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Assignment 3

CSC 349A
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1.a

$$\begin{aligned}
\text{fl}(f(x)) &= \text{fl}\left(\frac{\text{fl}\big(1+\text{fl}(\cos 3.155)\big)}{\text{fl}\Big(\text{fl}\big(3.155-\text{fl}(\pi)\big)^2\Big)}\right)\\
&= \text{fl}\left(\frac{\text{fl}\big(1+(-0.9999)\big)}{\text{fl}\big(0.01300\big)^2}\right)\\
&= \text{fl}\left(\frac{0.0001000}{0.0001690}\right)\\
&= 0.5917
\end{aligned}$$

Note that $|\varepsilon_t| = |1 - \frac{0.5917}{0.49999251}| \approx 0.1834 > 0.1$, as desired.

1.b

$$\begin{aligned}
\cos x &\approx \cos\pi - \sin\pi(x-\pi) - \frac{\cos\pi}{2}(x-\pi)^2 + \frac{\sin\pi}{6}(x-\pi)^3 + \frac{\cos\pi}{24}(x-\pi)^4\\
&= -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}
\end{aligned}$$

1.c

$$\begin{aligned}
f(x) &\approx \frac{1 + (-1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24})}{(x-\pi)^2}\\
&= \frac{1}{2} - \frac{(x-\pi)^2}{24}
\end{aligned}$$

1.d

Let ε such that $|\frac{\varepsilon}{3.155}|$ is small. We know that $\frac{1}{2} - \frac{(x-\pi)^2}{24}$ is a very good approximation for $f(x)$, so consider $\frac{1}{2} - \frac{(x+\varepsilon-\pi)^2}{24}$:

$$\frac{1}{2} - \frac{(x+\varepsilon-\pi)^2}{24} \approx \frac{1}{2} - \frac{(0.01341+\varepsilon)^2}{24}.$$

We assume "small" here means $|\frac{\varepsilon}{3.155}| < 0.01$, so $0.49999983 \leq \frac{1}{2} - \frac{(0.01341+\varepsilon)^2}{24} \leq 0.499999996$. In other words, the function evaluated at any small perturbation $x+\varepsilon$ of x gives a value very close to the exact value of the function when evaluated at x, so the problem of computing $f(3.155)$ is well-conditioned.

1.e

Again, we use our approximation to $f(x)$, $\frac{1}{2} - \frac{(x-\pi)^2}{24}$. Let ε such that $|\frac{\varepsilon}{3.155}| < 0.01$. Then

$$\mathrm{fl}\left(\frac{1}{2} - \frac{\mathrm{fl}(\mathrm{fl}(\mathrm{fl}(3.155-\pi)+\varepsilon)^2)}{24}\right) = \mathrm{fl}\left(\frac{1}{2} - \frac{\mathrm{fl}((0.013+\varepsilon)^2)}{24}\right)$$

and $0.4999 \leq \mathrm{fl}\left(\frac{1}{2} - \frac{\mathrm{fl}((0.013+\varepsilon)^2)}{24}\right) \leq 0.5$. So clearly no small perturbation $x+\varepsilon$ of x will give a value close to the calculated value of 0.5917 in a).

2

2.a

```
function root = Bisect (xl, xu, eps, imax, f,
    enablePlot)
x = [xl:0.01:xu];
if enablePlot
    hold on;
    y0 = yline(0, '--k', 'x = 0');
    y0.LabelHorizontalAlignment = 'left';
end
i = 1;
fl = f(xl);
fprintf ('iteration\tapproximation\n')
while i <= imax
    xr = (xl + xu)/2;
    fprintf ( '%5.0f %17.7f\n', i, xr )
    fr = f(xr);
    if enablePlot
        if ismember(i, [1,3,5,6])
            z = [xl, xr, xu];
            fz = f(z);
            xlabel('y');
            ylabel('x');
            plot(x, f(x), '-k', z, fz, '*b');
        end
    end
    if fr == 0 || (xu-xl)/abs(xu+xl) < eps
```

```
            root = xr;
            if enablePlot
                hold off;
            end
            return;
        end
        i = i + 1;
        if fl * fr < 0
            xu = xr;
        else
            xl = xr;
            fl = fr;
        end
end
if enablePlot
    hold off;
end
fprintf('Failed to converge in %g iterations.\n', imax);
root = NaN;
```

2.b

We have that

$$B = 2 + y$$

and

$$A_c = 2y + \frac{y^2}{2},$$

so since it must always be the case that

$$0 = 1 - \frac{Q^2}{gA_c^3}B,$$

we can simply find the roots of the function

$$\begin{aligned} f(y) &= 1 - \frac{Q^2}{gA_c^3}B \\ &= 1 - \frac{Q^2}{g(2y + \frac{y^2}{2})^3}(2+y) \\ &= 1 - \frac{18^2}{9.81(2y + \frac{y^2}{2})^3}(2+y) \end{aligned}$$

2.c

Below is the additional M-file:

```
f = @(y)1-((18.^2)./(9.81*(2*y+(y.^2)/2).^3)).*(2+y);
```

The MATLAB statements used to call Bisect, along with its output:

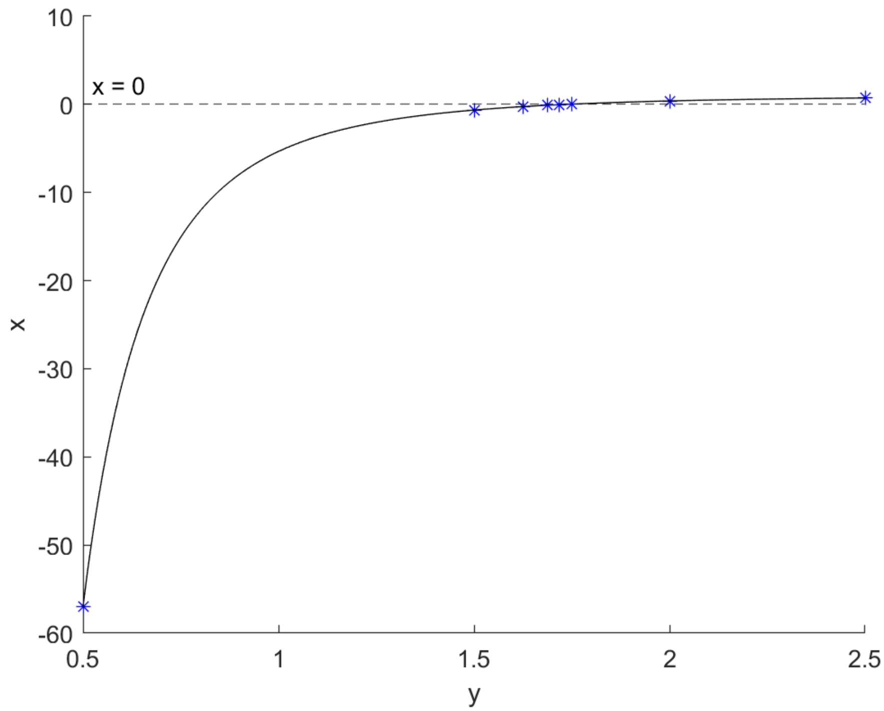
```
q2c
Bisect(0.5, 2.5, 0.01, 11, f, 1)
iteration        approximation
     1             1.5000000
     2             2.0000000
     3             1.7500000
     4             1.6250000
     5             1.6875000
     6             1.7187500
     7             1.7343750

ans =

     1.7344

diary off
```

Finally, below is the figure produced by the above call to Bisect:


10
x = 0
0
-10
-20
-30
-40
-50
-60
x
0.5
1
1.5
2
2.5
y