Exercises in Basic Algebra

1) Expand the following expressions:

a) $3x(yz - y^2 + x^2)$ b) $(3c - d)(a + b)$ c) $(c + d)^2$

d) $(c - d)^2$ e) $\left(\frac{1}{a} + a\right)^2$ f) $(c + d + e)^2$

g) $(c + d - e)^2$

2) Simplify the following expressions:

a) $(x + y)^2 - (x - y)^2$ b) $(s - t)(s + t) + \left(t - \frac{s^2}{2t}\right)^2$ c) $\frac{x^2 + 2xy + y^2}{x + y}$

d) $\frac{a^2 - b^2}{a - b}$ e) $\frac{ac^2 - az^2}{c + z}$

3) Simplify the following:

a) $s^2 s^4$ b) $\frac{w^5}{w^3}$ c) $x^{2.5}\sqrt{x}$

d) $(d^5)^8$ e) $\frac{(a^6)^{\frac{1}{3}}}{a}$ f) g^0

g) $z^{-2}\sqrt{z^3}$ h) $\frac{(ab)^3}{a^3}$ i) $(xy)^4 (yz)^{-3} (xz)^2$

4) Write out the following sums without a summation notation (without a Σ):

a) $\sum_{k=1}^{3} 3k$ b) $\sum_{k=4}^{5} 3k$ c) $\sum_{j=0}^{3} j^2$

d) $\sum_{m=5}^{8} (m - 3)$ e) $\sum_{l=-3}^{1} (l^2 + 1)$ f) $\sum_{k=2}^{4} \frac{k + 2}{k - 1}$

5) Calculate the following arithmetic sums:

a) $\sum_{k=1}^{10} k$ b) $\sum_{k=1}^{10} 2k$ c) $\sum_{k=1}^{4} 3$

d) $\sum_{k=1}^{4} (3 + k)$

6) Calculate the following geometric sums:

a) $\sum_{k=0}^{3} 2^k$ b) $\sum_{k=-1}^{2} 3^k$ c) $\sum_{j=0}^{4} \left(\frac{1}{3}\right)^j$

d) $\sum_{j=0}^{\infty} \left(\frac{1}{3}\right)^j$ e) $\sum_{k=1}^{4} (-2)^k$ f) $\sum_{m=0}^{\infty} (-0.2)^m$

g) $\sum_{k=1}^{3} 2 \cdot 3^k$ h) $\sum_{j=0}^{\infty} 3 \cdot 0.5^j$

7) Solve the following equations:

a) $x + 3 = 8$ b) $4x - 3 = 2x + 3$ c) $\frac{3+x}{x-1} = 2$

d) $3x = 2x$ e) $x - 2 = \frac{-4+x}{3}$

8) Solve the following equations:

a) $a^2 - 4a + 3 = 0$ b) $x^2 - 2x - 3 = 0$ c) $4x + 6 = 2x^2$

d) $x - 2 = \frac{3}{x-4}$ e) $\frac{y^2}{3} + 2y = -3$

9) Calculate the following:

a) $(3 + 4i) + (1 - 2i)$ b) $(3 + 4i)(1 - 2i)$ c) $(3 + 4i)(3 - 4i)$

Answers

1)a) $3xyz - 3xy^2 + 3x^3$ b) $3ac + 3bc - ad - bd$ c) $c^2 + 2cd + d^2$

d) $c^2 - 2cd + d^2$ e) $\frac{1}{a^2} + 2 + a^2$

f) $c^2 + d^2 + e^2 + 2cd + 2ce + 2de$ g) $c^2 + d^2 + e^2 + 2cd - 2ce - 2de$

2) a) $4xy$ b) $\frac{s^4}{4t^2}$ c) $x + y$

d) $a + b$ e) $a(c - z)$

3)a) s^6 b) w^2 c) x^3

d) d^{40} e) a f) 1

g) $z^{-\frac{1}{2}} = \frac{1}{\sqrt{z}}$ h) b^3 i) x^6yz^{-1}

4)a) 3 + 6 + 9 b) 12 + 15 c) 0 + 1 + 4 + 9

d) 2 + 3 + 4 + 5 e) 10 + 5 + 2 + 1 + 2 f) $\frac{4}{1} + \frac{5}{2} + \frac{6}{3}$

5)a) 55 b) 110 c) 12

d) 22

6)a) 15 b) $\frac{40}{3} = 13\frac{1}{3}$ c) $\frac{121}{81} \approx 1.4938$

d) 3/2 = 1.5 e) 10 f) $\frac{1}{1.2} = \frac{10}{12} = \frac{5}{6}$

g) 78 h) 6

7)a) x = 5 b) x = 3 c) x = 5

d) x = 0 e) x = 1

8)a) $a = 2 \pm 1 \Rightarrow a_1 = 1,\ a_2 = 3$ b) $x_1 = -1,\ x_2 = 3$ c) $x_1 = -1,\ x_2 = 3$

d) $x_1 = 1,\ x_2 = 5$ e) y = -3

9)a) 4 + 2i b) 11 - 2i c) 25