CSC 320 Assignment 3

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1

Yes. We provide constructions for TMs deciding the three languages. Let M_1 be a decider for L_1, and M_2 be a decide for L_2.

Concatenation ($L_1 \circ L_2$): Our Nondeterministic Turing Machine operates as follows:

Run like M_1 on the input. Whenever M_1 would enter an accepting state, branch and run like M_2 on the remainder of the input. Accept when any branch of computation finishes in an accepting state of M_2, reject otherwise.

Intersection ($L_1 \cap L_2$): Our Turing Machine operates as follows:

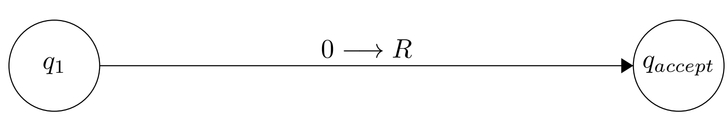
Run like M_1 on the input and reject if M_1 rejects. Then, run like M_2 on the input and accept if M_2 accepts, and reject if M_2 rejects.

Complement ($\overline{L_1}$): Our Turing Machine operates as follows:

Run like M_1 on the input. Accept if M_1 rejects, reject if M_1 accepts.

2

We will encode the following Turing Machine:



We assume that $\Sigma = \{0, 1\}$. Also recall that q_{accept} becomes q_2 for the purposes of our encoding.

Let $X_1 = 0, X_2 = 1$. Then our transition function contains the following:

$$\delta(q_1, X_1) = (q_2, X_1, D_2)$$

and we encode it as:

$$0^1 10^1 10^2 10^1 10^2 = 01010010100$$

3

Assume for a contradiction that L is decidable. Then there exists a Turing Machine that decides L, call it R.

For any $\langle M, w \rangle$, M_1 runs as follows:

- Run like M on input w and write a \$ when and only when M accepts w

We construct V to run on $\langle M, w \rangle$ as follows:

- Use $\langle M, w \rangle$ to output $\langle M_1 \rangle$
- Run R on $\langle M_1 \rangle$
- If R accepts, accept. If R rejects, reject.

Thus V decides A_{TM}, and so our assumption that L is decidable was false.

Assuming we are able to choose the encoding of our Turing Machine, it does not matter what symbol we choose to take the place of \$. That is, if we were to choose 1, for example, we could simply modify the Turing Machine to use 2 whenever it would otherwise use 1, and use 1 only in the specific case where we need it.

4

M has a finite number of states, so we can simply construct a Turing Machine to simulate M until it either writes a symbol to its tape, or returns to a state that has already been visited. We know that M's behaviour at a particular state will not change if its tape has not been changed. So our Turing Machine will have to simulate at most M states of M's computation before it is able to confirm whether or not M ever writes a symbol to its tape.

5

If $\langle M \rangle$ is in $\overline{E_{TM}}$, then there exists a string w accepted by M. Thus our Nondeterministic Turing Machine can simply run like M on every binary string and accept when one branch of computation ends in the accept state. Clearly w is one of the binary strings, so our constructed Nondeterministic Turing Machine always accepts if $L(M) \neq \emptyset$, and $\overline{E_{TM}}$ is thus recognizable.

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We know that E_{TM} is undecidable, so since $\overline{E_{TM}}$ is recognizable, E_{TM} clearly cannot be recognizable.

We define the following computable function:

$$\begin{aligned} f : \Sigma^* &\longrightarrow \Sigma^* \\ \langle M \rangle &\longmapsto \langle M, M \rangle \end{aligned}$$

We know that

$$L(M) \cap L(M) = L(M)$$

so it must be the case that

$$L(M) = \emptyset \Longleftrightarrow L(M) \cap L(M) = \emptyset$$

In other words,

$$\langle M \rangle \in E_{TM} \Longleftrightarrow \langle M, M \rangle \in \{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$$

Thus we have given the reduction

$$E_{TM} \leq_M \{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$$

We know that E_{TM} is not recognizable, and so $\{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$ is also not recognizable.