M_Z is small compared to $\mu \pm M_{1,2}$. (In cases like $\mu \approx \pm M_2 > M_Z$ the above expressions are not

reliable, but one can do almost-degenerate perturbation theory to find the neutralino mass eigenvalues.) If we also assume that $|\mu|$ is larger than $M_{1,2}$, then the LSP is $\tilde{N}_1$, since M_1 is typically about half of M_2. The physical neutralino masses are the absolute values of these quantities. In (6.8), the flipped neutralino is $\tilde{N}_4$ if μ is positive and is $\tilde{N}_3$ if μ is negative. The electroweak interactions split the degeneracy between the neutralinos $\tilde{N}_3$ and $\tilde{N}_4$. By comparing (6.8) with (5.3), we see that the chargino $\tilde{C}_1$ and the neutralino $\tilde{N}_2$ are exactly degenerate to this order in the expansion in M_Z^2:

$$ M_{\tilde{C}_1} = M_{\tilde{N}_2} + \mathcal{O}\left(\frac{M_Z^2}{\mu^2 - M_{1,2}^2}\right)^2 \quad . \tag{6.9} $$

Also, the neutralino $\tilde{N}_3$ is often quite close in mass to the other chargino $\tilde{C}_2$; they are exactly degenerate in the limit of no electroweak breaking and the corrections from this limit turn out to be similar. For example, in the large μ limit, one has

$$ M_{\tilde{C}_2} - M_{\tilde{N}_3} = \frac{M_Z^2}{\mu}\left[\cos^2\theta_W - \frac{1 + \sin 2\beta}{2}\right] \quad . \tag{6.10} $$

For $\tan\beta = $ a few, this happens to be numerically small. Numerical calculations have shown that these coincidences are quite good, even when the expansion in M_Z^2 is not so reliable.

7. DISCUSSION

Supersymmetry predicts[19] the existence of a light Higgs scalar, which should be discovered at LEPII if its mass is less than about 90 GeV (perhaps 118 GeV), and at the SSC or LHC otherwise. However, discovery of a light Higgs by itself will neither confirm nor deny the existence of supersymmetry, since it can also be a feature of non-supersymmetric models. The first definitive experimental signal of supersymmetry may very well turn out to be the discovery of the gluino at a hadron collider. Because the gluino is a color octet, it should be copiously produced at the SSC and LHC, and its mass measured.

In the following, we adopt the GUT assumption for the gaugino mass parameters. With the gluino mass known, this fixes the gaugino mass parameters M_2 and M_1 which appear in the chargino and neutralino sector, *and* the functions C_1, C_2, C_3 appearing in the formulas for the squark and slepton masses. Numerically, one typically has $M_1 \approx .17 M_{\tilde{g}}$, $M_2 \approx .33\, M_{\tilde{g}}$, and the ranges for C_1, C_2, C_3 are given by (3.15)-(3.18).

Squarks and Sleptons

Knowledge of the gluino mass determines the splittings in the squark and slepton $(\text{mass})^2$ spectrum. The overall scale in this spectrum is set by the universal parameter m_0^2, which does not appear in the splittings of the squared masses. The spinless sparticles of the first two families generally arrange themselves into three "bands":

• The lightest of these bands contains the three right-handed sleptons, $(\tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R)$. This is a consequence of $C_1 < C_2, C_3$. The mass scale for this band of right-handed sleptons is set by m_0 and C_1. For larger values of $\tan\beta$, $\tilde{\tau}_R$ is slightly lighter than $(\tilde{e}_R, \tilde{\mu}_R)$.

• The middle band contains the three degenerate left-handed charged sleptons, $(\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L)$ and the three sneutrinos $(\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$, with a slightly different mass determined by the sum rule (3.5). This splitting within the band is most pronounced if both m_0 and the gluino mass are in the lower part of their allowed ranges, because then the D-term contribution is relatively more significant compared to m_0^2 and C_2. The splitting between the light band of right-handed sleptons and the middle band of left-handed sleptons is governed by the value of C_2 via (3.9). The splitting between the two lower bands is more significant if the gluino mass is relatively large compared to m_0, as in "no-scale" models. For large $\tan\beta$, $\tilde{\nu}_\tau$ is lighter than $\tilde{\nu}_e$ and $\tilde{\tau}_L$ is slightly heavier than $\tilde{e}_L$.

