TMA4180 Optimisation: Form-finding of tensegrity structures

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1 Abstract

In this paper we will apply optimization techniques on the problem of form finding of a tensegrity structure. We will consider free and constrained optimisation on both convex and non-convex problems.

2 Introduction

2.1 Modeling of the structures

Tensegrity structures consist of bars and cables that are connected with joints. We will model the structures as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, ..., N\}$ is a set of vertices, and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is a set of edges. The vertices naturally represent the joints of the structure, and an edge $e_{ij} = (i, j)$ with $i < j$ indicates that the joints i and j are connected through either a cable or a bar.

The position of a node i is given by $x^{(i)} = (x_1^{(i)}, x_2^{(i)}, x_3^{(i)})$. Additionally, we will collect the position of all nodes in a vector $X = (x^{(1)}, ..., x^{(N)}) \in \mathbb{R}^{3N}$

The goal is to determine the position X of all the nodes. We rely on the fundamental physical principle that the structure will attain a stable resting position X^* only when the total potential energy of the system has a local or global minimum. This naturally gives us an optimization problem.

We will assume that all bars are made of the same material, and have identical thickness and cross section. However they can differ in length. A bar e_{ij} has a resting length $\ell_{ij} > 0$, where the internal elastic energy is 0. If the bar is stretched or compressed to a new length $L(e_{ij}) = \|x^{(i)} - x^{(j)}\|$, we will model the energy using a quadratic model

$$E_{elast}^{bar}(e_{ij}) = \frac{c}{2\ell_{ij}^2}(L(e_{ij}) - \ell_{ij})^2 = \frac{c}{2\ell_{ij}^2}(\|x^{(i)} - x^{(j)}\| - \ell_{ij})^2 \tag{1}$$

where the parameter $c > 0$ depends on the material and cross section of the bar. We also consider the potential energy of the bar, as it has a considerable mass.

$$E_{grav}^{bar}(e_{ij}) = \frac{\rho g \ell_{ij}}{2}(x_3^{(i)} + x_3^{(j)}) \tag{2}$$

Cables are modeled similarly, we only permit varying length. A cable has a resting length $\ell_{ij} > 0$, where the internal elastic energy is 0. Compression of a cable yields no energy, but stretching will be modeled similarly to a bar. This gives us

$$E_{elast}^{cable}(e_{ij}) = \begin{cases} \frac{k}{2\ell_{ij}^2}(\|x^{(i)} - x^{(j)}\| - \ell_{ij})^2 & \text{if} \quad \|x^{(i)} - x^{(j)}\| > \ell_{ij} \\ 0 & \text{if} \quad \|x^{(i)} - x^{(j)}\| \leq \ell_{ij} \end{cases} \tag{3}$$

where $k > 0$ is a material parameter, ρ is the mass density, and g is the gravitational acceleration. Additionally, we consider the weight of the cables negligible compared to the weight of the bars. That is:

$$E_{bar}^{cable}(e_{ij}) = 0 \tag{4}$$

We will also model external loads for a given node. If node i is loaded with mass $m_i \geq 0$, this will result in the total external energy

$$E_{ext}(X) = \sum_{i=1}^{N} m_i g x_3^{(i)} \tag{5}$$

We can express the total energy of the structure as

$$E(X) = \sum_{e_{ij}\in\mathcal{B}} (E_{elast}^{bar}(e_{ij}) + E_{grav}^{bar}(e_{ij})) + \sum_{e_{ij}\in\mathcal{C}} E_{elast}^{cable}(e_{ij}) + E_{ext}(X) \tag{6}$$

where $\mathcal{B}, \mathcal{C} \subset \mathcal{E}$ are the sets of bars and cables in the structure. This function is continuous, the only potential problem is the piecewise continuous function $E_{elast}^{cable}(e_{ij})$ at the point where $L(e_{ij}) = \|x^{(i)} - x^{(j)}\| = \ell_{ij}$. However, we see that it evaluates to 0, so this term is also continuous.

Note that minimizing (6) might not admit a solution, as the energy can be unbounded from below by letting all z-coordinates of the nodes tend to $-\infty$. We propose two solutions to this issue.

