MATH 200 Fall 2018 Assignment 3[1]

Thomas §14.4, 14.5, 14.6, 14.7

1. The length l, width w, and height h of a box change with time. At a certain instant the dimensions are $l = 1m$, and $w = h = 2m$, and l and w are increasing at a rate of $2m/s$ while h is decreasing at a rate of $3m/s$. At that instant find the rates at which the following quantities are changing.

    a) The volume

    b) The surface area

    c) The length of a diagonal

2. If we assume that all of the functions have continuous second order partial derivatives, show that any function of the form

$$z = f(x + at) + g(x - at)$$

is a solution to the wave equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$

where a is some constant.

3. Find an equation of the tangent plane to the given surface defined by

$$z = 3(x - 1)^2 + 2(y + 3)^2 + 7$$

at (2,-2, 12).

4. Find the directional derivative of

$$f(x, y) = \sqrt{xy}$$

at P(2,8) in the direction of Q(5,4)

5. Find the maximum rate of change of

$$f(x, y) = \sin(xy)$$

at (1,0), and state the direction in which this occurs.

[1] Due to the number of people in this class, a subset of the questions will be used to grade this assignment. This subset will be determined after all of the assignments have been handed in.

6. Find all directions in which the directional derivative of

$$f(x, y) = ye^{-xy}$$

at the point (0,2) has the value 1.

7. Find the local minimum and maximum values, as well as saddle point(s) of

$$f(x, y) = e^x \cos(y)$$

8. Consider

$$f(x, y) = x^2 + 4y^2 - 4xy + 2$$

Show that f has an infinite number of critical points and that $D = 0$ at each one. Then show that has a local (and absolute) minimum at each critical point.