Examples of FIR Filter Design with the Window Method

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February 16, 2016

lecture notes and example questions:
`https://github.com/otoolej/DSP_notes_FIR`

Review: window method for FIR filter design

- procedure for window method:
  1. derive ideal impulse response $h_d[n]$ from frequency response $H_d(\omega)$
  2. select window type $w[n]$ and window length M according to design specifications
  3. shift and apply window: $h[n] = h_d\left[n - \frac{M-1}{2}\right] w[n]$
  4. filter: $y[n] = \sum_{k=0}^{M-1} h[k]x[n-k]$
- frequency-domain characteristics of window:
  - width of main lobe determines the width of filter's transition band
  - sidelobes effect ripple in pass-band and stop-bands

Learning Objectives

- different windows for designing FIR filters
- can design filter to given specifications by adjusting:
  - window type and length (M)
- more control is possible for windows with adjustable parameters (e.g. Kaiser window)
- process of designing filter given specifications.

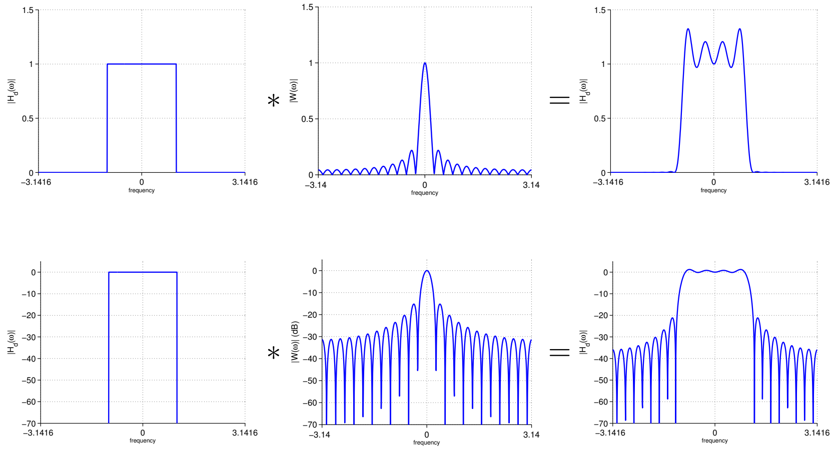
Effect of Window on Frequency Response of Filter

- why include window?
  - want $h[n]$ to be finite;
  - recall $h[n] = h_d\left[n - \frac{M-1}{2}\right] w[n]$
- consequence of limiting $h_d[n]$ in the time domain:
  - filter $y[n] = x[n] * h[n] \leftrightarrow Y(\omega) = X(\omega)H(\omega)$
  - but $h[n] = \bar{h}_d[n]w[n]$, and thus

$$y[n] = x[n] * (\bar{h}_d[n]w[n]) \leftrightarrow Y(\omega) = X(\omega)\left[\bar{H}_d(\omega) * W(\omega)\right]$$

(with $\bar{h}_d[n] = h_d[n - (M-1)/2]$)

$$H_d(\omega) * W(\omega) = H(\omega)$$



Many different types of window functions, but

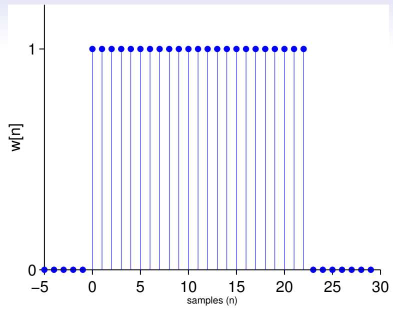
- window type determines main lobe and side lobes
- and M determines width of main lobe, only

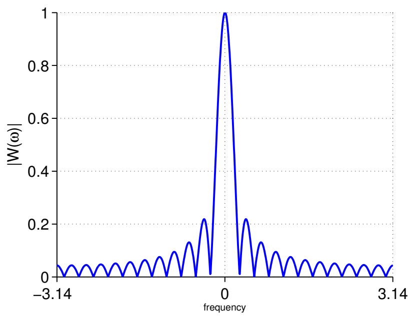
effect on frequency response $H(\omega)$:

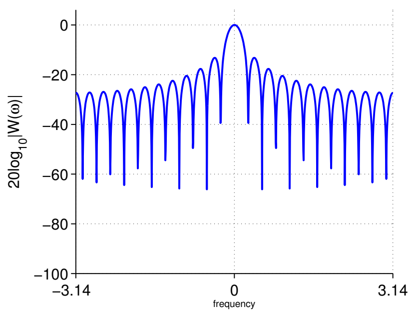
- main lobe of window $\Rightarrow$ transition bandwidth
  - increase in M (window length) results in a decrease in the transition band
- side lobe of window $\Rightarrow$ passband ripple and stopband attenuation
  - different window types have different levels of passband ripple

rectangular window

$$w[n] = \begin{cases} 1 & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$$



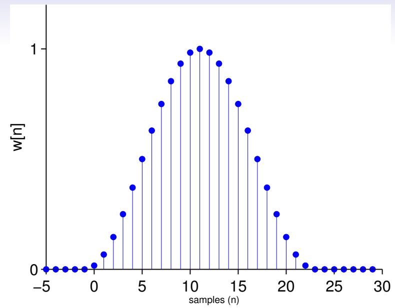


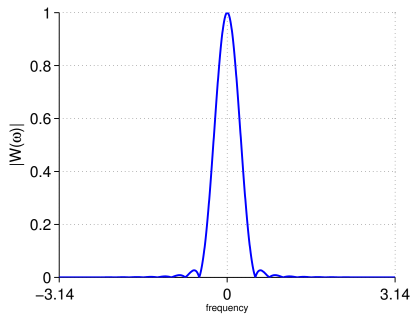


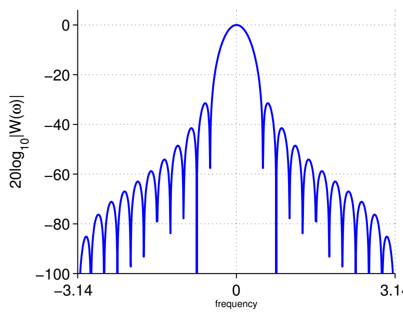
Hann (Hanning) window

$$w[n] = 0.5 - 0.5\cos\left(\frac{2\pi n}{M-1}\right)$$

for $0 \leq n \leq M-1$ ($w[n] = 0$ otherwise).



