CSC 320 Assignment 1

Oliver Tonnesen
V00885732

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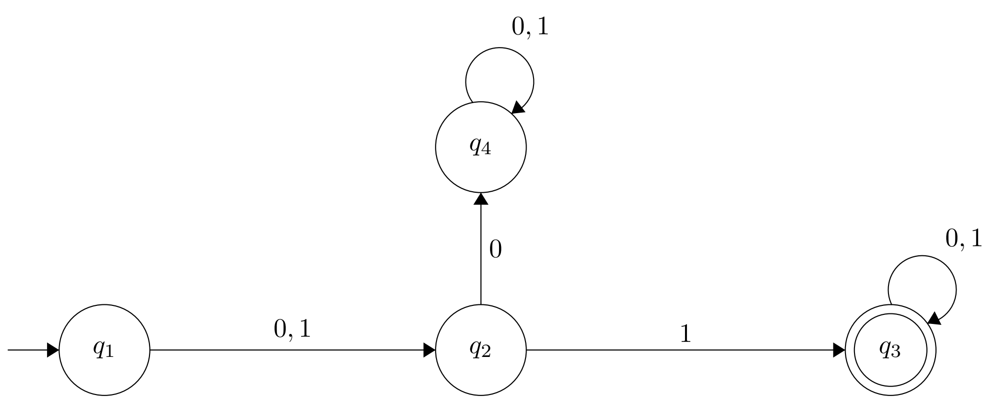
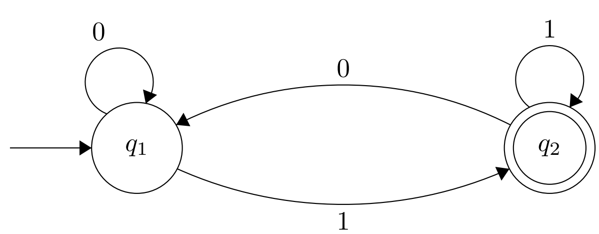
1