2.2 Fixing the position of a set of nodes

The first option is fixing some of the nodes such that

$$x^{(i)} = p^{(i)} \qquad \text{for } i = 1, ..., M \tag{7}$$

for some fixed $p^{(i)} \in \mathbb{R}^3$, and $1 \leq M < N$. This constraint is convenient because we still have a free optimization problem, where we have replaced some $x^{(i)}$ by $p^{(i)}$. The dimension of X is now $3(N-M)$

Theorem: If the graph $\mathcal{G}$ is connected, the objective function (6) with the constraint (7) admits a solution.

Proof: We start by showing coercivity: Using that $\mathcal{G}$ is connected, the entire structure is connected through either cables or bars. This means that for any free node in the structure, there exists some set of nodes $\mathcal{S}$ that defines a path to a fixed node $p^{(i)}$.

If we consider any possible combination of $x_1^{(i)} \to \pm\infty, x_2^{(i)} \to \pm\infty, x_3^{(i)} \to \pm\infty$, the average length of edges in $\mathcal{S}$ will also tend to ∞. For cables we will only consider the case when $\|x^{(i)} - x^{(j)}\| > \ell_{ij}$ as the energy will be equal. The elastic energy in $E(X)$ will be given by

$$\begin{aligned} \lim_{\|X\|\to\infty} \sum_{e_{ij}\in\mathcal{C}} E_{elast}^{cable}(e_{ij}) &= \lim_{\|X\|\to\infty} \sum_{e_{ij}\in\mathcal{C}} \frac{k}{2\ell_{ij}^2}(\|x^{(i)} - x^{(j)}\| - \ell_{ij})^2 = \infty \\ \lim_{\|X\|\to\infty} \sum_{e_{ij}\in\mathcal{B}} E_{elast}^{bar}(e_{ij}) &= \sum_{e_{ij}\in\mathcal{B}} \frac{c}{2\ell_{ij}^2}(\|x^{(i)} - x^{(j)}\| - \ell_{ij})^2 = \infty \end{aligned} \tag{8}$$

The fact that we allow $x_3^{(i)} \to -\infty$ could potentially result in

$$\lim_{\|X\|\to\infty} E_{ext}(X) = -\infty \quad \text{and additionally} \quad \lim_{\|X\|\to\infty} \sum_{e_{ij}\in\mathcal{B}} E_{grav}^{bar}(e_{ij}) = -\infty \quad \text{if } e_{ij} \text{ is a bar} \tag{9}$$

However, it's clear that the terms in (9) will be dominated by one of the terms in (8) because they contain quadratic terms. Hence, the total energy function (6) is coercive.

We have already shown that the function is continuous, therefore it's also lower semi-continuous and this implies that the minimisation problem admits a solution. □

If we have a disconnected graph, we would have to split the graph into subgraphs that are connected, and fix at least one node in every connected subgraph. We will not consider these situations in this paper, so we will not prove this.

2.3 Imposing positive z-values of the nodes

The second constraint models is a self-supported free standing structure, with the only condition being that it's above ground:

$$x_3^{(i)} \geq 0 \quad \forall \quad i = 1, ..., N \tag{10}$$

Note that coerciveness is not as immediate in this case. If we simultaneously move all the nodes horizontally in any direction, we see that the distance between the nodes do not change, and thus the energy is constant. This issue can be solved without a loss of generality by fixing the x_1 and x_2-position of a given node: $x^{(i)} = (p_1, p_2, x_3^{(i)})$. This simply disallows moving the entire structure horizontally.

Theorem: If the graph $\mathcal{G}$ is connected, the objective function (6) with the constraint (10) admits a solution.

With this setup, coerciveness mostly follows from the proof in the theorem above. Note that we do not allow $x_3 \to -\infty$ because of the constraint (10). Additionally, note that the energy from $E_{elast}^{cable}(e_{ij})$ and $E_{elast}^{bar}(e_{ij})$ does not tend to ∞ when we increase z simultaneously for all nodes. However, in this case the external force and bar weight will increase, and thus the total energy function is coercive. $\square$

Note that this restriction indeed creates a constrained optimization problem, unlike the constraint (7) where we had a free optimization problem in a lower dimension.

3 Cable net structures

In this section, we are analysing the situation where all members of the structure are cables, and where we fix certain nodes in order to ensure that a solution exists. This gives us the following optimization problem:

$$\min_X E(X) = \sum_{e_{ij} \in \mathcal{E}} E_{elast}^{cable}(e_{ij}) + E_{ext}(X) \quad \text{s.t. } x^{(i)} = p^{(i)}, i = 1, ..., M \tag{11}$$

In order to solve this optimization problem, we first have to show some properties about the function. We have already shown that the more general problem (6) is continuous, therefore (11) is continuous.