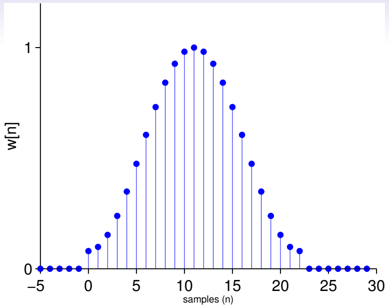


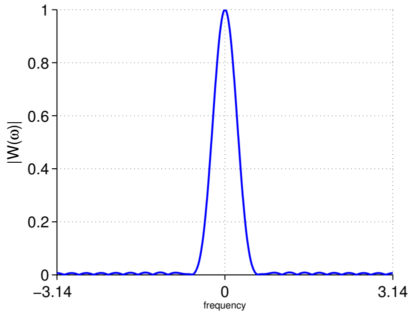


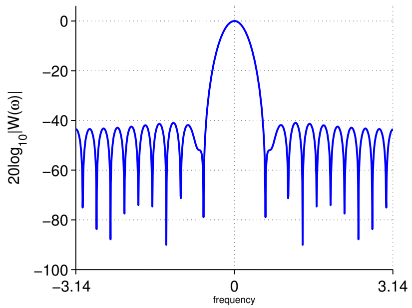
Hamming window

$$w[n] = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$$

for $0 \le n \le M-1$ ($w[n] = 0$ otherwise).



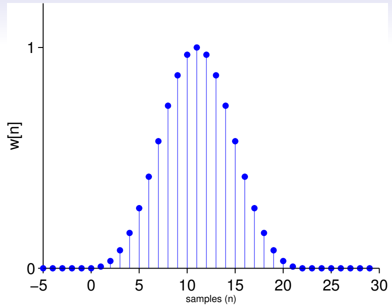


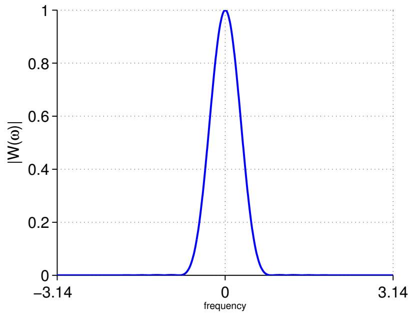


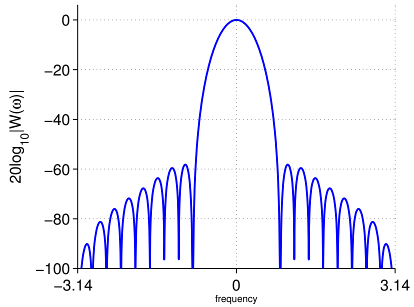
Blackman window

$$w[n] = 0.42 - 0.5\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos\left(\frac{4\pi n}{M-1}\right)$$

for $0 \le n \le M-1$ ($w[n] = 0$ otherwise).







Windows

- Several window functions have been proposed which lead to filters with varying transition band widths and stopband attenuation for a fixed filter length.
- Can relate the transition band width ($\triangle\omega$) to the filter length. For example the Blackman Window $\triangle\omega \approx -12\pi/M$.
- Can determine approximately the minimum stopband attenuation that can be achieved with a particular window. For example with the Hanning Window it is 44 dB.
- Can determine approximately the passband ripple. For example with the Hamming Window it is 0.0194 dB.

Window Specifications

window type	side-lobe amplitude (dB)	Approx. width of main lobe ($\approx \Delta\omega$)	Approx. stopband $20\log_{10}(\delta)$ (dB)	Approx. passband $20\log_{10}(1+\delta)$ (dB)
rectangular	-13	$4\pi/(M+1)$	-21	0.7416
Hanning	-31	$8\pi/M$	-44	0.0546
Hamming	-41	$8\pi/M$	-53	0.0194
Blackman	-57	$12\pi/M$	-74	0.0017

(Discrete-Time Signal Processing, Oppenheim *et al.*, 2nd Ed., pp. 471)

- transition width ($\Delta\omega$) for 4 windows (rectangular, Hanning, Hamming, and Blackman windows) $\approx$ main lobe of window
- passband and stopband ripple are equal $\delta = \delta_1 = \delta_2$

Kaiser Window

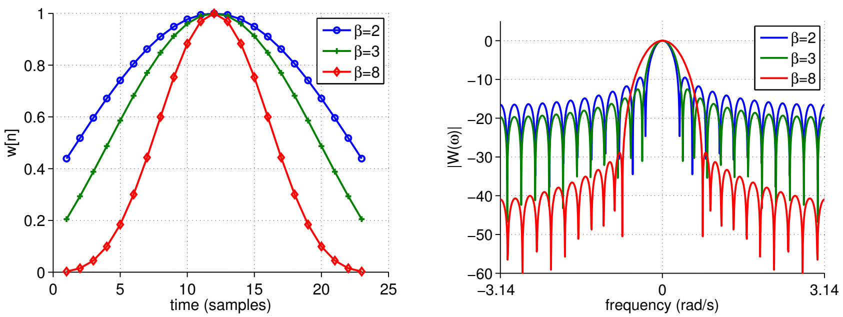
window function with parameter β to control shape of the window:

$$w[n] = \begin{cases} \frac{I_0[\beta(1-[(n-M_h)/M_h]^2)]^{1/2}}{I_0(\beta)} & 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases}$$

where $M_h = (M-1)/2$, I_0 is zero-order modified Bessel function of the first kind.

- two parameters M and β control *both* main lobe width and side lobe amplitude
- $\Rightarrow$ more precise than selecting a window (e.g. Hamming) which has set side lobe amplitude;

Kaiser window with $M = 23$



Parameter β controls both main and side lobes.

