**Problem 1:**

Using direct proof, prove : **If n is any even integer, then (−1)n = 1**

If n is any positive even integer

Then (-1)n = 1

Proof:

Since n is any positive even integer

Then n = 2r for some r E Z

Then (-1)n = (-1)2r

Since (ab)c = a bc

(-1)n = ((-1)2)r

(-1)n = (1)r

(-1)n = 1 proved

**Problem 2:**

Using induction proof, prove **for integer**  ***n*** > **5, 4*n* < 2*n***

For n as an integer, prove by induction, that for n=>5, 4n < 2n.

This is what I have:

Base case n= 5

4(5) < 25

20 < 32 --Correct

Induction

Assuming 4n < 2n then 4k < 2k

Prove

4(k+1) < 2k+1

  4k + 4 < 2k + 2k

4(5) + 4 <25 + 25 ----5 from base case

24 < 64 ----Correct

**Problem 3:**

Prove by induction that (11n – 6) is divisible by 5 for every positive integer n.

Induction hypothesis: Assume that P(k) is correct for some positive integer k.

11k −6 is divisible by 5 and hence 11k −6 = 5m for some integer m.

So 11k =5m+6.

Induction step: We will now show that P(k + 1) is correct.

In this case we want to show that 11k+1− 6 can be expressed as a multiple of 5.

so we will start with the formula 11k+1− 6 and we will rearrange it into something

involving multiples of 5. Also assume that 11k = 5m + 6 .

11k+1−6 =(11×11k)−6

= 11(5m + 6) − 6

= 11(5m) + 66 − 6

  = 5(11m) + 60  
   = 5(11m + 12)

As 11m + 12 is an integer we have that 11k+1− 6 is divisible by 5, so P(k + 1) is

correct.

Hence by induction, P(n) is correct for all positive integers n.

**Problem 4:**

Prove the following statement by Contradiction.

**The sum of a rational number and an irrational number is irrational.**

Suppose x is rational and y is irrational. So

x

Now suppose that sum of x and y is rational, so we can write

x + y =

+ y =

y**=**  -

y **=**

This shows that **y** is rational, but we have assumed that **y** is irrational.

Hence we get contradiction. So, sum must be irrational.