2 Preliminaries: math

2.1 Basic calculus

2.1.1

$$\frac{\partial y}{\partial x} = 2ax + b$$

2.1.2

$$\frac{\partial y}{\partial x} = \cos^2 x - \sin^2 x$$

2.1.3

$$\frac{\partial y}{\partial x} = \frac{e^{-x}}{(1 + e^{-x})^2}$$

2.1.4

$$\begin{aligned}\frac{\partial y}{\partial x} &= \frac{e^x + e^{-x}}{e^x + e^{-x}} - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2} \\ &= 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2\end{aligned}$$

2.2 Taylor expansion

2.2.1

$$y = e^b + ae^b x + \frac{a^2 e^b x^2}{2} + \frac{a^3 e^{a\xi + b} x^3}{6}, \xi \in [0, x]$$

2.2.2

$$y = \cos b + a \cos bx + \frac{a^2 \cos b}{2} x^2 + \frac{a^3 \cos(a\xi + b)}{6} x^3, \xi \in [0, x]$$

2.3 Matrix multiplication

2.3.1

$$\begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

2.3.2

$$\begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 4 & -2 \\ 1 & -2 & 1 \\ -2 & 4 & -2 \end{pmatrix}$$

2.4 Applying chain rule on vectors and matrices

$$\begin{aligned} \frac{\partial y}{\partial \mathbf{x}} &= \frac{\partial \|\mathbf{A}^T\mathbf{x} - \mathbf{b}\|_2^2}{\partial \mathbf{x}} \\ &= \frac{\partial (\mathbf{A}^T\mathbf{x} - \mathbf{b}) \cdot (\mathbf{A}^T\mathbf{x} - \mathbf{b})}{\partial \mathbf{x}} \\ &= 2(\mathbf{A}^T\mathbf{x} - \mathbf{b})^T \end{aligned}$$

3 Preliminaries: programming

3.2 Reading, display, and save an image

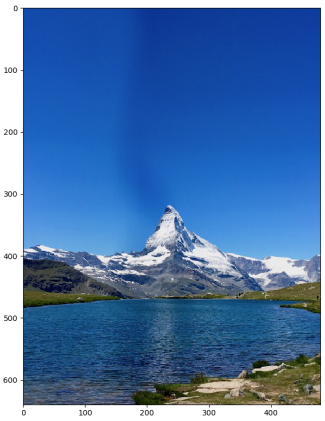

Figure 1: Original image.

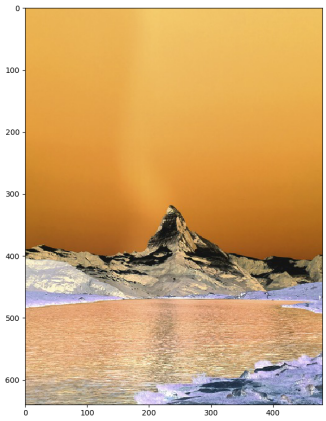

Figure 2: Inverted image.

In this section I learned to open, manipulate, and display image files in Python. The inverted image was obtained by subtracting the original image from a white image of the same size.

3.3 Finding the edges of an image

This image was obtained by taking the difference of two Gaussian filtered copies of the original image.

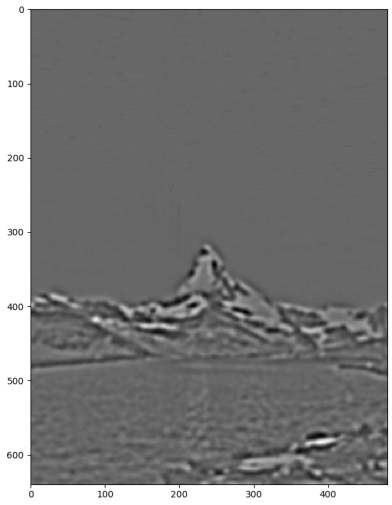

Figure 3: Difference of Gaussian filters.

3.4 Thresholding

The image from the previous section was used to create the following image by turning the least intense 95% of the pixels black.

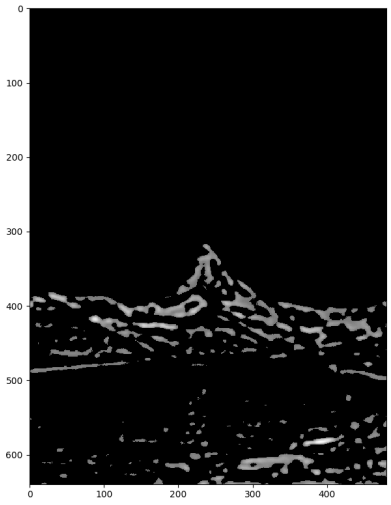

Figure 4: Thresholded image.