Filter Design with Kaiser Window

- given filter specifications: transition width $\Delta\omega$ and pass-band ripple δ_1 (or minimum stopband attenuation δ_2, with $\delta = \delta_1 = \delta_2$), then find M and β
- first let $A = -20 \log_{10} \delta_p$, then

$$\beta = \begin{cases} 0.1102(A-8.7) & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & 21 \le A \le 50 \\ 0 & A < 21 \end{cases}$$

- and

$$M = \frac{A-8}{2.285\Delta\omega}$$

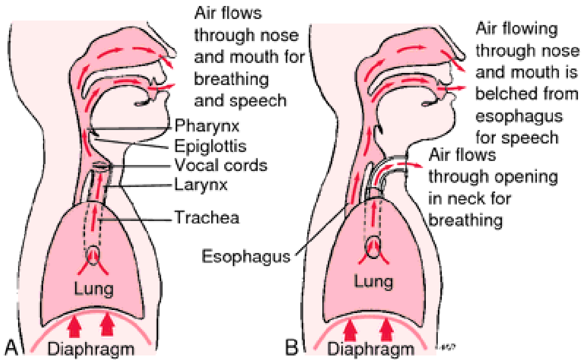
Comparison of Windows

window type	peak side-lobe amplitude (dB)	Approx. width of main lobe ($\approx \Delta\omega$)	Approx. stopband $20\log_{10}(\delta)$ (dB)	Kaiser parameter β	Transition width of Kaiser $\Delta\omega$
rectangular	-13	$4\pi/(M+1)$	-21	0	$1.81\pi/M$
Hanning	-31	$8\pi/M$	-44	3.86	$5.01\pi/M$
Hamming	-41	$8\pi/M$	-53	4.86	$6.27\pi/M$
Blackman	-57	$12\pi/M$	-74	7.04	$9.19\pi/M$

Filter Design Example: Enhancing Oesophageal Speech

- cancer of the larynx accounts for 3% of all cancers
  - main risk factors: male, alcohol, and smoking;
- treatment includes removal of larynx:
  - no vocal-folds: how to speak? (vocal-folds vibrate to produce glottal waveform)
- possible modes of speech:
  - electro-larynx
  - insert prosthetic device between trachea and oesophagus (trache-oesophageal puncture)
  - oesophageal phonation

- oesophageal speech:
  - suck air down through the oesophagus, then expel to mimic glottal waveform
- problems with oesophageal speech: intelligibility and quality
- possible solution: signal processing to enhance speech



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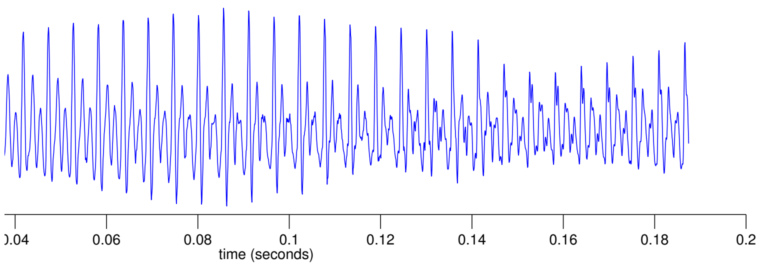
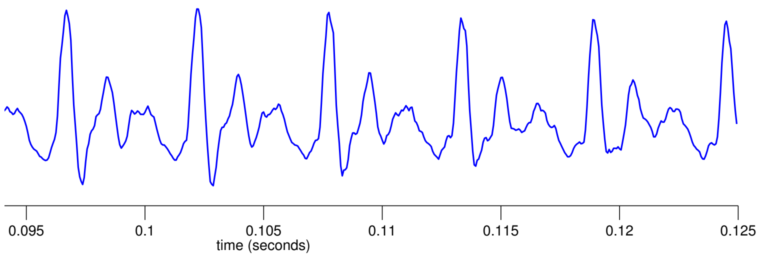
example of voiced speech (time-domain)





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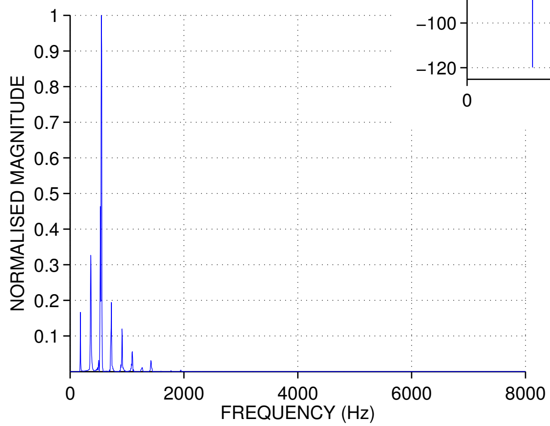
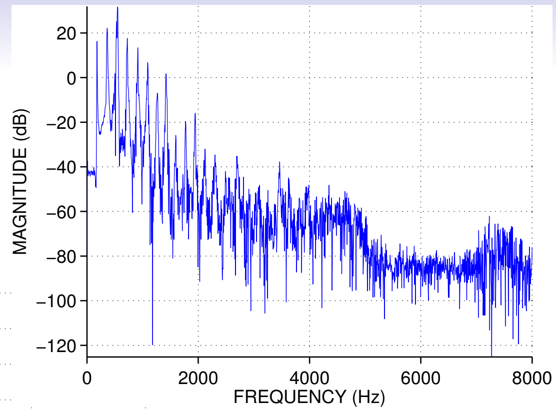
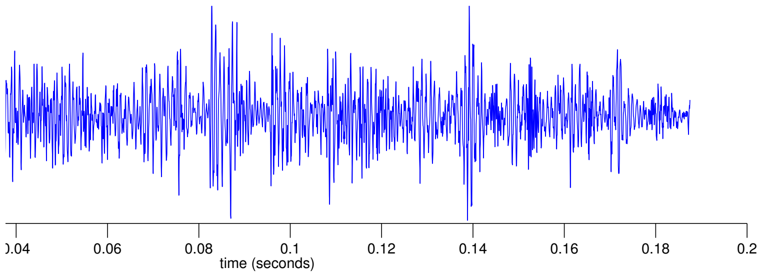
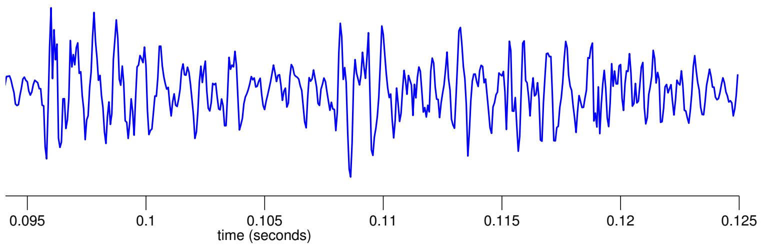
frequency domain