Theorem: The function given in (11) is C^1.

We will consider this function term by term. The gradient of $E_{ext}(X)$ is

$$\nabla E_{\mathbf{ext}}(X) = \nabla \sum_{i=1}^{N} m_i g x_3^{(i)} = (0, 0, m_1 g, 0, 0, m_2 g, ..., 0, 0, m_N g) \tag{12}$$

which is continuous. In fact, it's clear that $E_{ext}(X) \in C^\infty$.

On the other hand, the term $E_{elast}^{cable}(e_{ij})$ is obviously differentiable when $\|x^{(i)} - x^{(j)}\| \neq \ell_{ij}$. If $E_{elast}^{cable}(e_{ij})$ is to be different from zero in it's entire domain we need

$$\lim_{\|x^{(i)} - x^{(j)}\| \to \ell_{ij}^+} \nabla E_{elast}^{cable}(e_{ij}) = 0 \tag{13}$$

We will calculate the partial derivative with respect to $x_s^{(i)}$ where s represents one of the three directions: $s = 1, 2, 3$. As we take the limit from above, we only consider the expression of $E_{elast}^{cable}(e_{ij})$ when $\|x^{(i)} - x^{(j)}\| > \ell_{ij}$.

$$\frac{\partial}{\partial x_s^{(i)}} E_{elast}^{cable}(e_{ij}) = \frac{\partial}{\partial x_s^{(i)}} \frac{k}{2\ell_{ij}^2} (\|x^{(i)} - x^{(j)}\| - \ell_{ij})^2 = \frac{k}{\ell_{ij}^2} (1 - \frac{\ell_{ij}}{\|x^{(i)} - x^{(j)}\|})(x_s^{(i)} - x_s^{(j)}) \tag{14}$$

Similarly, we have

$$\frac{\partial}{\partial x_s^{(j)}} E_{elast}^{cable}(e_{ij}) = \frac{k}{\ell_{ij}^2}(1 - \frac{\ell_{ij}}{\|x^{(i)} - x^{(j)}\|})(x_s^{(j)} - x_s^{(i)}) \tag{15}$$

where the only difference is a factor of -1.

We take the limit:

$$\lim_{\|x^{(i)}-x^{(j)}\| \to \ell_{ij}^+} \frac{\partial}{\partial x_s^{(i)}} E_{elast}^{cable}(e_{ij}) = \lim_{\|x^{(i)}-x^{(j)}\| \to \ell_{ij}^+} \frac{k}{\ell_{ij}^2}(1 - \frac{\ell_{ij}}{\|x^{(i)} - x^{(j)}\|})(x_s^{(i)} - x_s^{(j)}) = 0 \tag{16}$$

This holds for all partial derivatives, which shows that the function is C^1. Note that $\|x^{(i)} - x^{(j)}\| > \ell_{ij}$, so we never divide by zero. Thus we have a sum of C^1 functions, which is C^1. □

However, the function is not C^2. Again consider the situation when $\|x^{(i)} - x^{(j)}\| > \ell_{ij}$:

$$\frac{\partial}{\partial x_2^{(i)}} \frac{\partial}{\partial x_1^{(i)}} E_{elast}^{cable}(e_{ij}) = \frac{\partial}{\partial x_2^{(i)}} \left(\frac{k}{\ell_{ij}^2}(1 - \frac{\ell_{ij}}{\|x^{(i)} - x^{(j)}\|})(x_1^{(i)} - x_1^{(j)}) \right) = \frac{k}{\ell_{ij}} \frac{(x_1^{(i)} - x_1^{(j)})(x_2^{(i)} - x_2^{(j)})}{\|x^{(i)} - x^{(j)}\|^3} =$$

Now look at the limit:

$$\lim_{\|x^{(i)}-x^{(j)}\| \to \ell_{ij}^+} \frac{k}{\ell_{ij}} \frac{(x_1^{(i)} - x_1^{(j)})(x_2^{(i)} - x_2^{(j)})}{\|x^{(i)} - x^{(j)}\|^3} = \frac{k(x_1^{(i)} - x_1^{(j)})(x_2^{(i)} - x_2^{(j)})}{\ell_{ij}^4} \neq 0$$

, and therefore the function is not C^2 in general. (Note that the problem is C^2 if we have $\sum\limits_{e_{ij} \in \mathcal{E}} E_{elast}^{cable}(e_{ij}) = 0$, but that is only possible if we have $\mathcal{E} = \emptyset$, or if no cables are stretched past ℓ_{ij}. These are not particularly interesting situations.) **Kanskje slett dette? idk**

3.1 Convexity

Convexity is another property that is of great importance when considering the choice of optimization algorithm.

