Exercises, Calculus

1. Simplify the following expressions:

a) $e^a e^{2-a}$ b) e^{x-y}/e^{x+y} c) $\frac{e^{(x+y)^2}}{e^{x^2}}$

2. Simplify the following expressions:

a) $\ln(x^3) - \ln(x^2)$ b) $\ln(a/b) + \ln(b/c)$ c) $\frac{1}{4}\ln\left(x^2\right)$

d) $a\ln(x) - b\ln(x^2)$

3. Calculate the derivative, $f'(x)$, of the following functions:

a) $f(x) = 5x - 3$ b) $f(x) = 24 + 3x^3$ c) $f(x) = e^x$

d) $f(x) = 2\cos(x)$ e) $f(x) = -\sin(x)$ f) $f(x) = \ln(x)$

4. Calculate the derivative, $f'(x)$, of the following functions:

a) $f(x) = \sin(x) - 3x^2$ b) $f(x) = \frac{3}{x^{2.5}}$ c) $f(x) = e^{2x}$

d) $f(x) = e^{2x-4}$ e) $f(x) = e^{-\sin(x)}$ f) $f(x) = \frac{\ln(x)}{x - 3x^3}$

g) $x\ln(x) - x$

5. Linearize:

a) $3x^2 - 5$ at $x = 2$ b) e^{2x} at $x = 0$ c) $\ln(x + 5)$ at $x = -1$

d) $\sin(-x) - 2x$ at $x = 0$

Extra: Write an R script that plots these function together with their linearizations.

6. Find the maximal value of:

a) xe^{-x} b) $5y - y^2 + 3$ c) e^{t-t^2}

7. Calculate the primitive function of:

a) x b) $\frac{1}{x}$ c) e^{5x}

d) $\cos(2x)$ e) $x^3 - 5x$ f) $\ln(x)$ (compare **4g**!)

Answers

1. a) e^2 b) e^{-2y} c) e^{2xy+y^2}

2. a) ln(x) b) ln(a/c) = ln(*a*) – ln(*c*) c) $\frac{1}{2}\ln(x)=\ln\left(x^{\frac{1}{2}}\right)$

d) (*a* – 2*b*)ln(*x*)

3. a) 5 b) $9x^2$ c) e^x

d) –2sin(*x*) e) –cos(*x*) f) $\frac{1}{x}$

4. a) cos(*x*) – 6*x* b) $-7.5x^{-3.5}$ c) $2e^{2x}$

d) $2e^{2x-4}$ e) $-\cos(x)e^{-\sin(x)}$

f) $\frac{1-3x^2-\ln(x)\left(1-9x^2\right)}{\left(x-3x^3\right)^2}$ g) ln(*x*)

5. a) 7 + 12(*x* – 2) = 12*x* – 17 b) 1 + 2*x* c) $\ln(4)+\frac{1}{4}(x+1)$

d) –3*x*

6. a) e^{-1} b) $\frac{37}{4}=9\frac{1}{4}$ c) $e^{\frac{1}{4}}$

7. a) $\frac{x^2}{2}+C$ (*C* is arbitrary constant) b) ln(*x*) + *C* c) $\frac{1}{5}e^{5x}+C$

d) $\frac{1}{2}\sin(2x)+C$ e) $\frac{x^4}{4}-5\frac{x^2}{2}+C$ f) $x\ln(x)-x+C$