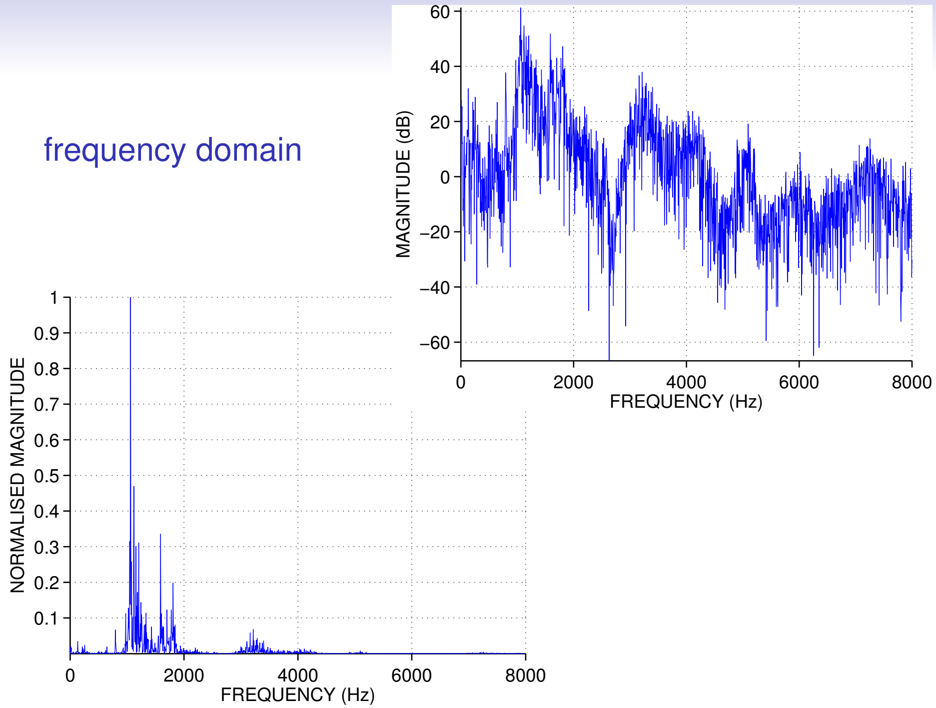


example of oesophageal (voiced) speech





frequency domain



Design low-pass FIR filter

find impulse response $h[n]$ coefficients if:

- pass-band edge frequency: $f_p = 2.5$ kHz
- transition width 200 Hz
- sampling frequency 16 kHz

select: M (filter length) and window type $w[n]$.

ideal low pass filter:

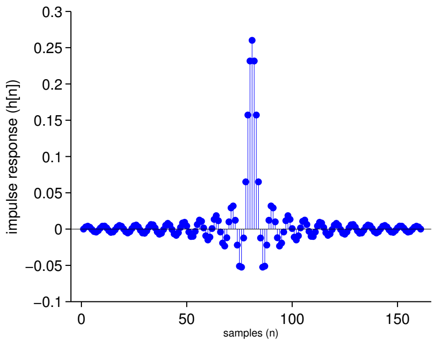
$$H_d(\omega) = \begin{cases} 1 & 0 \leq |\omega| \leq w_c \\ 0 & \text{otherwise} \end{cases}$$

inverse DTFT:

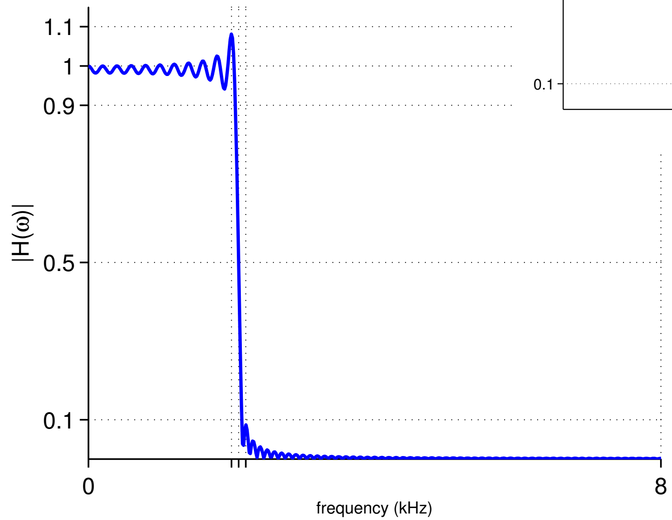
$$\begin{aligned} h_d[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n}\mathrm{d}\omega \\ &= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\mathrm{d}\omega \\ &= \begin{cases} \frac{\sin(\omega_c n)}{\pi n} & n \neq 0 \\ \frac{\omega_c}{\pi} & n = 0 \end{cases} \end{aligned}$$

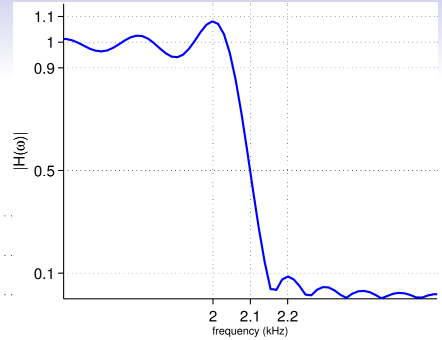
impulse response function $h[n]$

$$M = \frac{4\pi}{\Delta\omega} - 1$$
$$= \left\lceil \frac{4\pi}{(200/16000)(2\pi)} \right\rceil_{\text{odd}}$$
$$= 159$$

