MATH 322 Assignment 5

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1

1.a

A generator matrix for C:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

A parity check matrix for C:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1.b

$dim(C) = 4$, $dim(C^{\perp}) = 1$.

1.c

$mindist(C) = 2$, $mindist(C^{\perp}) = 5$.

2

($\Longrightarrow$) C has minimum distance d, so there exists a codeword $w \in C$ with weight d. Since $H \cdot w^{\mathsf{T}} = 0$, we can use w to construct a linear combination of d columns in H which sums to 0, thus there exists a set of d linearly dependent columns of H. Similarly, since there does <u>not</u> exist a codeword of weight $d-1$ in C, such a linear combination cannot be constructed using only $d-1$ columns of H.

($\Longleftarrow$) There exists a set of d linearly dependent columns of H. Thus there exists a linear combination of these columns summing to 0. We can simply construct a codeword of weight d using this linear combination. Any set of $d-1$ columns of H are linearly dependent, so a codeword cannot have weight less than d.

3

By definition, $C^{\perp} = \{v \in H_n \mid u \cdot v = 0 \; \forall u \in S\}$. $C = C^{\perp}$, so we have that $w \cdot w = 0$ for any $w \in C$. Thus x must have even weight, and so all codewords in C must have even weight.

4

4.a

We row reduce H_1 and H_2, then, using their standard form (or equivalent permuted standard form) we construct their corresponding generator matrices:

$$H_1 : \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\implies G_1 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ the generator matrix for } C_1$$

$$H_2 : \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \xrightarrow{\sigma = (34657)} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\implies G_2' = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

We see that $G_1 = G_2'$, and so C_1 and C_2 are indeed equivalent.

4.b

$$G_2' = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\sigma^{-1} = (75643)} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5

We begin with the given parity check matrix and permute it to standard form:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{\sigma = (143)} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

We then construct the corresponding generator matrix and apply the inverse permutation to get the desired generator matrix:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\sigma^{-1} = (341)} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$