Theorem: The cable net objective function (11) is convex, but not strictly convex

We will look at the convexity term by term. $E_{ext}(X) = \sum_{i=1}^N m_i g x_3^{(i)}$ is convex, but not strictly convex:

$$E(\lambda X + (1-\lambda)Y) = \sum_{i=1}^N m_i g(\lambda x_3^{(i)} + (1-\lambda) y_3^{(i)}) = \sum_{i=1}^N \lambda m_i g x_3^{(i)} + (1-\lambda) m_i g y_3^{(i)} = \lambda E(X) + (1-\lambda) E(Y)$$

Now we show that $\sum\limits_{e_{ij} \in \mathcal{E}} E_{elast}^{cable}(e_{ij})$ in (11) is convex.

Let $\mu > 0, \kappa > 0$ be constants, $g : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ where

$$g(x^{(i)}, x^{(j)}) := \kappa f(\|x^{(i)} - x^{(j)}\|) \qquad\qquad f(t) := \begin{cases} (t-\mu)^2 & , t > \mu \\ 0 & , t \leq \mu \end{cases}$$

By definition of norms, g is a convex function. Next, we must show that f is convex. By differentiating f, we obtain

$$f'(t) := \begin{cases} 2(t-\mu) & , t > \mu \\ 0 & , t \leq \mu \end{cases}$$

This shows for all $t > \mu$, $f'(t)$ is non-negative and the function value increases. Hence, $f(t)$ is a convex function. By setting $t = \|x^{(i)} - x^{(j)}\|, \mu = l_{ij}$ and $\kappa = \frac{k}{2l_{ij}^2}$ where $k > 0$ is material parameter from (3), we obtain

$$E_{elast}^{cable}(e_{ij}) = g(e_{i,j}) = g(x^{(i)}, x^{(j)}) = \kappa f(t) \implies f(t) = \frac{1}{\kappa} E_{elast}^{cable}(e_{ij}).$$

Since κ is a constant and $f(t)$ is a convex function, it implies that $E^{cable}_{elast}(e_{ij})$ is a convex function. Thus proving that (11) is a convex function as it is a sum of convex functions. $\square$

The fact that this energy expression is convex means that Quasi-Newton methods are a good candidate. As our function is not C^2, Newton's Method is not an option.

3.2 Necessary and sufficient optimality conditions

As we have a convex function that is differentiable, the necessary and sufficient optimality condition for a solution X^* is simply

$$\nabla E(X^*) = 0 \tag{17}$$

This will be a global minimizer, again due to convexity. It will not necessarily be unique, as that would require strict convexity.

4 Tensegrity domes

We will now consider the situation with added bars, but still using the constraint of fixed nodes. Our new optimisation problem has the following objective function:

$$\begin{aligned} E(X) &= \sum_{e_{ij}\in\mathcal{B}} \left(E^{bar}_{elast}(e_{ij}) + E^{bar}_{grav}(e_{ij})\right) + \sum_{e_{ij}\in\mathcal{C}} E^{cable}_{elast}(e_{ij}) + E_{ext}(X) \\ \text{s. t. } & x^{(i)} = p^{(i)}, i = 1, ..., M \end{aligned} \tag{18}$$

which still leaves us with a free optimization problem. However, the situation will be slightly more complicated.

The objective function (18) is not generally differentiable. We see that the elastic energy model for bars is the same as cables for $\|x^{(i)} - x^{(j)}\| > \ell_{ij}$ with a different material parameter, which gives us similar partial derivatives:

$$\frac{\partial}{\partial x^{(i)}_s} E^{bar}_{elast}(e_{ij}) = \frac{c}{\ell^2_{ij}}(1 - \frac{\ell_{ij}}{\|x^{(i)} - x^{(j)}\|})(x^{(i)}_s - x^{(j)}_s)$$

where $c > 0$ is the material parameter.