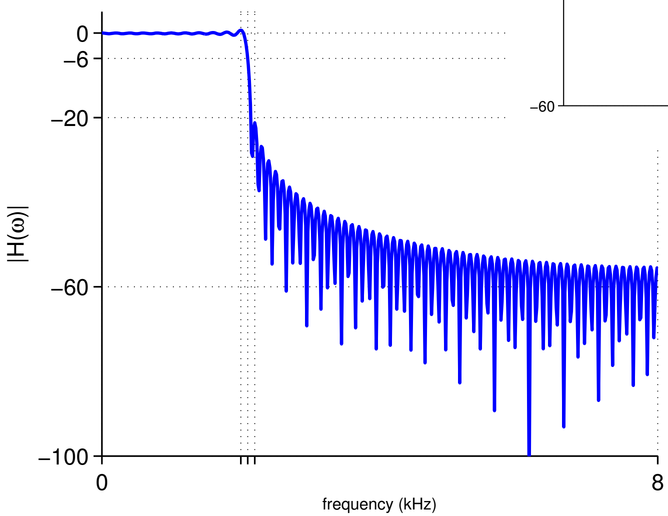


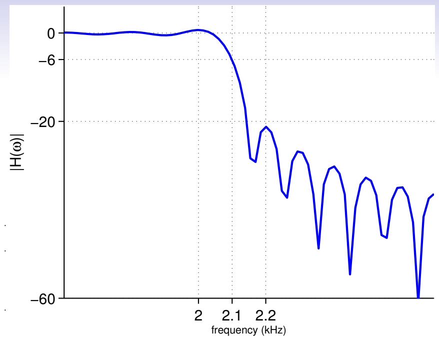
frequency response $|H(\omega)|$



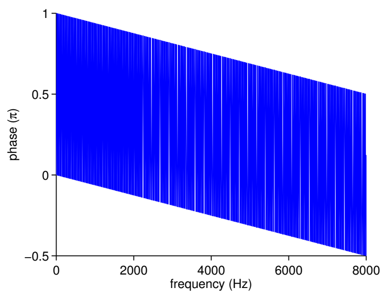


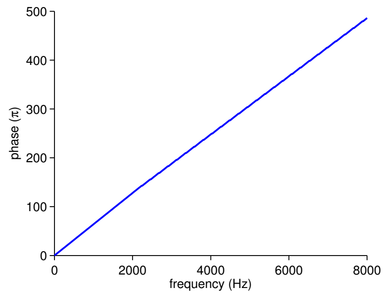
frequency response $|H(\omega)|$





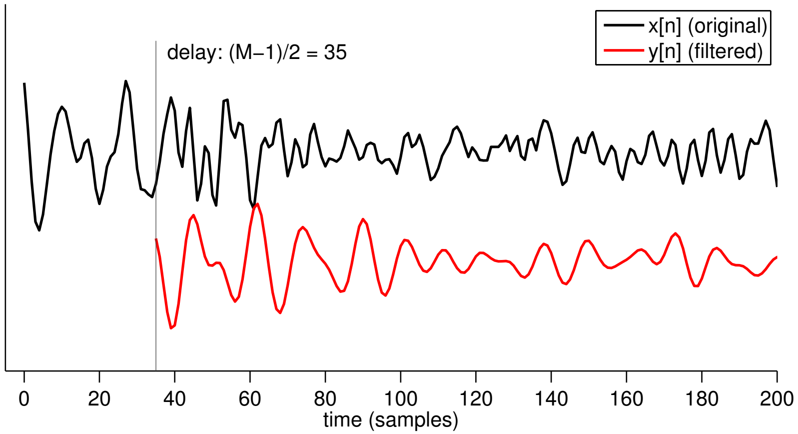
phase $\theta(\omega)$ $(M = 71)$



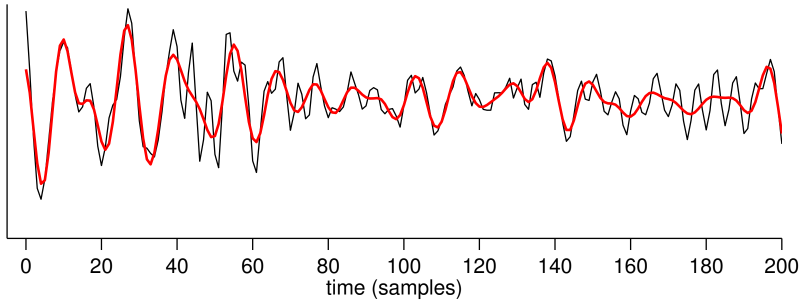


use impulse response $h[n]$ to filter oseophageal speech signal $x[n]$ ($M = 71$):

$$y[n] = \sum_{k=0}^{70} h[k]x[n-k]$$

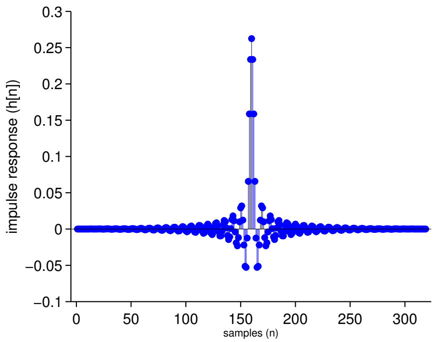


shift $y[n]$ to compare:

