CSC 425/520 FALL 2020
ANALYSIS OF ALGORITHMS
MIDTERM EXAMINATION
UNIVERSITY OF VICTORIA

1. Student ID: _____

2. DATE: 22 OCTOBER 2020
   DURATION: 75 MINUTES
   INSTRUCTOR: V. SRINIVASAN

3. THIS QUESTION PAPER HAS FIVE PAGES INCLUDING THE COVER PAGE.

4. THIS QUESTION PAPER HAS 4 QUESTIONS. MARKS FOR EACH ARE INDICATED ON THE QUESTION SHEET.

Q1 (5)	
Q2 (5)	
Q3 (5)	
Q4 (5)	
TOTAL (20) =	

1. Consider the following input instance to the stable matching problem. In this example, $\{H_1, H_2, H_3\}$ represent the set of hospitals and $\{S_2, S_2, S_3\}$ represent the set of students. Their preference lists are as follows:

   $H_1 : S_1, S_2, S_3$
   $H_2 : S_2, S_1, S_3$
   $H_3 : S_3, S_1, S_2$

   $S_1 : H_2, H_1, H_3$
   $S_2 : H_1, H_2, H_3$
   $S_3 : H_3, H_2, H_1$

   (a) Find two stable matchings in this input instance.

   (b) What is the stable matching output by Gale-Shapley algorithm in this input instance?

2. In the interval partitioning problem, there are n requests labeled $1, \ldots, n$ with each request i specifying a start time s_i and a finish time f_i. The goal in this problem is to schedule all the requests using minimum number of machines.

   (a) Using a counter-example, show that a greedy algorithm that uses the "earliest finish time" rule does not find the optimal solution.

   (b) In the class, we proved that the greedy algorithm that uses the "earliest start time" rule always finds the optimal solution. Outline the main steps of the proof.

3. Suppose that your goal is to come up with a new algorithm for matrix multiplication whose running time is better than the $O(n^{2.807})$ running time of Strassen's matrix multiplication algorithm.

   Your plan to achieve this by coming up with a new method for multiplying two 3×3 matrices using as few multiplications as possible. Show that in order to beat Strassen's algorithm, your method must use 21 multiplications or less. *(Hint: Write down the recurrence equation for your method similar to the equation for Strassen's algorithm in the slides and use Master Theorem to solve your recurrence).*

4. Use the sequence alignment algorithm we learnt in the class to compute the edit distance between two strings: PALETTE and PALATE. Assume a gap penalty δ of 2 and mismatch penalty α_{pq} of 1 when $p \neq q$. Also assume that the matches have penalty 0. Show the computation of the algorithm by filling up the table below. What is the final answer?

		P	A	L	A	T	E
P							
A							
L							
E							
T							
T							
E							

END OF EXAM