Note that this expression is valid for all values of $\|x^{(i)} - x^{(j)}\|$, which means that the function is not differentiable when $\|x^{(i)} - x^{(j)}\| = 0$. However, the distance being 0 would imply two nodes being in the same position which would never happen in practice, so this is not a problem. However, our function has such little smoothness causes us some other problems.

4.1 Optimality conditions for tensegrity domes

For C^2 optimisation problems, the general necessary optimality conditions for a solution X^* is

$$\begin{aligned} &\nabla E(X^*) = 0 \\ &H_f(X^*) \text{ is positive semi-definite} \end{aligned}$$

Unfortunately our objective function is not C^2. For all practical purposes as long as all nodes have unique positions, we have the necessary condition $\nabla E(X^*) = 0$, but we have no neccesary second order conditions.

Similarly, the typical sufficient condition of $H_f(X^*)$ being positive definite is not applicable. For convex functions, the one condition we actually do have of $\nabla E(X) = 0$ would be sufficient. However it turns out that our objective function is no longer convex.

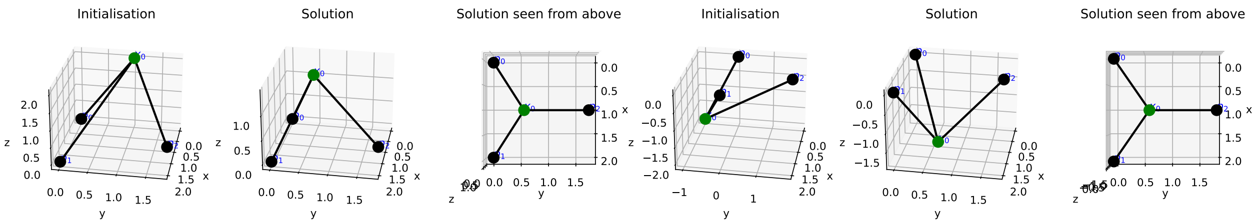

Figure 1: Local optimizer to the left, global optimizer to the right. The free node $x^{(0)}$ is connected through bars to three fixed nodes $p^{(0)}$, $p^{(1)}$, and $p^{(2)}$. Numerics will be discussed later in the paper.

4.2 Non-convexity

In order for a function to be convex we need it to hold for all $X, Y \in \mathbb{R}^{3N}$ and any $0 \leq \lambda \leq 1$. Therefore, we will consider $\lambda = \frac{1}{2}$, and X, Y such that

$$\|x^{(i)} - x^{(j)}\|, \|y^{(i)} - y^{(j)}\| = \ell_{ij} \quad \forall \quad i, j \in N \tag{19}$$

Now let $Y = -X$, that is: $x^{(i)} - x^{(j)} = -(y^{(i)} - y^{(j)}) = y^{(j)} - y^{(i)}$ such that

$$E^{bar}_{elast}(\lambda X + (1-\lambda)Y) = E^{bar}_{elast}(\frac{1}{2}X + \frac{1}{2}(-X)) = E^{bar}_{elast}(0) = \sum_{e_{ij} \in \mathcal{B}} \frac{c}{2\ell_{ij}^2}(\|0\| - \ell_{ij})^2 = \sum_{e_{ij} \in \mathcal{B}} \frac{c}{2} \tag{20}$$

which is greater than zero as $c > 0$. On the other hand with assumption (19), we get the following result,

$$\begin{aligned} &\lambda E^{bar}_{elast}(X) + (1-\lambda)E^{bar}_{elast}(Y) = \frac{1}{2}E^{bar}_{elast}(X) + \frac{1}{2}E^{bar}_{elast}(-X) \\ &= \frac{1}{2}\sum_{e_{ij} \in \mathcal{B}} \frac{c}{2\ell_{ij}^2}(\|x^{(i)} - x^{(j)}\| - \ell_{ij})^2 + \frac{1}{2}\sum_{e_{ij} \in \mathcal{B}} \frac{c}{2\ell_{ij}^2}(\|x^{(i)} - x^{(j)}\| - \ell_{ij})^2 = 0 \end{aligned} \tag{21}$$

$$E^{bar}_{elast}(\lambda X + (1-\lambda)Y) = \sum_{e_{ij} \in \mathcal{B}} \frac{c}{2} \nleq 0 = \lambda E^{bar}_{elast}(X) + (1-\lambda)E^{bar}_{elast}(Y)$$

Thus showing that the objective function is not convex. □

Since the function is non-convex, it means that we need the general optimality conditions, as well as restricting our choice of algorithms. This also means that there can exist local minima that are not global, as seen in figure 1.

