Given the 2D-to-3D corresponding points, calculate the projection matrix

First, we create a zero matrix with size $2d \times 12$, where d is the amount of data. We have $d = 298$ in our dataset.

We have 298-pair (x_i, y_i) and (X_i, Y_i, Z_i). We also know that $x_i = P(X_i)$ and thus

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

We can rewrite this equation to

$$\mathbf{AP} = \mathbf{0}$$

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ \dots \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \end{bmatrix} \begin{bmatrix} P_{11} \\ P_{12} \\ \dots \\ P_{33} \\ P_{34} \end{bmatrix} = \mathbf{0}$$

To solve the projection matrix $\mathbf{P}$ based on known matrix $\mathbf{A}$, our target is to minimize the distance between $\mathbf{AP}$ and $\mathbf{0}$.

$$\min ||\mathbf{AP} - \mathbf{0}|| = \min \mathbf{P}^T \mathbf{A}^T \mathbf{AP}$$

Therefore, if we use the corresponding eigenvector with the minimum eigenvalue of $\mathbf{A}^T\mathbf{A}$, then we can minimize the objective function to λ_{min}.

We have the eigenvector $\mathbf{e_{min}} = \mathbf{P}$ and reshape it to the original form

$$\begin{bmatrix} 0.6265 & 0.0137 & -0.3748 & -0.0604 \\ -2.2751e-4 & -0.6241 & -0.2715 & -0.0012 \\ -4.8611e-8 & 3.0957e-5 & -9.3982e-4 & 1.5004e-5 \end{bmatrix}$$

Based on the projection matrix, calculate the calibration matrix、rotation matrix, and translation matrix.

$$\mathbf{P}_{3\times4} = \mathbf{K}_{3\times3}[\mathbf{R}_{3\times3}|\mathbf{T}_{3\times1}]$$

where $\mathbf{K}$ is the upper triangular calibration matrix, $\mathbf{R}$ is the orthogonal rotation matrix, and $\mathbf{T}$ is the translation vector.

Let $\mathbf{M}_{3\times3} = \mathbf{KR}$, then $\mathbf{P}_{3\times3}(\text{the left-most part}) = \mathbf{M}^{-1}_{3\times3} = (\mathbf{KR})^{-1} = \mathbf{R}^{-1}\mathbf{K}^{-1}$.

Next, we perform QR-decomposition to $\mathbf{M}^{-1}$, then we have

$$\mathbf{K} = \begin{bmatrix} -0.6265 & -0.0016 & -0.3750 \\ 0 & 0.6327 & -0.2508 \\ 0 & 0 & -9.4033e-4 \end{bmatrix}$$

and

$$\mathbf{R} = \begin{bmatrix} -1.0000 & 3.3722e-4 & 6.2831e-5 \\ -3.3911e-4 & -0.9995 & -0.0329 \\ 5.1696e-5 & -0.0329 & 0.9995 \end{bmatrix}$$

For translation vector $\mathbf{T}$, we have the right-most part of $\mathbf{P}_{3\times1}$ and $\mathbf{K}$ and hence $\mathbf{T} = \mathbf{K}^{-1}\mathbf{P}_{3\times1}$, we have

$$\mathbf{T} = \begin{bmatrix} 0.1060 \\ -0.0083 \\ -0.0160 \end{bmatrix}$$

Use the projection matrix to calculate the projected 2D points from 3D points. Calculate the average projection error

$$\begin{aligned} \text{Error} &= \frac{1}{N}\sum_{i=1}^{N} ||\mathbf{x}_{gt} - \mathbf{P}\mathbf{X}_i|| \\ &= \frac{1}{289}\sum_{i=1}^{289} ||\mathbf{x}_{gt} - \mathbf{P}\mathbf{X}_i|| \\ &= 0.4271 \end{aligned}$$