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Assignment 6

1

1.a

$$
\begin{aligned}
P(x) &= f(0)\frac{(x-h)(x-2h)}{0-h)(0-2h)} + f(h)\frac{(x-0)(x-2h)}{(h-0)(h-2h)} + f(2h)\frac{(x-0)(x-h)}{(2h-0)(2h-h)} \\
&= f(0)\frac{x^2-3hx+2h^2}{2h^2} + f(h)\frac{x^2-2hx}{-h^2} + f(2h)\frac{x^2-hx}{2h^2}
\end{aligned}
$$

1.b

$$
\begin{aligned}
\int_0^{2h} f(x)dx \approx \int_0^{2h} P(x)dx& \\
&= \frac{f(x)}{2h^2}\left(\frac{x^3}{3} - \frac{3hx^2}{2} + 2h^2x\right) + \frac{f(h)}{-h^2}\left(\frac{x^3}{3} - \frac{2hx^2}{2}\right) + \frac{f(2h)}{2h^2}\left(\frac{x^3}{3} - \frac{hx^2}{2}\right)\Bigg|_0^{2h} \\
&= \frac{f(x)}{2h^2}\left(\frac{(2h)^3}{3} - \frac{3h(2h)^2}{2} + 2h^2(2h)\right) - \frac{f(h)}{h^2}\left(\frac{(2h)^3}{3} - \frac{2h(2h)^2}{2}\right) \\
&\qquad + \frac{f(2h)}{2h^2}\left(\frac{(2h)^3}{3} - \frac{h(2h)^2}{2}\right) \\
&= \frac{f(x)}{2h^2}\left(\frac{8h^3}{3} - \frac{12h^3}{2} + 4h^3\right) - \frac{f(h)}{h^2}\left(\frac{8h^3}{3} - \frac{8h^3}{2}\right) + \frac{f(2h)}{2h^2}\left(\frac{8h^3}{3} - h^3\right) \\
&= f(0)\left(\frac{4h}{3} - 3h + 2h\right) - f(h)\left(\frac{8h}{3} - \frac{8h}{2}\right) + f(2h)\left(\frac{4h}{3} - h\right) \\
&= \frac{hf(0)}{3} + \frac{4hf(h)}{3} + \frac{hf(2h}{3} \\
&= \frac{h}{3}\left(f(0) + 4f(h) + f(2h)\right)
\end{aligned}
$$

1.c

$$
\begin{aligned}
\int_0^{0.2} f(x)dx \approx \int_0^{2h} P(x)dx& \qquad \text{with } h = 0.1 \\
&= \frac{h}{3}\left(f(0) + 4f(h) + f(2h)\right) \\
&= \frac{0.1}{3}\left(0.5 + 4 \cdot 0.50125 + 0.50503\right) \\
&\approx 0.1003343
\end{aligned}
$$

2

2.a

$f(x)$	$\int_{-1}^{1} f(x)dx$	$\frac{6}{7}f\left(-\sqrt{\frac{2}{5}}\right) + \frac{2}{7}f(0) + \frac{6}{7}f\left(\sqrt{\frac{2}{5}}\right)$
1	2	2
x	0	0
x^2	$\frac{2}{3}$	$\frac{24}{35}$

So the degree of precision of the quadrature formula is 1.

2.b

If $f(x) = e^{-x}\sqrt{x+1}$, then $f\left(-\sqrt{\frac{2}{5}}\right) = 1.141108375$, $f(0) = 1$, and $f\left(\sqrt{\frac{2}{5}}\right) \approx 0.6788107828$, so the quadrature formula gives

$$\frac{6}{7}f\left(-\sqrt{\frac{2}{5}}\right) + \frac{2}{7}f(0) + \frac{6}{7}f\left(\sqrt{\frac{2}{5}}\right) \approx 1.845644992$$

3

3.a

```
function trap(a, b, maxiter, tol, f)
m = 1;
x = linspace(a, b, m+1);
y = f(x);
approx = trapz(x, y);
fprintf('  \tm  \tintegral approximation\n');
fprintf(' %5.0f %16.10f\n', m, approx);
for i = 1 : maxiter
    m = m * 2;
    oldapprox = approx;
    x = linspace(a, b, m+1);
    y = f(x);
    approx = trapz(x, y);
    fprintf(' %5.0f %16.10f\n', m, approx);

    if abs(1-oldapprox/approx) < tol
        return
    end
```

```
end
fprintf('Did not converge in %g iterations\n', maxiter);
```

3.b

```
f = @(x) cos(1./x)

f =

    @(x)cos(1./x)

g = @(x) (exp(2.*x))./sqrt(x.^2+1)

g =

    @(x)(exp(2.*x))./sqrt(x.^2+1)

trap(0.5, 2, 20, 10^-5, f);
          m          integral approximation
      1       0.3460767940
      2       0.6955684290
      4       0.8096147845
      8       0.8405732779
     16       0.8483519280
     32       0.8502886073
     64       0.8507719513
    128       0.8508927293
    256       0.8509229200
    512       0.8509304675
trap(0, 2, 20, 10^-10, g);
          m          integral approximation
      1      25.4170349840
      2      17.9333691661
      4      15.7530628094
      8      15.1827950682
     16      15.0385646992
     32      15.0024015477
     64      14.9933541353
    128      14.9910918677
```

```
    256      14.9905262749
    512      14.9903848751
   1024      14.9903495250
   2048      14.9903406875
   4096      14.9903384781
   8192      14.9903379258
  16384      14.9903377877
  32768      14.9903377532
  65536      14.9903377445
  131072      14.9903377424
  262144      14.9903377418
diary off
```