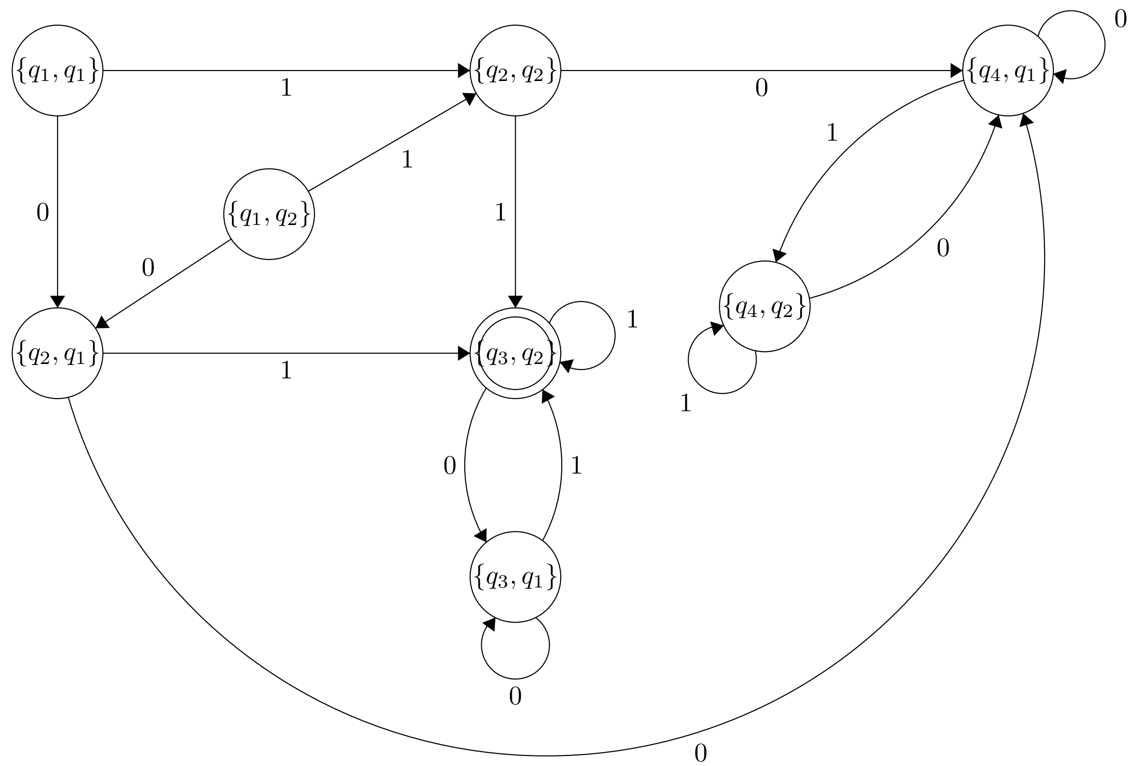
1.a



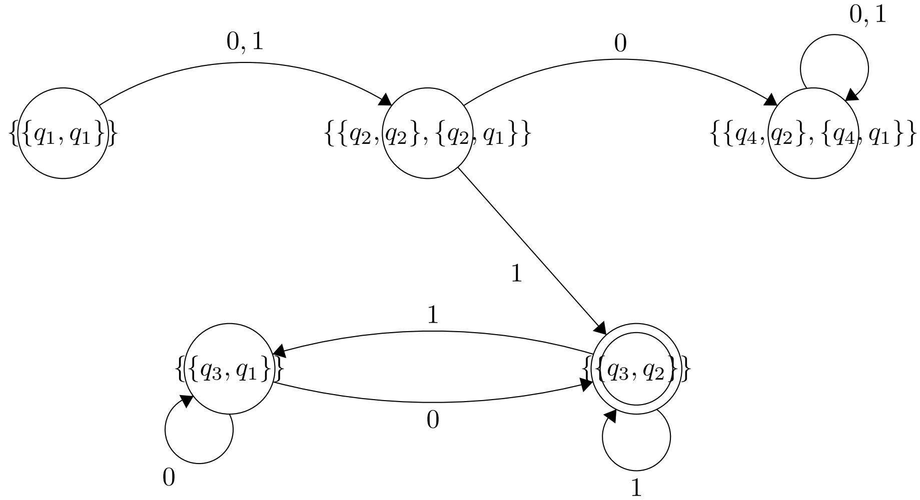
1.b



1.c



1.d



To prove minimality, we provide a distinguishing string w for each pair of states in the given DFA:

$$\begin{aligned}
\{\{q_1,q_1\}\} \text{ and } \{\{q_2,q_2\},\{q_2,q_1\}\} &: 01\\
\{\{q_1,q_1\}\} \text{ and } \{\{q_4,q_2\},\{q_4,q_1\}\} &: 01\\
\{\{q_1,q_1\}\} \text{ and } \{\{q_3,q_1\}\} &: 001\\
\{\{q_1,q_1\}\} \text{ and } \{\{q_3,q_2\}\} &: 001\\
\{\{q_2,q_2\},\{q_2,q_1\}\} \text{ and } \{\{q_4,q_2\},\{q_4,q_1\}\} &: 1\\
\{\{q_2,q_2\},\{q_2,q_1\}\} \text{ and } \{\{q_3,q_1\}\} &: 01\\
\{\{q_2,q_2\},\{q_2,q_1\}\} \text{ and } \{\{q_3,q_2\}\} &: 01\\
\{\{q_4,q_2\},\{q_4,q_1\}\} \text{ and } \{\{q_3,q_1\}\} &: 1\\
\{\{q_4,q_2\},\{q_4,q_1\}\} \text{ and } \{\{q_3,q_2\}\} &: 1\\
\{\{q_3,q_2\}\} \text{ and } \{\{q_3,q_1\}\} &: 1
\end{aligned}$$

We clearly see each pair of states is distinguishable, and so our DFA is minimal.

2

We construct an NFA recognizing One-off(L) given the DFA recognizing L, M. We first define M' to be the same as M, but with non-accepting states in place of accepting states. Our NFA has a start state with ε-transitions to M''s start state, and to another state with a transition arrow for 0 and 1 going to the start state of M. We now create another accepting state with transition arrows for 0 and 1 coming from every state in M' that was an accepting state in M.

We now have an NFA that accepts any string beginning with a 0 or a 1 followed by a string in L, and also accepts any string beginning with a string in L followed by a 0 or a 1. Thus, our NFA accepts One-off(L).

3

3.a

We shall construct an NFA recognizing A^R given the DFA recognizing A. We first change all accepting states in the DFA to non-accepting states in the NFA. Then, we reverse all transitions in the DFA, and create a new state with ε-transtions to all former accepting states, and let it be the start state. The former start state is now the single accepting state. We have now constructed an NFA recognizing A^R, and so A^R is regular.

3.b

i.

We showed above that A^R is regular if A is regular, and we know that the regular languages are closed under concatenation. Thus, if A and B are regular, then $A^R B^R = \{w^R y^R | w \in A, y \in B\}$ is regular.

ii.