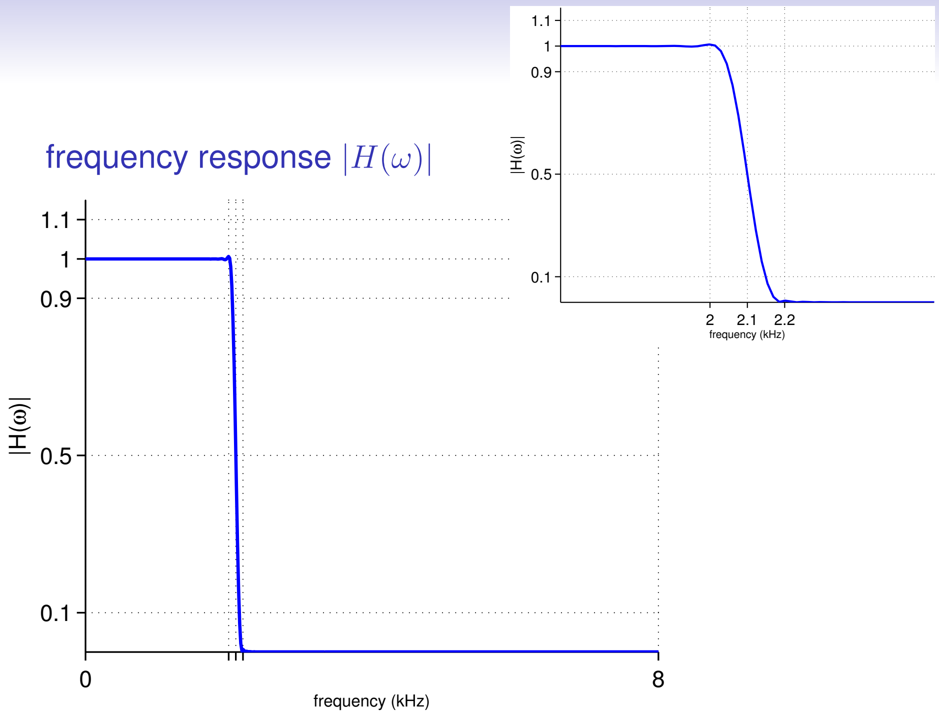


impulse function $h[n]$ with Hann (Hanning) window

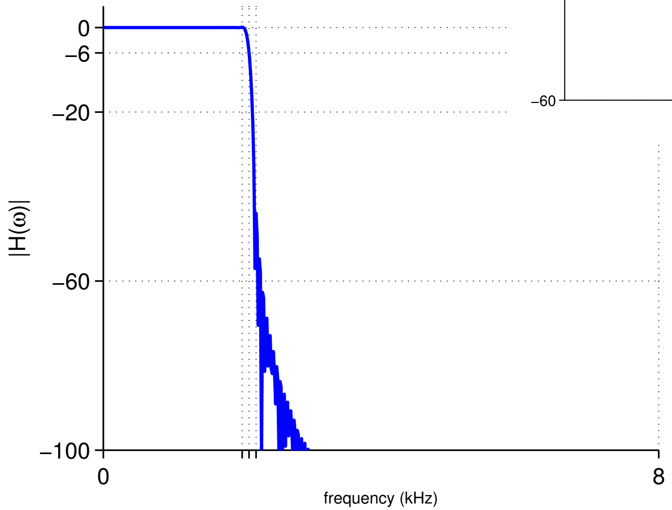
$$\begin{aligned} M &= \frac{8\pi}{\Delta\omega} \\ &= \left\lceil \frac{8\pi}{(200/16000)2\pi} \right\rceil_{\text{odd}} \\ &= 319 \end{aligned}$$

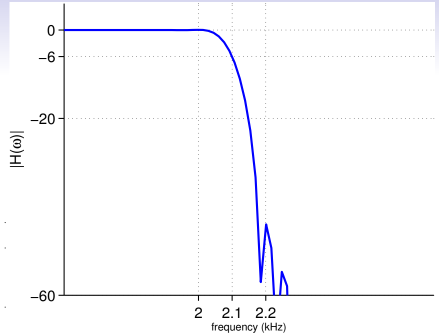


frequency response $|H(\omega)|$

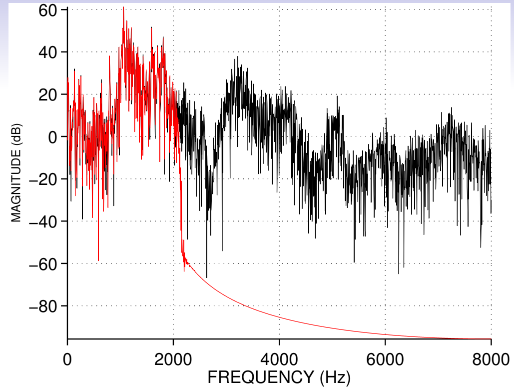


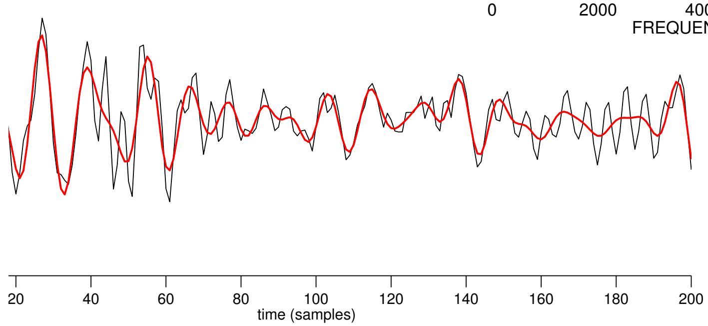
frequency response $|H(\omega)|$





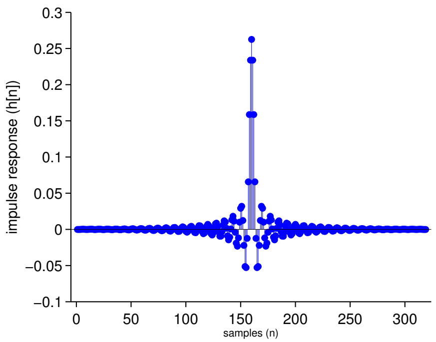
filtered oesophageal signal using Hann window



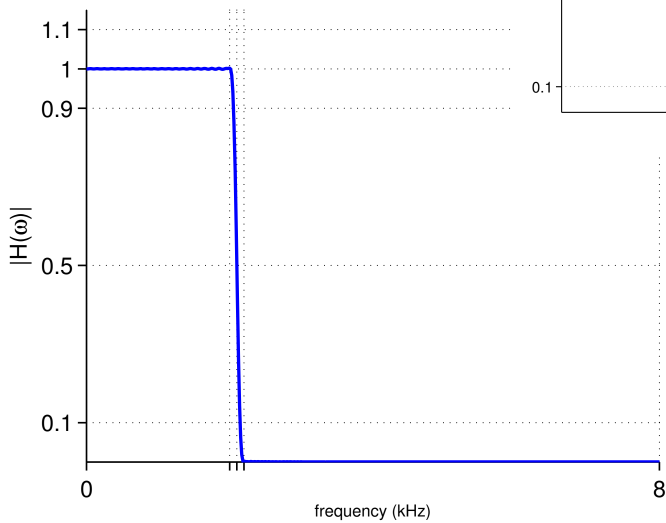


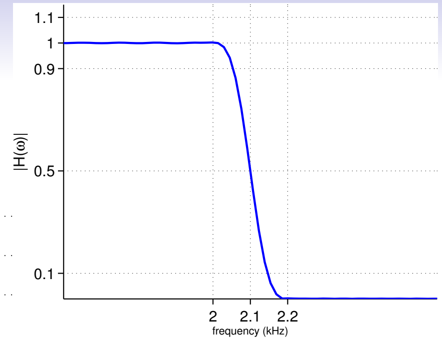
impulse function $h[n]$ with Hamming window

$$
\begin{aligned}
M &= \frac{8\pi}{\Delta\omega} \\
&= \left\lceil \frac{8\pi}{(200/16000)2\pi} \right\rceil_{\text{odd}} \\
&= 319
\end{aligned}
$$

