Contents

Introduction

The format of this document is that there will be one section per geometric concept. Within each section, we will talk about some mathematical mathematical theory and discuss how it will be visualised. There will then be an interactive visualisation where you get to play around with the concept for yourself, and see first hand whatever is being described.

Rotations in $\mathbb{R}^4$

In this section, we will be exploring the group structure of rotations in $\mathbb{R}^4$. Specifically, we will show that it can be decomposed into two copies of $\mathrm{SO}(3)$, and furthermore, we will visualise this fact.

Mathematical Theory

Steps:

- show that SO(4) has a lie algebra with dynkin diagram that is 2 points.
- As SO(4) is semisimple, it is determined by its dynkin diagram.
- Since SO(3) is a simple lie algebra whos dynkin diagram is a single point, we get that SO(4) is the direct sum of SO(3) + SO(3).
- For simply connected lie groups, there is a bijective correspondence between lie groups and their lie algebras.
- SO(4) is not simply connected.
- Lie algebras and Dynkin Diagrams

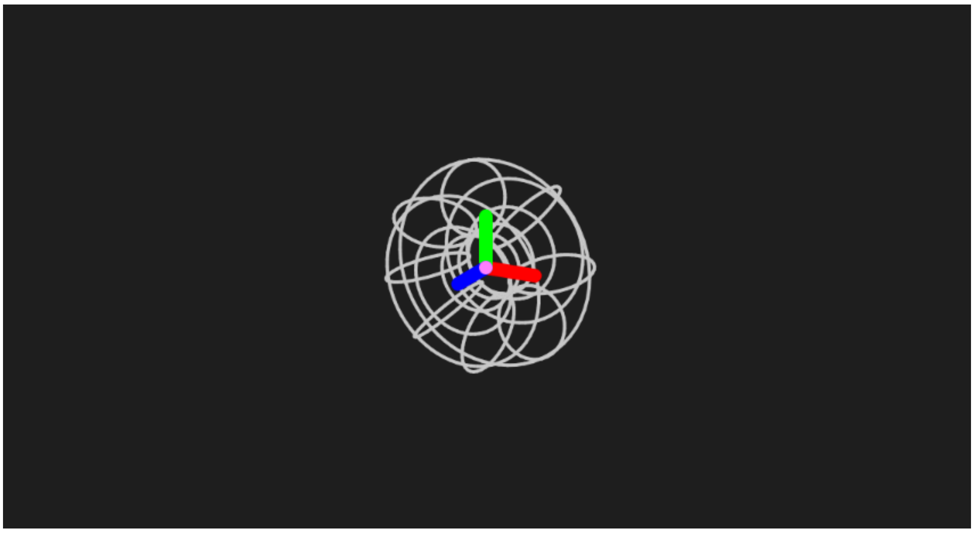
Visualisation Theory

The visualisation here allows the user to pick out 2 elements of $\mathrm{SO}(3)$ (rotations in $\mathbb{R}^3$), which are then combined in an intuitive way to form an element of $\mathrm{SO}(4)$.

When thinking about the decomposition of $\mathrm{SO}(4) \cong \mathrm{SO}(3) \oplus \mathrm{SO}(3)$, it is helpful to utilise both of these approaches in order to create the most intuitive visualisation. Remember that the goal is to visualise a single element of $\mathrm{SO}(4)$, as well as the unique pair of elements of $\mathrm{SO}(3)$ that it is associated to.

We break the visualisation down into 3 parts. The first is a visualisation of our element of $\mathrm{SO}(4)$ via it's action on a coloured hypercube with vertices at $(a, b, c, d) \in \mathbb{R}^4$ where $a, b, c, d \in \{-1, 1\}$.

Visualisation



Curved Space

Spherical Geometry

Hyperbolic Geometry

Hyperbolic 2-Space

Hyperbolic 3-Space

Cayley Graph Quotients

In this section we will show how maps from a cayley graph into $\mathbb{R}^2$ can be used to define quotients of free groups.

Mathematical Theory

Visualisation Theory

Visualisation

How to make interactive visuals like these

Bibliography