Exercises in Linear Algebra

1. Calculate : a) $(1 \;\; 2 \;\; 3)^T + (2 \;\; -1 \;\; 1)^T$

b) $(1 \;\; 5) \cdot (2 \;\; 0)$

c) $\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 5 \\ 1 & 0 \end{pmatrix}$

2. Calculate: a) $\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

b) $\begin{pmatrix} 3 & 0 & -2 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

c) $\begin{pmatrix} 5 & 10 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} -2 & 2 & 5 \\ 1 & 2 & 3 \end{pmatrix}$

3. Calculate: a) $\begin{pmatrix} 5 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

b) $\begin{pmatrix} 4 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 7 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

4. Calculate: a) $\begin{pmatrix} 2 & 1 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

b) $\begin{pmatrix} 3 & 4 & 10 \\ 1 & -2 & 20 \\ 7 & -1 & 30 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

c) $\begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

d) $\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

5. Expand: a) $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$

b) $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$

c) $(A + B)(A - B)$

Answers

1. a) $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ b) 2 c) $\begin{pmatrix} 3 & 8 \\ 0 & 2 \end{pmatrix}$

2. a) $\begin{pmatrix} 11 \\ 4 \end{pmatrix}$ b) $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 30 & 55 \\ 1 & -10 & -19 \end{pmatrix}$

3. a) 23 b) $\begin{pmatrix} 20 & 4 \\ 15 & 3 \end{pmatrix}$ c) 24

4. a) $\begin{pmatrix} 2x+y \\ 5x-2y \end{pmatrix}$ b) $\begin{pmatrix} 3x+4y+10z \\ x-2y+20z \\ 7x-y+30z \end{pmatrix}$ c) $\begin{pmatrix} ax+by \\ bx+cy \end{pmatrix}$ d) $ax^2 + 2bxy + cy^2$

5. a) $\mathbf{u}\cdot\mathbf{u} + 2\mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v} = |\mathbf{u}|^2 + 2\mathbf{u}\cdot\mathbf{v} + |\mathbf{v}|^2$ b) $|\mathbf{u}|^2 - |\mathbf{v}|^2$
   c) $AA - AB + BA - BB = A^2 - AB + BA - B^2$