Fix a three nodes $p^{(0)}$, $p^{(1)}$, and $p^{(2)}$, and consider a free node $x^{(0)}$ that initially is placed above the xy-plane. The position of $x^{(0)}$ will be determined by the equilibrium between external load, gravity, and elastic energy of the bar. However, if we initialize the At this starting position the bar will not have any elastic energy, so the gravity will bring the node downwards until the gravity is equally strong as the elastic force from the bar, which will be a local minima. However, it will not be a global minima as we will have lower energy if we instead placed $x^{(2)}$ below $p^{(1)}$ as seen in the rightmost illustration.

5 Tensegrity domes in constrained optimization

We will now consider the full problem (18) with the constraints given by

$$x_3^{(i)} \geq 0 \quad \forall \quad i = 1, ..., N \tag{22}$$

As this is a constrained optimization problem, we define the Lagrangian

$$\mathcal{L}(X,\lambda) = E(X) - \sum_{i\in \mathcal{I}} \lambda_i c_i(X) \tag{23}$$

Where $c_i(X) = x_3^{(i)}$.

The first order optimality conditions for a given (X^*, λ^*) are

$$\begin{aligned}
&\nabla_x \mathcal{L}(X, \lambda^*) = 0 \\
&x_3^{*(i)} \geq 0, \quad i = 1, ..., N \\
&\lambda_i^* \geq 0, \quad i = 1, ..., N \\
&\lambda_i^* x_3^{*(i)} = 0, \quad i = 1, ..., N
\end{aligned} \tag{24}$$

As for LICQ we have

$$\begin{aligned}
\nabla c_1(X) = \left(\frac{\partial c_1}{\partial x^{(1)}}, \frac{\partial c_1}{\partial x^{(2)}}, ..., \frac{\partial c_1}{\partial x^{(i)}}, ..., \frac{\partial c_1}{\partial x^{(N)}}\right) &= \Big((0,0,1),(0,0,0),...,(0,0,0),...,(0,0,0)\Big) \\
\nabla c_2(X) &= \Big((0,0,0),(0,0,1),...,(0,0,0),...,(0,0,0)\Big) \\
&\vdots \\
\nabla c_i(X) &= \Big((0,0,0),(0,0,0),...,\underbrace{(0,0,1)}_{i^{\text{th}}\text{ term}},...,(0,0,0)\Big) \\
&\vdots \\
\nabla c_N(X) &= \Big((0,0,0),(0,0,0),...,(0,0,0),...,(0,0,1)\Big)
\end{aligned}$$

It's clear that the inequality constraints $\{\nabla c_i(X), i = 1, 2, ..., N\}$ are linearly independent as there are no vectors with non-zero terms in the same dimension. This means LICQ holds, and therefore the KKT conditions are neccesary, but they are not sufficient as our problem is not convex.

6 Numerical

6.1 Methods

The next section include a number of numerical experiments and results. This section is used to explain our programmatic approach to the problem. The code used to solve the optimization problems and to generate the plots used can be found in this Github repository. There are three parts of the code.

6.1.1 BFGS

One part is a completely generic implementation of the BFGS method in the algoritmer.py file. This implementation follows closely that of algorithm 6.1 in [NW06], the only difference being the choice of steplength. Our implementation use a line search method to find a steplength that satisfies the *strong* Wolfe conditions, rather than the regular ones used in the book.

The linesearch method is based on algorithm 3.5 in [NW06], with a bisecting interpolation implementation of zoom (algorithm 3.6). The next steplength is chosen according to $\alpha_{k+1} = \rho\alpha_k$. The number ρ together with c_1 and c_2 completely specify the algorithm, and are preset as

ρ	c_1	c_2
2	0.01	0.9

The BFGS iterations stop either when some predetermined maximum iterations has been reached, or when the norm of the gradient is below a threshold of 1×10^{-10}

6.1.2 Tensegrity

The second part is the generation of objective and gradient functions, and can be found in the tensegrity.py file.

When creating a TensegrityStructure, the functions gen_E and gen_grad_E are called with the corresponding cables, bars, free weights, fixed points and rest lengths and returns the objective and gradient function of the setup according to the equations in the previous sections. The returned functions take as input only the position of the free points, which are the variables to be optimized. This is neat, as it allows us to use any generic optimization algorithm to solve the problem.