• The heaviest band contains all of the squarks of the first two families (and $\tilde{b}_R$ if $\tan\beta$ is not too large). The essential reason they are all heavier than the sleptons, and why they congregate in a band, is because they all obtain a large common contribution from the RG equation which is $C_3 \gg C_1, C_2, M_Z^2$. Within this band, there is a small splitting between the groups $(\tilde{u}_L, \tilde{c}_L)$ and $(\tilde{d}_L, \tilde{s}_L)$ as mandated by the sum rule (3.4). The splitting (within the band) between $(\tilde{u}_R, \tilde{c}_R)$ and $(\tilde{d}_R, \tilde{s}_R)$ is small, giving a measure of the value of C_1 after the D-term contribution in (3.10) is taken into account. This can be interpreted in terms of the custodial symmetry of the standard model. For $\tan\beta > 1$, the D-term contribution to the splitting between right-handed up and down-type squarks happens to have the opposite sign from the RG contribution from C_1, increasing their tendency to be degenerate in mass. Numerically one has (in GeV^2)

$$M_{\tilde{u}_R}^2 - M_{\tilde{d}_R}^2 \approx (.1\, M_{\tilde{g}})^2 + (43)^2 \cos(2\beta)\,.$$

Knowing the value of C_3 tells us the approximate splitting of the heaviest band of squarks from the lighter bands of left-handed sleptons and of right-handed sleptons through (3.19).

These qualitative features of the spectrum of the squarks and sleptons of the first two families change drastically depending on the relative values of the gluino mass and the input parameter m_0.

In the "no-scale" limit $m_0 \ll M_{\tilde{g}}$, the three bands should be well separated in mass, with a discernable structure within each band. In this case, the sum rules (3.4), (3.5), and (3.6), which do not rely on the input parameters, can be tested. In addition, the measurement of the separation between the bands directly tests the hypothesis of equal input gaugino masses.

In the opposite "anti-no-scale" limit, $m_0 \gg M_{\tilde{g}}$, m_0 dominates the mass spectrum, all the bands are bunched together, and any hint of the structure within the bands disappears. The most extreme versions of this limit are already ruled out, because of lower limits on the mass of the gluino.

The squarks of the third family are not degenerate with those of the first two families, because the Yukawa couplings are significant.

The values of the stop masses $\tilde{t}_1$ and $\tilde{t}_2$ are the result of several competing effects. For one, the term proportional to $y_t^2 \Sigma_t^2$ in the RG equations pushes the masses lower compared to their counterparts from the first two families. There is also a positive contribution for the top squarks of magnitude m_t^2. Finally, the left-right cross-terms for the top squarks introduces a mixing depending on $A_t + \mu \cot \beta$, which increases one eigenvalue and lowers the other.

The bottom squark mass eigenstates are also different from their counterparts $\tilde{d}_L$ and $\tilde{d}_R$ because of three effects. First, $m_{\tilde{b}_L}^2$ is smaller than $m_{\tilde{d}_L}^2$ because of the term proportional to $y_t^2 \Sigma_t^2$ in the RG equations. Second, the terms proportional to $y_b^2 \Sigma_b^2$ in the RG equations push both $m_{\tilde{b}_L}^2$ and $m_{\tilde{b}_R}^2$ lower than $m_{\tilde{d}_L}^2$ and $m_{\tilde{d}_R}^2$. Finally, the left-right cross term introduces a mixing of $\tilde{b}_L$ and $\tilde{b}_R$ depending on $m_b(A_b + \mu \tan \beta)$, so that the splitting between the true bottom squark mass eigenstates is larger than the splitting between $\tilde{d}_L$ and $\tilde{d}_R$. The latter two effects are only significant if $\tan \beta$ is comparable to m_t/m_b, which we have noted is difficult to reconcile with the radiative electroweak breaking mechanism. In the usual case where $\tan \beta$ is at most about 10, $\tilde{b}_R$ is degenerate with $\tilde{d}_R$, and the bottom squark mixing is negligible.