We construct a DFA recognizing the language as follows:

Let $D_A = (Q_A, \Sigma_A, \delta_A, q_{0_A}, F_A)$ and $D_B = (Q_B, \Sigma_B, \delta_B, q_{0_B}, F_B)$ be DFAs recognizing A and B, respectively.

$$
\begin{aligned}
&Q = \{(q_i, q_j, x) | q_i \in A, q_j \in B, x \in \{0, 1, 2 \ldots, 2n\}\} \\
&\Sigma = \Sigma_B \cup \Sigma_A \\
&\delta((q_i, q_j, x), a) = \begin{cases} (\delta_A(q_i, a), q_j, x + 1) \text{ if } x \equiv 0 \pmod 2 \\ (q_i, \delta_B(q_j, a), x + 1) \text{ if } x \equiv 1 \pmod 2 \end{cases} \\
&q_0 = (q_{0_A}, q_{0_B}, 0) \\
&F = \{(q_i, q_j, 2n) | q_i \in F_A, q_j \in F_B\}
\end{aligned}
$$

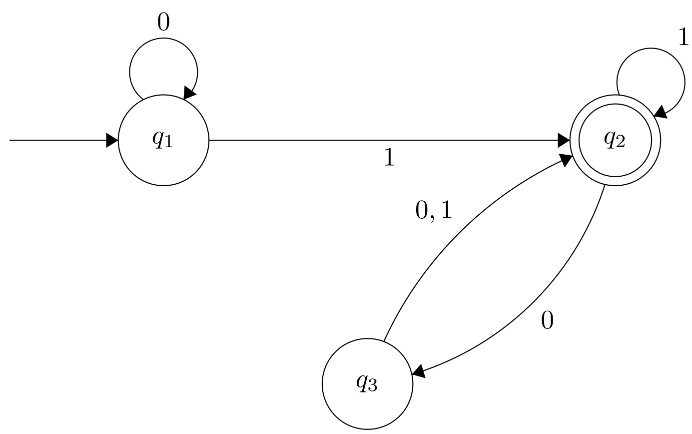
Our DFA follows both D_A and D_B in an alternating fashion and accepts only when it has found an accepting string of length n from both languages.

4

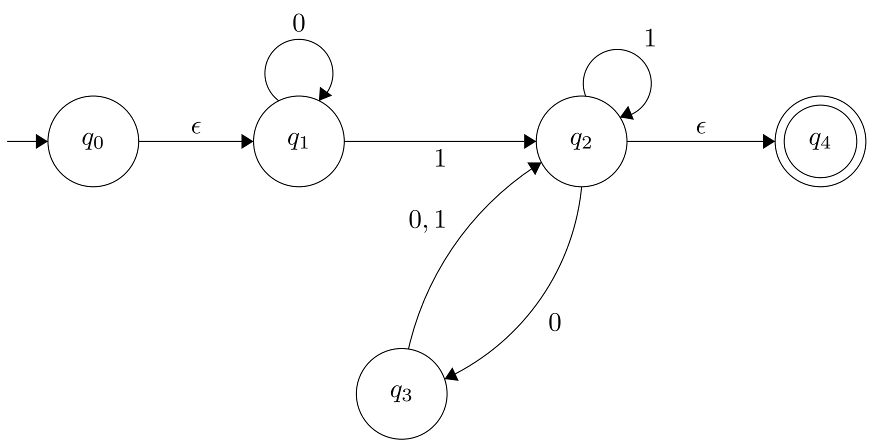
4.a

Only 2.

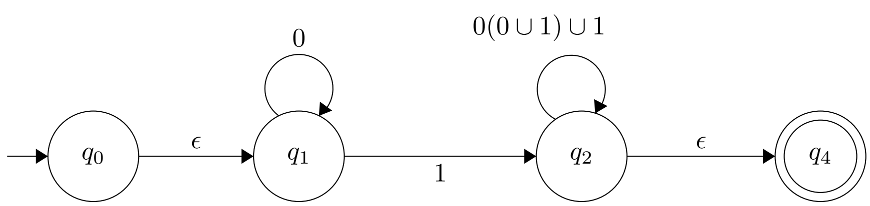
4.b



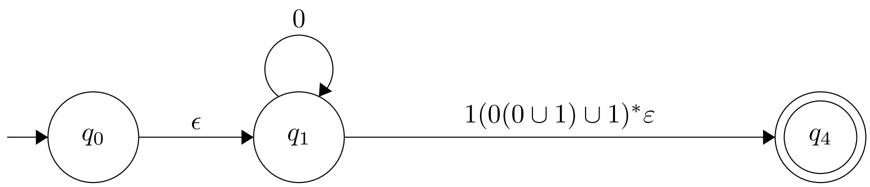
0
1
q_1
1
q_2
0, 1
0
q_3

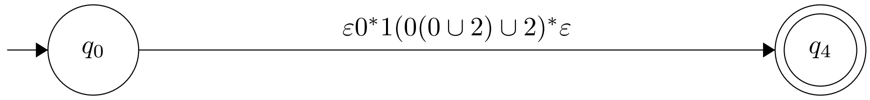


0
1
q_0
ϵ
q_1
q_2
ϵ
q_4
1
0, 1
0
q_3



0
$0(0 \cup 1) \cup 1$
q_0
ϵ
q_1
q_2
ϵ
q_4
1





So the regular expression is $\varepsilon 0^* 1(0(0 \cup 2) \cup 2)^* \varepsilon$.