All tests can be found in the tests.py file, but the Tensegrity setup for all the numerical experiments below is given before the corresponding results.

6.1.3 Freestanding structures

Instead of solving (18) subject to (22), we added quadratic penalization to the energy and gradient functions. The objective function thus gained a term of the from

$$E_{qp} = \sum_{i \in \mathcal{N}} \frac{1}{2} \mu (x_3^{(i)})^2 \tag{25}$$

where $\mathcal{N}$ is the set of all points with z-component smaller than zero.

μ is initialized as 1, an

6.2 Experiments

6.2.1 Cable nets

For the first experiment we will consider 4 free and 4 fixed nodes along with following parameters:

$$\text{4 fixed nodes } p^{(1)} = (5,5,0),\ p^{(2)} = (-5,5,0),\ p^{(3)} = (-5,-5,0),\ p^{(4)} = (5,-5,0)$$
$$\mathcal{E} = \{(1,5),\ (2,6),\ (3,7),\ (4,8),\ (5,6),\ (6,7),\ (7,8),\ (8,5)\}$$
$$k = 3, \quad \ell_{ij} = 3 \quad \forall \quad (i,j) \in \mathcal{E}, \quad m_i g = \frac{1}{6}, \quad i = 5,6,7,8$$

This problem has a analytical solution for the free nodes:

$$x^{(5)} = (2,2,-\frac{3}{2}),\ x^{(6)} = (-2,2,-\frac{3}{2}),\ x^{(5)} = (-2,-2,-\frac{3}{2}),\ x^{(5)} = (2,-2,-\frac{3}{2})$$

We see from (2) that we indeed reach this configuration of nodes.

Noe om at den ender opp i samme løsning uansett startkonfigurasjon?

6.2.2 Tensegrity domes

We now consider bars as well, and will use the 4 fixed nodes and the following parameters:

$$p^{(1)} = (1,1,0),\ p^{(2)} = (-1,1,0),\ p^{(3)} = (-1,-1,0),\ p^{(4)} = (1,-1,0)$$
$$\ell_{15} = \ell_{26} = \ell_{37} = \ell_{48} = 10, \qquad \ell_{18} = \ell_{25} = \ell_{36} = \ell_{47} = 8, \qquad \ell_{56} = \ell_{67} = \ell_{78} = \ell_{58} = 1$$
$$c = 1, \quad k = 0.1, \quad g\rho = 0, \quad m_i g = 0, \quad i = 5,6,7,8$$

