Oliver Tonnesen
V00885732

Assignment 2

CSC 349A
September 19, 2019

1

1.a

$(+0.4003 \times 5^{+02})_5 = 40.03_5 = 20.12_{10}$

1.b

$(-0.1102 \times 5^{+03})_5 = -110.2_5 = 30.4_{10}$

1.c

$$\begin{aligned} 01000144 &\rightarrow (0.1000 \times 5^{-44})_5 \\ &= (1.000 \times 5^{-100})_5 \\ &= (5^{-100})_5 \\ &= (5^{-25})_{10} \\ &= 0.0000000000000000033554432_{10} \\ &= (0.33554432 \times 10^{-17})_{10} \end{aligned}$$

1.d

Our range is $[5^3, 5^4]$, so two consecutive numbers are represented as 01000004 and 01001004. Thus the space between the two numbers is 00001004, or $(0.0001 \times 5^4)_5 = 1_5 = 1_{10}$

2

2.a

$$
\begin{aligned}
\mathrm{fl}\left(\frac{-2c}{b-\sqrt{b^2-4ac}}\right) &= \mathrm{fl}\left(\frac{\mathrm{fl}(-2.000 \times 0.5810)}{\mathrm{fl}\Big(-35.63 - \mathrm{fl}\big(\sqrt{\mathrm{fl}(\mathrm{fl}(-35.63 \times -35.63) - \mathrm{fl}(\mathrm{fl}(4.000 \times 1.000) \times 0.5810))}\big)\Big)}\right) \\
&= \mathrm{fl}\left(\frac{-1.162}{\mathrm{fl}\Big(-35.63 - \mathrm{fl}\big(\sqrt{\mathrm{fl}(1269 - \mathrm{fl}(4 \times 0.5810))}\big)\Big)}\right) \\
&= \mathrm{fl}\left(\frac{-1.162}{\mathrm{fl}\Big(-35.63 - \mathrm{fl}\big(\sqrt{\mathrm{fl}(1269 - 2.324)}\big)\Big)}\right) \\
&= \mathrm{fl}\left(\frac{-1.162}{\mathrm{fl}\Big(-35.63 - \mathrm{fl}\big(\sqrt{1266}\big)\Big)}\right) \\
&= \mathrm{fl}\left(\frac{-1.162}{\mathrm{fl}\Big(-35.63 - 35.58\Big)}\right) \\
&= \mathrm{fl}\left(\frac{-1.162}{-71.21}\right) \\
&= 0.01631
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{fl}\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) &= \mathrm{fl}\left(\frac{\mathrm{fl}\Big(35.63 - \mathrm{fl}\big(\sqrt{\mathrm{fl}(\mathrm{fl}(-35.63 \times -35.63) - \mathrm{fl}(\mathrm{fl}(4.000 \times 1.000) \times 0.5810))}\big)\Big)}{\mathrm{fl}(2.000 \times 1.000)}\right) \\
&= \mathrm{fl}\left(\frac{\mathrm{fl}\Big(35.63 - \mathrm{fl}\big(\sqrt{\mathrm{fl}(1269 - \mathrm{fl}(4.000 \times 0.5810))}\big)\Big)}{2.000}\right) \\
&= \mathrm{fl}\left(\frac{\mathrm{fl}\Big(35.63 - \mathrm{fl}\big(\sqrt{\mathrm{fl}(1269 - 2.324)}\big)\Big)}{2.000}\right) \\
&= \mathrm{fl}\left(\frac{\mathrm{fl}(35.63 - 35.58)}{2.000}\right) \\
&= \mathrm{fl}\left(\frac{0.05000}{2.000}\right) \\
&= 0.02500
\end{aligned}
$$

2.b

$\left|1 - \frac{0.01631}{0.01631395}\right| \approx 0.00024$

$\left|1 - \frac{0.02500}{0.01631395}\right| \approx 0.53$

2.c

polynomial	(i) is more accurate	(ii) is more accurate
$0.01x^2 - 125x + 0.05$	X	
$0.03x^2 + 125x + 0.025$		X

3

3.a

$$\begin{aligned}
f(x) &= (x+1)^{\frac{1}{2}}, \qquad & f(3) &= 2 \\
f'(x) &= \frac{1}{2}(x+1)^{-\frac{1}{2}}, \qquad & f'(3) &= \frac{1}{4} \\
f''(x) &= -\frac{1}{4}(x+1)^{-\frac{3}{2}}, \qquad & f''(3) &= -\frac{1}{32} \\
f'''(x) &= \frac{3}{8}(x+1)^{-\frac{5}{2}}, \qquad & f'''(3) &= \frac{3}{256} \\
R_2 &= \frac{f'''(\xi)}{3!}(x-3)^3 = \frac{3}{3!8}(\xi+1)^{-\frac{5}{2}}(x-3)^3, \xi \in [x,3] \\
&\implies f(x) \approx 2 + \frac{1}{4}(x-3) - \frac{1}{32}\frac{(x-3)^2}{2!} + \frac{3}{8}\frac{(\xi+1)^{-\frac{5}{2}}}{3!}(x-3)^3
\end{aligned}$$

3.b

$$\begin{aligned}
f(3.12) &\approx 2 + \frac{1}{4}(3.12-3) - \frac{1}{32}\frac{(3.12-3)^2}{2!} \\
&= 2.029775 \\
&\approx \sqrt{4.11998655}
\end{aligned}$$

3.c

$$R_2 = \frac{3}{3!8}(\xi+1)^{-\frac{5}{2}}(x-3)^3$$

This error term is clearly maximal with large x and small ξ, so if we let $x = 3.2, \xi = 3$, we have that

$$\begin{aligned}
R_2 &\leq \frac{3}{3!8}(3+1)^{-\frac{5}{2}}(3.2-3)^3 \\
&= \frac{1}{16}(3+1)^{-\frac{5}{2}}(3.2-3)^3 \\
&= \frac{4^{-\frac{5}{2}}}{16}(0.2)^3 \\
&= 0.15625 \times 10^{-4}
\end{aligned}$$