The two sum rules (4.5) and (4.6) allow us to analyze the qualitative features of the

spectrum. When the first two family squarks are clumped together, we can rewrite (4.5) in the form

$$M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2 = 2M_{\tilde{b}_L}^2 + 2m_t^2 + (M_{\tilde{b}_L}^2 - M_{\tilde{q}}^2)$$

where $\tilde{q}$ is a generic squark from the first two families. We see that the location of the center of mass squared of $\tilde{t}_1$ and $\tilde{t}_2$ is determined by the amount by which $\tilde{b}_L$ is lower than the main squark band. Similarly, the mass squared difference sum rule (4.5) becomes effectively

$$\left(M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2\right)^2 = 4m_t^2(A_t + \mu \cot \beta)^2 + \left(M_{\tilde{q}}^2 - M_{\tilde{b}_L}^2\right)^2$$

indicating a lower bound for the splitting between $\tilde{t}_1$ and $\tilde{t}_2$ which is determined by that between $\tilde{b}_L$ and the main squark band. Thus for a small difference between $\tilde{b}_L$ and the main squark band, the split between $\tilde{t}_1$ and $\tilde{t}_2$ may be small, if $A + \mu \cot \beta$ is small. However, for large values of A or μ, the difference may be substantial. One then expects $\tilde{t}_2$ to be above the main squark band, and $\tilde{t}_1$, $\tilde{b}_L$ below. In the "anti-no-scale" limit, $\tilde{b}_L$'s mass can be much lower than the main squark band. In this case, the center of mass squared of $\tilde{t}_1$ and $\tilde{t}_2$ is lower than $\tilde{b}_L$ which is itself much lower than the rest of the squarks. Also, the split between $\tilde{t}_1$ and $\tilde{t}_2$ may be very large, depending on the crossing term. If it is large enough, $\tilde{t}_2$ will be heavier than $\tilde{b}_L$.

Charginos and Neutralinos

The masses of the charginos and neutralinos are highly correlated with each other, and are primarily determined by the input parameters μ and $\sin 2\beta$, as well as by the gluino mass. In the limit when $M_W^2 \ll \mu^2 - (.1 \, M_{\tilde{g}}^2)$, one of the charginos is degenerate with a neutralino, from eq. (6.9). The other chargino is also usually close in mass to another neutralino, especially if $\tan \beta$ is in a range near 3 or 4, as we see from eq. (6.10).

The lightest of the neutralinos (LSP) is absolutely stable. In order to avoid cosmological problems, μ and $M_{\tilde{g}}$ cannot both be arbitrarily large. The center of masses of the neutralino is smallest when the flipped neutralino is the LSP, as we see from the trace sum rule (6.4). If μ is large compared to $M_1 \approx .17 \, M_{\tilde{g}}$ and $M_2 \approx .33 \, M_{\tilde{g}}$, then $\tilde{N}_1$ in (6.8) is the LSP, and the trace sum rule still tells us about the spread of the neutralino masses, and tests the idea of gaugino mass unification. The sum rule (6.7) indicates that the center of mass squared of the charginos is higher than that of the neutralinos.

We have mentioned in Section 4 that μ and $\sin 2\beta$ are likely to be measured by the

21

chargino masses [see eqs. (5.1) and (5.2)]. Knowing μ and $\sin 2\beta$ enables us to evaluate the product of the neutralino masses through the determinant equation (6.3). Then we can further bracket the neutralino masses by invoking the near degeneracies with the chargino masses.

When μ is comparable to or greater than $M_{\tilde{g}}$, then we see from (5.3) that one of the charginos is lighter than the gluino, and one is heavier. When μ is smaller than the gluino mass, then applying the present bound on the gluino mass (100 GeV) to eqs. (5.1) and (5.2), we see that one of the charginos is still lighter than the gluino. Thus in all cases, at least one chargino is lighter than the gluino.