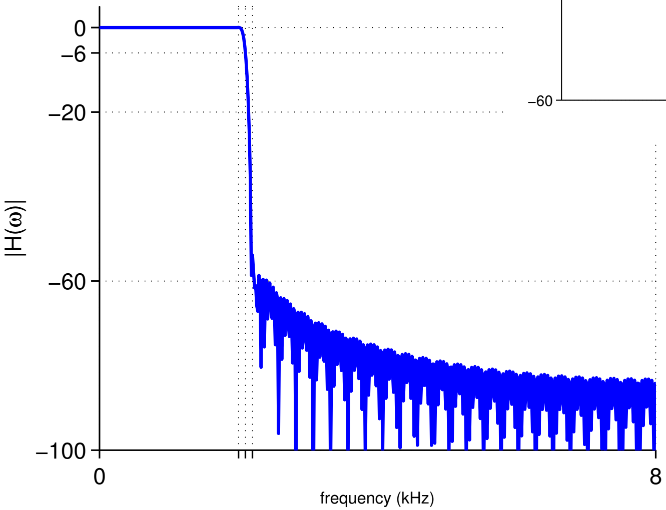


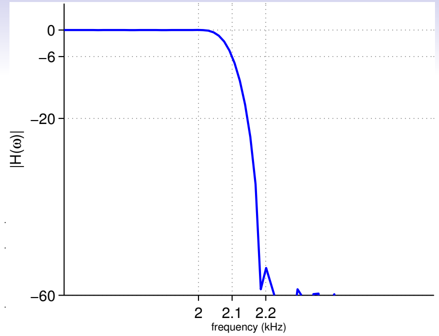
frequency response $|H(\omega)|$



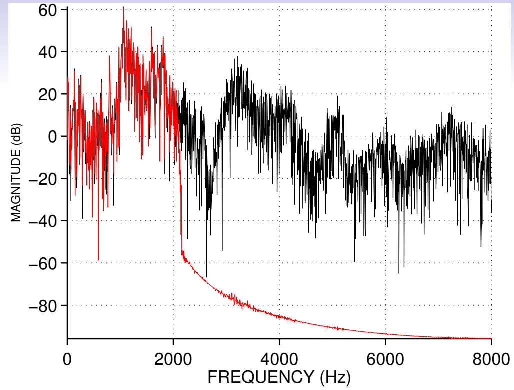


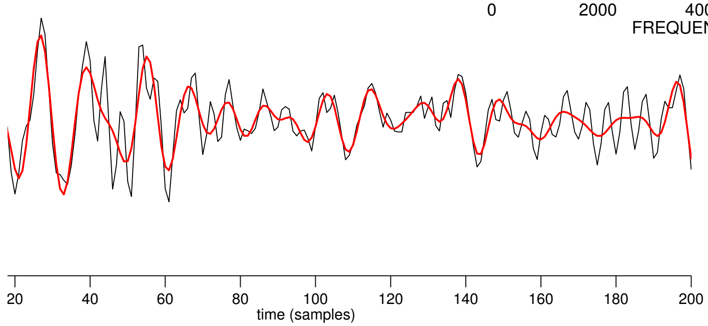
frequency response $|H(\omega)|$





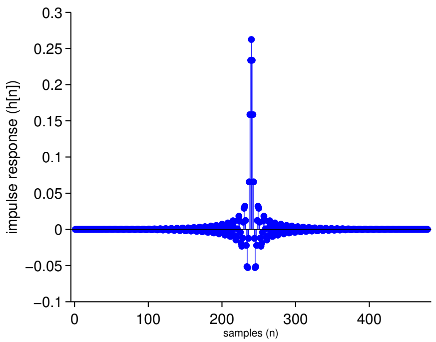
filtered oesophageal signal using Hamming window



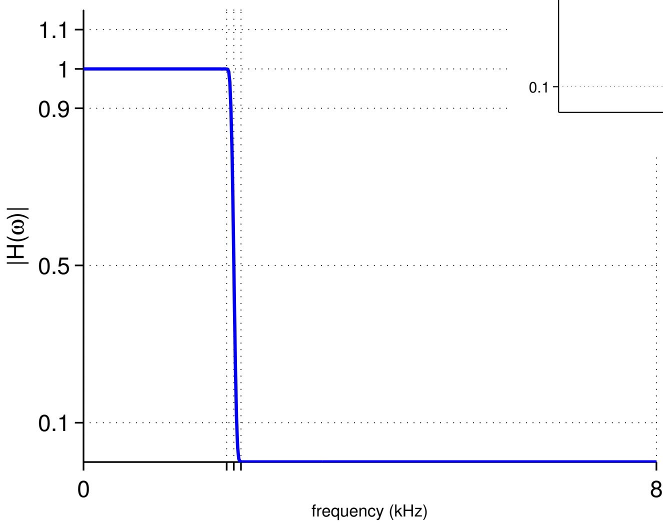


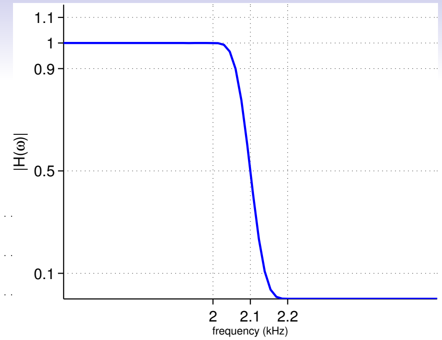
impulse function $h[n]$ with Blackman window

$$M = \frac{12\pi}{\Delta\omega} = \left\lceil \frac{12\pi}{(200/16000)2\pi} \right\rceil_{\text{odd}} = 479$$

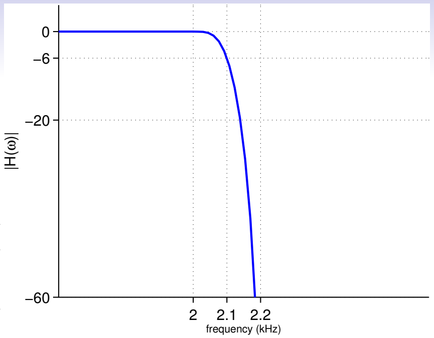


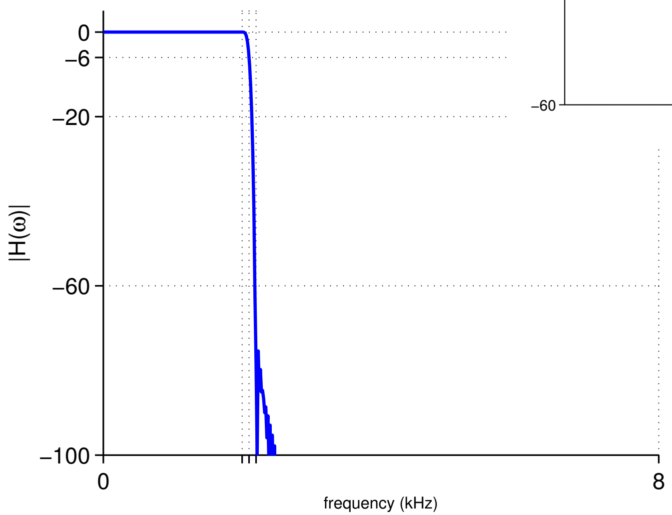
frequency response $|H(\omega)|$



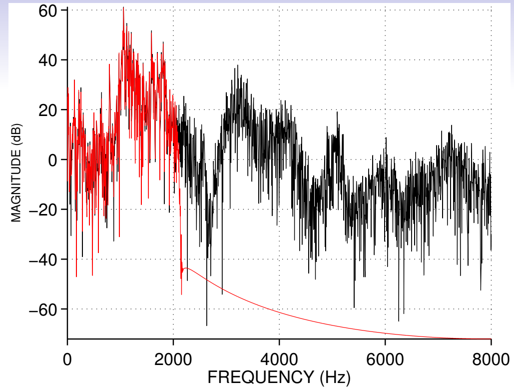


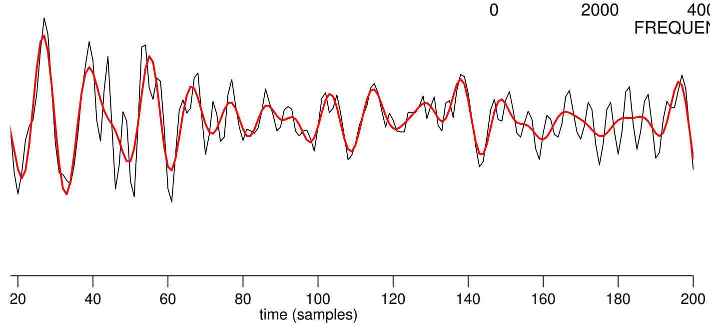
frequency response $|H(\omega)|$





filtered oesophageal signal using Blackman window





which window to use and why?

should consider:

- transition width, but can adjust M accordingly
  - increasing M increases computational load and delay, but not a problem here (max. 15 ms delay)
- stop band attenuation and pass/stop-band ripple:
  - Hann, Hamming, and Blackman seem adequate
- which one: either Hann, Hamming, and Blackman?
- what about Kaiser window?
  - if $\beta = 7.04$ for example, then min. stopband attenuation is -74 dB, which is equal to the Blackman window.
  - but, transition width differ ($12\pi/M$ vs. $9.19\pi/M$)
  - therefore, for Kaiser $M = 369$ vs. $M = 480$ for Blackman

precise design with Kaiser window:

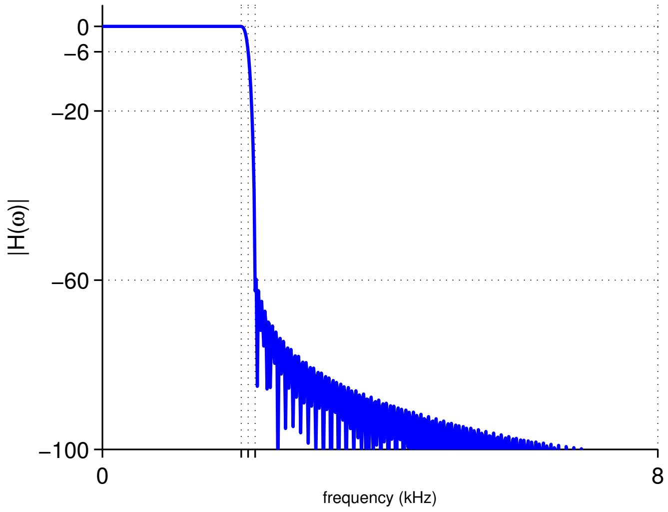
- what if we required stopband attenuation of -60 dB and $M < 300$
  - could use Blackman window (with stopband -74 dB), but $M = 479$ (see previous slides)
- solution: use Kaiser window with

$$\beta = 0.1102(A - 8.7) = 5.633$$

where $A = -20\log_{10}(\delta) = 60$ and

$$M = \frac{A - 8}{2.285\Delta\omega} = 289.7525 = \lceil 289.7525 \rceil_{\text{odd}} = 291$$

Low-pass filter with Kaiser window ($M = 291$ and $\beta = 5.633$):



Summary: Window Method FIR Filter Design

- Pro: simplicity
- Con: lack of flexibility: peak pass-band and peak stop-band ripples are approx. equal
  - may not suit design
- Con: convolution of spectral window with ideal frequency response means that pass-band and stop-band edge frequencies are not precisely specified
- Con: may not be able to determine $h_d[n]$ from $H_d(\omega)$
- Con: for a given window, stop-band attenuation is fixed (i.e. independent of M)
  - need to find suitable window
- Kaiser window includes a parameter which allows a trade off between transition width and ripple.