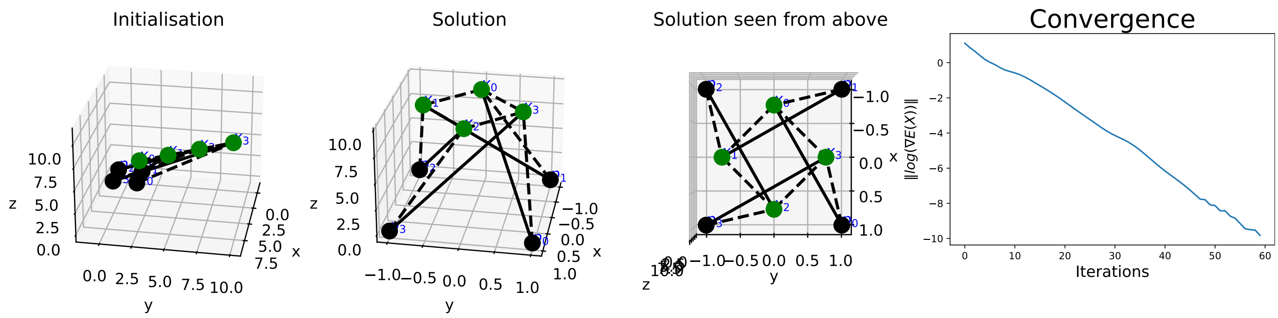

Figure 2: Cable net structure

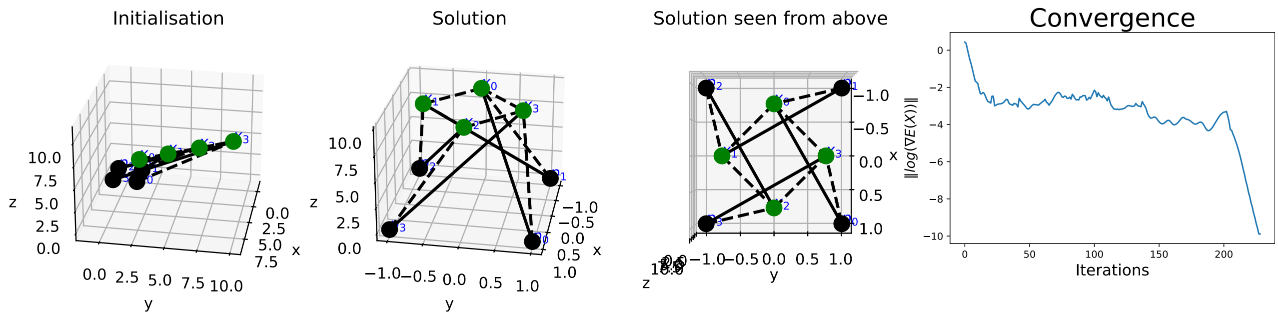

Figure 3: Tensegrity dome

Again, we have an analytical solution to this problem, namely

$$x^{(5)} = (-s, 0, t), x^{(6)} = (0, -s, t), x^{(7)} = (s, 0, t), x^{(8)} = (0, s, t), \text{with } s \approx 0.70970,\ t \approx 9.54287$$

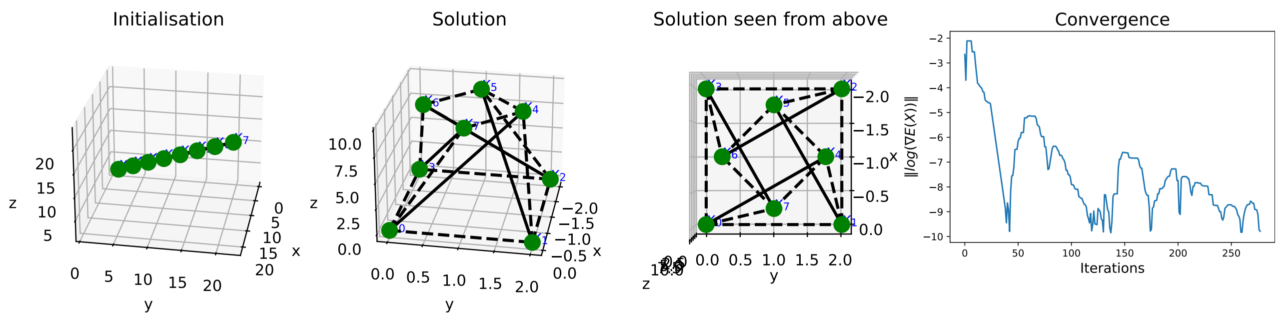

Figure 4: Free standing

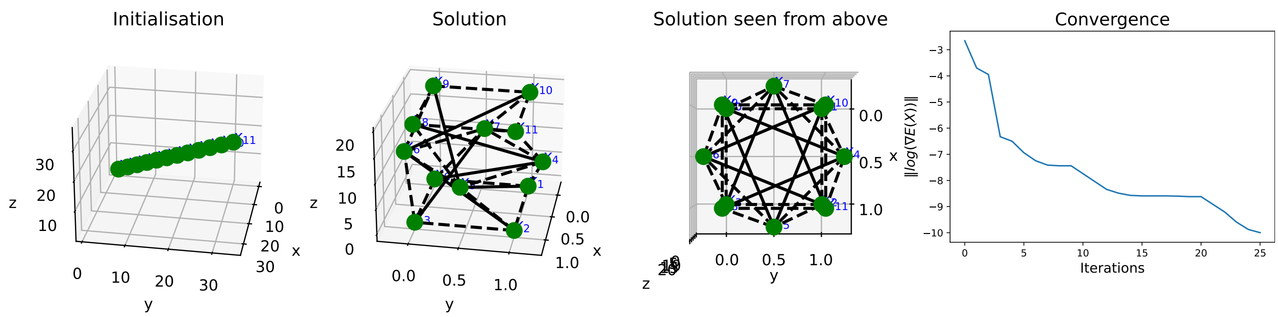

Figure 5: 2 free standing structures stacked

7 Conclusion

References

[NW06] Jorge Nocedal and Stephen J. Wright. *Numerical Optimization*. Second. Springer, 2006. ISBN: 0387303030.