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Assignment 4

CSC 349A
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1

1.a

The fourth column of A^{-1} is x such that $Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, so we solve:

$$\left(\begin{array}{cccc|c} 0 & -1 & -2 & -4 & 0 \\ -2 & -1 & 1 & 0 & 0 \\ -1 & 2 & -2 & 0 & 0 \\ 1 & 3 & -1 & 0.5 & 1 \end{array}\right)$$

Partial pivoting swap rows 1 and 2: $\left(\begin{array}{cccc|c} -2 & -1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -4 & 0 \\ -1 & 2 & -2 & 0 & 0 \\ 1 & 3 & -1 & 0.5 & 1 \end{array}\right)$

Forward elminiation: $\left(\begin{array}{cccc|c} -2 & -1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -4 & 0 \\ 0 & 2.5 & -2.5 & 0 & 0 \\ 0 & 2.5 & -0.5 & 0.5 & 1 \end{array}\right)$

Partial pivoting: swap rows 2 and 3: $\left(\begin{array}{cccc|c} -2 & -1 & 1 & 0 & 0 \\ 0 & 2.5 & -2.5 & 0 & 0 \\ 0 & -1 & -2 & -4 & 0 \\ 0 & 2.5 & -0.5 & 0.5 & 1 \end{array}\right)$

Forward elimination: $\left(\begin{array}{cccc|c} -2 & -1 & 1 & 0 & 0 \\ 0 & 2.5 & -2.5 & 0 & 0 \\ 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 2 & 0.5 & 1 \end{array}\right)$

Partial pivoting: no swap. Forward elimination: $\left(\begin{array}{cccc|c} -2 & -1 & 1 & 0 & 0 \\ 0 & 2.5 & -2.5 & 0 & 0 \\ 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & -\frac{13}{6} & 1 \end{array}\right)$

Back substitution:

$$
\begin{aligned}
x_4 &= \frac{b_4}{a_{44}} = \frac{1}{-\frac{13}{6}} && = -\frac{6}{13} \\
x_3 &= \frac{b_3 - a_{34}x_4}{a_{33}} = \frac{0 - (-4)(-\frac{6}{13})}{-3} && = \frac{8}{13} \\
x_2 &= \frac{b_2 - a_{23}x_3 - a_{24}x_4}{a_{22}} = \frac{0 - (-2.5)(\frac{8}{13}) - (0)(-\frac{6}{13})}{2.5} && = \frac{8}{21} \\
x_1 &= \frac{b_1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4}{a_{33}} = \frac{0 - (-1)(\frac{8}{21}) - (1)(\frac{8}{13}) - (0)(-\frac{6}{13})}{-2} && = 0
\end{aligned}
$$

So we get $x = \begin{pmatrix} 0 \\ \frac{8}{13} \\ \frac{8}{13} \\ -\frac{6}{13} \end{pmatrix}$.

1.b

We performed an even number of row swaps, so we need not flip the sing of our number at the end, so we have

$$\det A = (-2)(2.5)(-3)(-\frac{13}{6}) = -32.5,$$

the product of the diagonal entries in the upper triangular matrix we obtained from the Gaussian elimination.

2

2.a

function SLEDiag(A, b)
  $n \leftarrow \text{rank } A$
  $x_1 \leftarrow \frac{b_1}{a_{11}}$
  $x_2 \leftarrow \frac{b_2 - a_{21}x_1}{a_{22}}$
  for $i \leftarrow 3, \ldots, n$ **do**
    $x_i \leftarrow \frac{b_i - a_{i,i-2}x_{i-2} - a_{i,i-1}x_{i-1}}{a_{ii}}$
  end for
  return $(x_1, \ldots, x_n)$
end function

2.b

Computing x_1 takes one division, for a total of one flop, and computing x_2 takes one multiplication, one subtraction, and one division, for a total of three flops. For each i from 3 to n, computing x_i takes two multiplications, two subtractions, and one division, for a total of five flops. Thus the total flop count of our algorithm is

$$1 + 3 + 5(n-2) = 5n - 6 \in \Theta(n),$$

so our algorithm solves such systems in linear time.

2.c

```
function SLEDiag(A, b)
x = zeros(1, size(A, 1));
x(1) = b(1)/A(1,1);
x(2) = (b(2)-A(2,1)*x(1))/A(2,2);

for i = 3:size(A,1)
    x(i) = (b(i)-A(i,i-2)*x(i-2)-A(i,i-1)*x(i-1))/A(i,i);
end

disp(x);
```

The MATLAB statements used to call SLEDiag, along with its output:

```
A=[9,0,0,0;8,7,0,0;6,5,4,0;,0,3,2,1]

A =

     9     0     0     0
     8     7     0     0
     6     5     4     0
     0     3     2     1

b=[2,8,24,32]

b =

     2     8    24    32
```

```
SLEDiag(A, b)
    0.2222    0.8889    4.5556    20.2222

diary off
```

3

3.a

We solve: $\left(\begin{array}{cc|c} 3.102 & -0.155 & -2.012 \\ -2.534 & 0.1234 & 1.007 \end{array}\right)$

Our factor is $\mathrm{fl}\left(\frac{a_{21}}{a_{11}}\right) = \mathrm{fl}\left(\frac{-2.534}{3.102}\right) = -0.8169$. After forward elimination, we have $\left(\begin{array}{cc|c} 3.102 & -0.155 & -2.012 \\ 0 & 0.2500 & 1.644 \end{array}\right)$

Now, performing back substitution, we get $x_2 = \mathrm{fl}\left(\frac{b_2}{a_{22}}\right) = \mathrm{fl}\left(\frac{1.644}{0.2500}\right) = 6.576$

$$\begin{aligned} x_1 &= \mathrm{fl}\left(\frac{\mathrm{fl}\left(b_1 - \mathrm{fl}\left(a_{12}x_2\right)\right)}{a_{11}}\right) \\ &= \mathrm{fl}\left(\frac{\mathrm{fl}\left(-2.012 - \mathrm{fl}\left((-0.155)(6.576)\right)\right)}{3.102}\right) \\ &= \mathrm{fl}\left(\frac{\mathrm{fl}\left(-2.012 - (-1.102)\right)}{3.102}\right) \\ &= \mathrm{fl}\left(\frac{-0.9100}{3.102}\right) \\ &= -0.2934 \end{aligned}$$

So we get $a_{22} = 0.2500$, $b_2 = 1.644$, $x_1 = -0.2934$, and $x_2 = 6.576$.

3.b

```
A = [3.102,-0.155;-2.534,0.1234]

A =

    3.1020   -0.1550
   -2.5340    0.1234
```

```
cond(A)

ans =

    1.6110e+03

diary off
```