By using a panoply of sum rules, some of which are new, we have been able to analyze the qualitative features of the spectrum of squarks, sleptons, charginos, and neutralinos.

Mass Orderings

We repeat the main features of the spectrum:

• The squark and slepton spectrum is determined by m_0, which sets the overall scale, and $M_{\tilde{g}}$ which sets their splitting into bands.

• The chargino and neutralino masses are determined by μ and $M_{\tilde{g}}$.

Thus it is fortunate that, because of its strong interactions, it is quite likely that the gluino will be the first sparticle to be found. Below we assume knowledge of $M_{\tilde{g}}$ and proceed to discuss several possibilities.

As we have seen, there is at least one chargino which is lighter than the gluino. However, the lightest chargino may not be the lightest charged sparticle. There is a competition between the lightest chargino and the right-handed sleptons for the honor of being the lightest charged supersymmetric (odd R-parity) particle. When m_0 is large, the chargino certainly wins, but in the "no-scale"-type models, the answer is less clear and depends most crucially on the value of the parameter μ.

On the other hand, the relative value of the squark and gluon masses is not determined, since it depends directly on the input parameter m_0; if m_0 is greater than $M_{\tilde{g}}$, the squarks are heavier, and if m_0 is less than roughly .5 $M_{\tilde{g}}$ (see eq. (3.18)), the squarks are lighter than the gluinos. In the strict "no-scale" limit $m_0 = 0$, we find from (3.17) that the squark band is centered at a mass between .8 $M_{\tilde{g}}$ and .9 $M_{\tilde{g}}$. We see from eq. (3.2) that this is

22

the lightest the squark band can be relative to the gluino. As we have discussed earlier, a large m_0 implies more clumping between sleptons and squarks. The parameter m_0 is determined independently if the right-handed selectron is found at a relatively low mass.

We can summarize the relative positions of the lightest chargino, the right-handed selectron, the main squark band, and the gluino. For small m_0,

$$M_{\tilde{e}_R}, M_{\tilde{C}_1} < M_{\tilde{q}} < M_{\tilde{g}} . \qquad (m_0 \approx 0)$$

For intermediate values of m_0, the situations

$$M_{\tilde{C}_1} < M_{\tilde{e}_R} < M_{\tilde{q}} < M_{\tilde{g}} \qquad (m_0 < .5M_{\tilde{g}})$$

$$M_{\tilde{C}_1} < M_{\tilde{e}_R} < M_{\tilde{g}} < M_{\tilde{q}} \qquad (.5M_{\tilde{g}} < m_0 < M_{\tilde{g}})$$

can occur. However, for large enough m_0, one chargino and the gluino are lightest:

$$M_{\tilde{C}_1} < M_{\tilde{g}} < M_{\tilde{e}_R} < M_{\tilde{q}} . \qquad (m_0 > M_{\tilde{g}})$$

The lightest chargino and the right-handed selectron are both fine candidates to be pair-produced and studied at an e^+e^- collider like the NLC or LEPII if they are light enough. The chargino mass spectrum depends on the parameters μ and $\sin(2\beta)$, as well as on the gaugino mass parameter M_2. However, in our scenario for which the gluino is discovered and well studied at a hadron collider, the value of M_2 follows from knowledge of the gluino mass and α_3 at that scale. Then, knowledge of the chargino masses allows us to determine μ and $\sin(2\beta)$. From these two parameters one can in principle derive the whole neutralino spectrum as well, since M_1 is also known once we measure the gluino mass. In the end, the consistency of this picture becomes a numerical question of putting constraints on the input parameters of the theory through equations (5.1) and (5.2) for the charginos and (6.3) and (6.6) for the neutralinos.

References

1) For reviews, see H. P. Nilles, *Phys. Rep.* **110** (1984) 1 and H. E. Haber and G. L. Kane, *Phys. Rep.* **117** (1985) 75.

2) A. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49** (1982) 970; H. P. Nilles, *Phys. Lett.* **115B** (1982) 193; L. E. Ibáñez, *Phys. Lett.* **118B** (1982) 73; R. Barbieri, S. Ferrara, and C. Savoy, *Phys. Lett.* **119B** (1982) 343; L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27** (1983) 2359; P. Nath, R. Arnowitt and A. H. Chamseddine, *Nuc. Phys.* **B227** (1983) 121.

3) L. E. Ibáñez and G. G. Ross, *Phys. Lett.* **110B** (1982) 215; K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, *Prog. Theor. Phys.* **68** (1982) 927; L. Alvarez-Gaumé, M. Claudson, and M. Wise, *Nucl. Phys.* **B207** (1982) 16; J. Ellis, J. S. Hagelin, D. V. Nanopoulos, and K. Tamvakis, *Phys. Lett.* **125B** (1983) 275.

4) See for instance, KEK Report 92-16 (JLC-I), and references contained therein.

5) U. Amaldi, W. de Boer, and H. Furstenau, *Phys. Lett.* **B260** (1991) 447; J. Ellis, S. Kelley and D. Nanopoulos, *Phys. Lett.* **260B** (1991) 131; P. Langacker and M. Luo, *Phys. Rev.* **D44** (1991) 817.

6) J. C. Pati and A. Salam, *Phys. Rev.* **D10** (1974) 275; H. Georgi and S. Glashow, *Phys. Rev. Lett.* **32** (1974) 438; H. Georgi, in *Particles and Fields-1974*, edited by C.E.Carlson, AIP Conference Proceedings No. 23 (American Institute of Physics, New York, 1975) p.575; H. Fritzsch and P. Minkowski, *Ann. Phys. NY* **93** (1975) 193; F. Gursey, P. Ramond, and P. Sikivie, *Phys. Lett.* **60B** (1975) 177.

7) A. Font, L. E. Ibáñez and F. Quevedo, *Phys. Lett.* **B228** (1989) 79; L. E. Ibáñez and G. G. Ross, *Phys. Lett.* **B260** (1991) 291, *Nucl. Phys.* **B368** (1992) 3.

8) S. P. Martin *Phys. Rev. D* **46** (1992) 2769.

9) H. Arason, D. J. Castaño, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond, and B. D. Wright, *Phys. Rev. Lett.* **67** (1991) 2933; A. Giveon, L. J. Hall, and U. Sarid, *Phys. Lett.* **271B** (1991) 138.

10) L. Alvarez-Gaumé, J. Polchinski, and M. Wise, *Nucl. Phys.* **B221** (1983) 495; L. E. Ibáñez, C. López, and C. Muñoz, *Nucl. Phys.* **B256** (1985) 218;

11) J. S. Hagelin and S. Kelley, *Nucl. Phys.* **B342** (1990) 95; A. E. Faraggi, J. S. Hagelin, S. Kelley, and D. V. Nanopoulos, *Phys. Rev.* **D45** (1992) 3272.

12) L. E. Ibáñez and G. G. Ross, "Electroweak Breaking in Supersymmetric Models", CERN-TH.6412/92, in Perspectives in Higgs Physics, G. Kane, editor.

13) R. Arnowitt and P. Nath, SSCL-Preprint-229; *Phys. Rev. Lett.* **69** (1992) 725; *Phys. Rev.* **D46** (1992) 3981.

14) R. Arnowitt and P. Nath, *Phys. Lett.* **B289** (1992) 368.

15) G. G. Ross and R. G. Roberts, *Nuc. Phys.* **B377** (1991) 571.

16) P. Ramond, "Renormalization Group Study of the Minimal Supersymmetric Standard Model: No Scale Models", invited talk at workshop "Recent Advances in the Superworld", Houston TX, April 1993.

17) D. J. Castaño, E. J. Piard, and P. Ramond, to appear.

18) P. Nath, A. Chamseddine, and R. Arnowitt, "Applied $N = 1$ Supergravity", ICTP Series-Vol. 1 (World Scientific 1983).

19) See for instance G. L. Kane, C. Kolda, and J. D. Wells, "Calculable Upper Limit on the Mass of the Lightest Higgs Boson in Any Perturbatively Valid Supersymmetric Theory", Michigan preprint UM-TH-93-24, to appear in Phys. Rev. Lett., and references therein.