CSC 226 Problem Set 3 Written Part

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1 Listing vertices of a negative cycle

The following is a slight variation of the Bellman-Ford algorithm.

```
function FindNegCycle(G)
    d ← int array
    π ← vertex array
    for v in V do
        d[v] ← ∞
        π[v] ← null
    end for
    d[0] ← 0
    for i ← 0, 1, ..., |V| − 2 do
        for e in E do
            if d[e.u] + e.w < d[e.v] then
                d[e.v] ← d[e.u] + e.w
                π[e.v] ← e.u
            end if
        end for
    end for
    for e in E do
        if d[e.u] + e.w < d[v] then
            π[e.v] ← e.u
        end if
    end for
    cycle ← vertex aray
    i ← 0
    s ← e.u
    while π[s] ≠ e.v do
        cycle[i++] ← s
        s ← π[s]
    end while
    return cycle
end function
```

2 Dijkstra's algorithm and negative edge-weights

In the proof of correctness, we show that once a vertex is marked as visited, our path to it must have minimum weight. To do this, we show that our path s v is at most equal to any other arbitrary path s y. Here, we rely on the fact that each edge in the y v path must be at least 1 to show that the path s y v has weight strictly greater than that of our s v path. If our graph can have negative edge weights, we cannot make such a guarantee, and the weight of our s v path might be greater than that of s y v, and so the proof is invalid.

3 Eulerian circuits

Consider, WLOG, one of the connected subgraphs of G, $H = (V_H, E_H)$. Each vertex $v \in V_H$ has an even number of edges, so each time it is "entered" in some cycle, there must be an edge our of which to exit. Thus, H has an Eulerian circuit, and therefore has a cycle. This is true of all connected subgraphs of G, and so the claim holds.

4 Graph coloring

4.a

The subgraph induced by taking the vertices of any pair of independent sets is a complete bipartite graph, so there exist no edges between any two vertices $v_i, v_j \in V_k$. Any vertex $v_i \in V_j$ also shares edges with every vertex $v_k \notin V_j$. From these two observations, it is clear to see that all vertices in any independent set can be coloured the same colour. So we can consider each independent set as a single vertex for the purpose of colouring. Thus if we compute $\chi(K_5)$, we will have found $\chi(G)$. $\chi(K_5) = 5$, so $\chi(G) = 5$.

4.b

The graph contains C_5 as a subgraph. We know that $\chi(C_5) = 3$, so this graph's chromatic number must be at least 3. We now provide a 3-colouring of the graph: